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ABSTRACT

Teaching procedures of Project Physics Unit 1 are presented in this manual to help teachers make effective use of learning materials. Curriculum objectives are discussed in connection with instructional materials, suggested year time schedules, multi-media schedules, schedule blocks, resource charts, and experiment summaries. Brief analyses are included for transparencies, reader units, programmed instruction, and 16mm films. Also included is information about the background and development of each text section, procedures and equipments used and an explanation of film loops. Solutions to the study guide are given in detail, and brief answers to test items are provided along with proportions of correctly answering test samples. The first unit of the text, with marginal notes, is also compiled in the manual. The work of Harvard Project Physics has been financially supported by: the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education, and Harvard University. (CC)

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Project Physics **Teacher Guide** 1

An Introduction to Physics

Concepts of Motion

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Project Physics **Text**

An Introduction to Physics **1** **Concepts of Motion**

Authorized Interim Version 1968-69

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Welcome to the study of physics. This volume, more of a student's guide than a text of the usual kind, is part of a whole group of materials that includes a student handbook, laboratory equipment, films, programmed instruction, readers, transparencies, and so forth. Havrard Project Physics has designed the materials to work together. They have all been tested in classes that supplied results to the Project for use in revisions of earlier versions.

The Project Physics course is the work of about 200 scientists, scholars, and teachers from all parts of the country, responding to a call by the National Science Foundation in 1963 to prepare a new introductory physics course for nationwide use. Harvard Project Physics was established in 1964, on the basis of a two-year feasibility study supported by the Carnegie Corporation. On the previous pages are the names of our colleagues who helped during the last six years in what became an extensive national curriculum development program. Some of them worked on a full-time basis for several years; others were part-time or occasional consultants, contributing to some aspect of the whole course; but all were valued and dedicated collaborators who richly earned the gratitude of everyone who cares about science and the improvement of science teaching.

Harvard Project Physics has received financial support from the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education and Harvard University. In addition, the Project has had the essential support of several hundred participating schools throughout the United States and Canada, who used and tested the course as it went through several successive annual revisions.

The last and largest cycle of testing of all materials is now completed; the final version of the Project Physics course will be published in 1970 by Holt, Rinehart and Winston, Inc., and will incorporate the final revisions and improvements as necessary. To this end we invite our students and instructors to write to us if in practice they too discern ways of improving the course materials.

The Directors
Harvard Project Physics

An Introduction to Physics 1 Concepts of Motion

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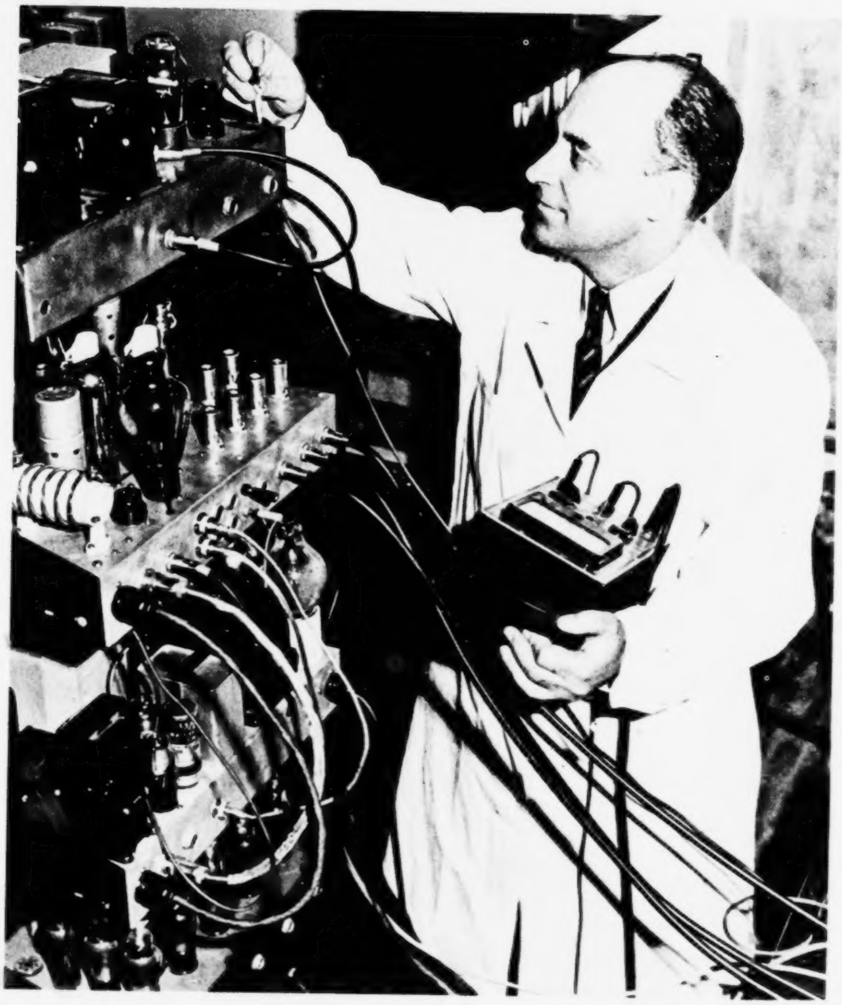
The physicist, Enrico Fermi (1901-1954), at different stages of his career in Italy and America. The photographs were kindly supplied by Mrs. Laura Fermi, shown also in the top photograph.



Prologue It is January of 1934, a dreary month in the city of Paris, and a husband and wife are bombarding a bit of aluminum with what are called alpha particles. Does this seem like a momentous event? Certainly not when stated so baldly. But let us look at it more closely, for it is momentous indeed.

Never mind the technical terms.• They will not get in the way of the story. It begins as something of a family affair. The husband and wife, French physicists, were Frédéric Joliot and Irene Curie, and the alpha particles they used in their experiment came shooting out of a radioactive metal, polonium, discovered 36 years before by none other than Irene's illustrious parents, Pierre and Marie Curie, who also discovered radium. What Frederic and Irene found was this: when bombarded by alpha particles, the commonplace bit of aluminum became radioactive.

•Reaffirm this sentence to your students. Assure them that the purpose of the prologue is not to teach the physics content; rather it is to be read for enjoyment as an example of a scientist at work.

Nothing like this had ever been observed before: a familiar, everyday substance becoming radioactive. The news was exciting to scientists—though it made few, if any newspaper headlines. The news traveled rapidly: by cablegram and letter. In Rome, Enrico Fermi, a young physicist on the staff at the University of Rome, became intrigued by the possibility of repeating the experiment of Fréderic and Irene—repeating it with one significant alteration. The story is told in the book Atoms in the Family by Enrico Fermi's wife, Laura. She writes:

All quotations in the Prologue are from Laura Fermi, Atoms in the Family: My Life With Enrico Fermi, University of Chicago Press, 1954 (available as a paperback book in the Phoenix Books series). Fermi was one of the major physicists of this century.

> ...he decided he would try to produce artificial radioactivity with neutrons [instead of alpha particles]. Having no electric charge, neutrons are neither attracted by electrons nor repelled by nuclei; their path inside matter is much longer than that of alpha particles; their speed and energy remain higher; their chances of hitting a nucleus with full impact are much greater. Against these unquestionable advantages, neutrons present a decidedly strong drawback. Unlike alpha particles, they are not emitted spontaneously by radioactive substances, but they are produced by bombarding certain elements with alpha particles, a process yielding approximately one neutron for every hundred thousand alpha particles. This very low yield made the use of neutrons appear questionable.

Only through actual experiment could one tell whether or not neutrons were good projectiles for triggering artificial radioactivity of the target nuclei. Therefore, Fermi, at the age of 33 and already an outstanding theoretical physicist, decided to design some experiments that could settle the issue. His first task was to obtain suitable instruments for detecting the particles emitted by radioactive materials. By far the best such instruments were what are called Geiger counters, but in 1934, Geiger counters were still relatively new and not readily available. Therefore, Fermi constructed his own.

The counters were soon finished, and tests showed that they could detect the radiation from radioactive materials. But Fermi also needed a source of neutrons. This he made by enclosing beryllium powder and the radioactive gas radon in a glass tube; the alpha particles from radon, on striking the beryllium, caused it to emit neutrons.

> Now Enrico was ready for the first experiments. Being a man of method, he did not start by bombarding substances at random, but proceeded in order, starting from the lightest element, hydrogen, and following the periodic table of elements. Hydrogen gave no results: when he bombarded water with neutrons, nothing happened. He tried lithium next, but again without luck. He went on to beryllium, then to boron, to carbon, to nitrogen. None were activated. Enrico wavered, discouraged, and was on the point of giving up his researches, but his stubbornness made him refuse to yield. He would try one more element. That oxygen would not become radioactive he knew already, for his first bombardment had been on water. So he irradiated flourine. Hurrah! He was rewarded. Fluorine was strongly activated, and so were other elements that came after fluorine in the periodic table.
>
> This field of investigation appeared so fruitful that Enrico not only enlisted the help of Emilio Segré and of Edoardo Amaldi but felt justified in sending a cable to Rasetti [a colleague who had gone to Morocco], to inform him of the experiments and to advise him to come back at once. A short while later a chemist, Oscar D'Agostino, joined the group, and systematic investigation was carried on at a fast pace.

Emilio Segré (and O. Chamberlain) won the Nobel Prize in 1959 for the discovery of the antiproton. He is now at the University of California at Berkeley. Rasetti is professor of physics at Johns Hopkins University.

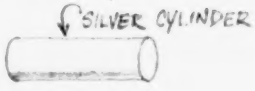
With the help of his colleagues, Fermi's work at the laboratory was pursued with high spirit, as Laura Fermi's account shows:

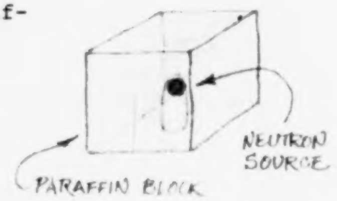
> ...Irradiated substances were tested for radioactivity with Geiger counters. The radiation emitted by the neutron source would have disturbed the measurements had it reached the counters. Therefore, the room where substances were irradiated and the room with the counters were at the two ends of a long corridor.
>
> Sometimes the radioactivity produced in an element was of short duration, and after less than a minute it could no longer be detected. Then haste was essential, and the time to cover the length of the corridor had to be reduced by swift running. Amaldi and Fermi prided themselves on being the fastest runners, and theirs was the task of speeding short-lived substances from one end of the corridor to the other. They always raced, and Enrico claims that he could run faster than Edoardo....

Again, follow the story to get a feeling for the atmosphere of important experiments—don't worry about details now.

And then, on the morning of October 22, 1934, a fateful discovery was made. Two of Fermi's co-workers were irradiating a hollow cylinder of silver with neutrons from a source placed at the center of the cylinder, to make it artificially radioactive. They found that the amount of radioactivity induced in the silver depended on other objects in the room!

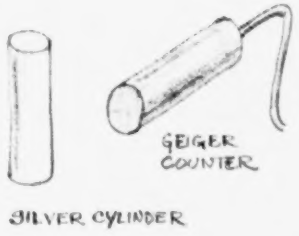
> ...The objects around the cylinder seemed to influence its activity. If the cylinder had been on a wooden table while being irradiated, its activity was greater than if it had been on a piece of metal. By now the whole group's interest had been aroused, and everybody was participating in the work. They placed the neutron source outside the cylinder and interposed objects between them. A plate of lead made the activity increase slightly. Lead is a heavy substance. "Let's try a light one next," Fermi said, "for instance, paraffin." [The most plentiful element in paraffin is hydrogen.] The experiment with paraffin was performed on the morning of October 22.
>
> They took a big block of paraffin, dug a cavity in it, put the neutron source inside the cavity, irradiated the silver cylinder, and brought it to a Geiger counter to measure its activity. The counter clicked madly. The halls of the physics building resounded with loud exclamations: "Fantastic! Incredible! Black Magic!" Paraffin increased the artificially induced radioactivity of silver up to one hundred times.



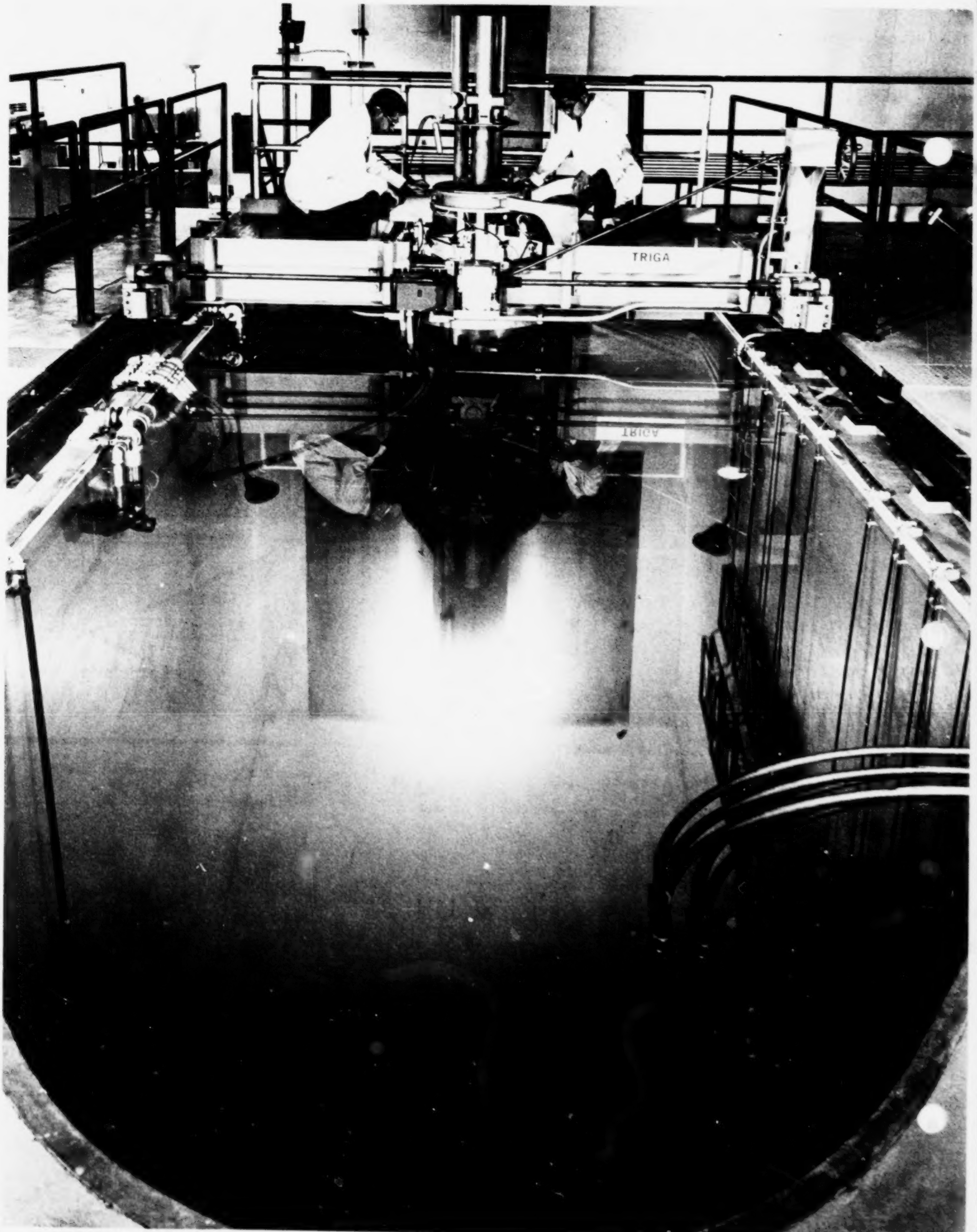


By the time Fermi came back from lunch, he had already formulated a theory to account for the strange action of paraffin.

> Paraffin contains a great deal of hydrogen. Hydrogen nuclei are protons, particles having the same mass as neutrons. When the source is inclosed in a paraffin block, the neutrons hit the protons in the paraffin before reaching the silver nuclei. In the collision with a proton, a neutron loses part of its energy, in the same manner as a billiard ball is slowed down when it hits a ball of its same size [whereas it loses little speed if it is reflected off a much heavier ball, or a solid wall]. Before emerging from the paraffin, a neutron will have collided with many protons in succession, and its velocity will be greatly reduced. This slow neutron will have a much better chance of being captured by a silver nucleus than a fast one, much as a slow golf ball has a better chance of making a hole than one which zooms fast and may bypass it.



> If Enrico's explanations were correct, any other substance containing a large proportion of hydrogen should have the same effect as paraffin. "Let's try and see what a considerable quantity of water does to the silver activity," Enrico said on the same afternoon.
>
> There was no better place to find a "considerable quantity of water" than the goldfish fountain...in the garden behind the laboratory....


TRIGA

> In that fountain the physicists had sailed certain small toy boats that had suddenly invaded the Italian market. Each little craft bore a tiny candle on its deck. When the candles were lighted, the boats sped and puffed on the water like real motor-boats. They were delightful. And the young men, who had never been able to resist the charm of a new toy, had spent much time watching them run in the fountain.
>
> It was natural that, when in need of a considerable amount of water, Fermi and his friends should think of that fountain. On that afternoon of October 22, they rushed their source of neutrons and their silver cylinder to that fountain, and they placed both under water. The goldfish, I am sure, retained their calm and dignity, despite the neutron shower, more than did the crowd outside. The men's excitement was fed on the results of this experiment. It confirmed Fermi's theory. Water also increased the artificial radioactivity of silver by many times.

The same process by which neutrons were slowed down in the fountain is used in today's large nuclear reactors. An example is the "pool" research reactor pictured on the opposite page.

This discovery—that slowed-down neutrons can produce much stronger effects in the transmutation of certain atoms than fast neutrons—turned out to be a crucial step toward further discoveries that, years later, led Fermi and others to the extraction of atomic energy from uranium.

The reason for presenting a description of Fermi's discovery of slow neutrons here was not to instruct you on the details of the nucleus. It was, instead, to present a quick, almost impressionistic, view of scientists in action. No other discovery in science was made or will be made in just the way Fermi and his colleagues made this one. Nevertheless, the episode does illustrate some of the characteristics—and some of the drama—of modern science.

Like religion, science probably began as a sense of awe and wonder. In its highest form its motive power has been sheer curiosity—the urge to explore and to know. This urge is within us all. It is vividly seen in the intense absorption of a child examining a strange sea shell tossed up from the ocean or a piece of metal found in the gutter. Who among us has resisted the temptation to explore the slippery properties of the mud in a rain puddle? Alas, everyday cares and the problems of growing up overtake us all too soon, and many of us lose our early sense of curiosity or channel it into more practical paths. Fortunately, a few preserve their childlike, wide-eyed wonderment and it is among such people that one often finds the great scientists and poets.

Science gives us no final answers. But it has come upon wondrous things, and some of them may renew our childhood delight in the miracle that is within us and around us. Take, for example, so basic a thing as size...or time.

Our Place in Space

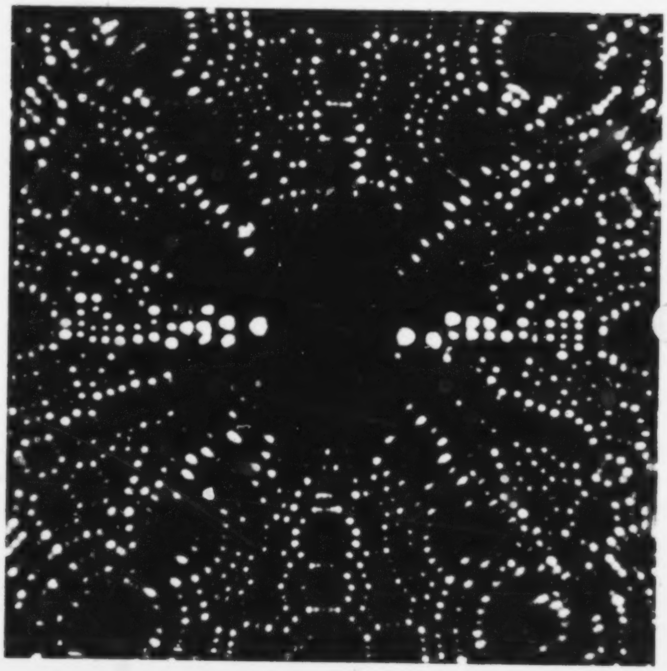
This is a good opportunity for a discussion of order of magnitude. Human height is given as 10^0 m = 1 meter or about 3 feet. This is obviously closer than 10^{-1} m (0.3ft) or 10^1 m (30 ft).

	(orders of magnitude)
Size of universe	10^{26} m
Distance to nearest galaxy	10^{22} m
Distance to nearest star	10^{17} m
Distance to the sun	10^{11} m
Diameter of earth	10^{7} m
One mile	10^{3} m
Human height	10^{0} m
Postage stamp	10^{-2} m
Paper thickness	10^{-4} m
Atomic diameter	10^{-10} m
Nuclear diameter	10^{-14} m

A globular star cluster

The estimated size of the universe now extends to more than a million, million, million, million times a man's height.

Atomic sites in tungsten



The smallest known constituent units of the universe are less in size than 1,000 million millionths of a man's height.

man's height × 1,000,000,000,000,000,000,000,000,000

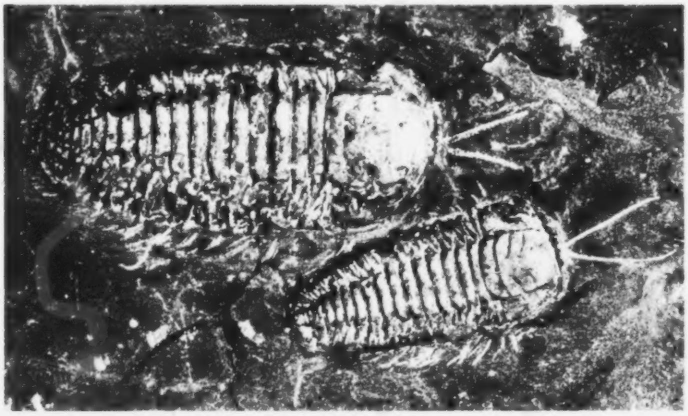
man's height × 0.000,000,000,000,001

Our Place in Time

	(orders of magnitude)
Expected life of sun	10^{18} sec
Age of earth	10^{16} sec
Human life span	10^{9} sec
One day	10^{5} sec
Light from sun to earth	10^{3} sec
Time between heartbeats	10^{0} sec
One beat of fly's wings	10^{-3} sec
Average lifetime of a muon	10^{-6} sec
Average lifetime of pi meson	10^{-15} sec
Shortest-lived particles	10^{-23} sec

You can learn to do rapid mental calculations by using order of magnitude approximations. Fermi, for example, watching the explosion of the first nuclear bomb in New Mexico, quickly computed the energy yield by tossing bits of paper into the air and measuring the distance they were blown by the shock wave from the blast.

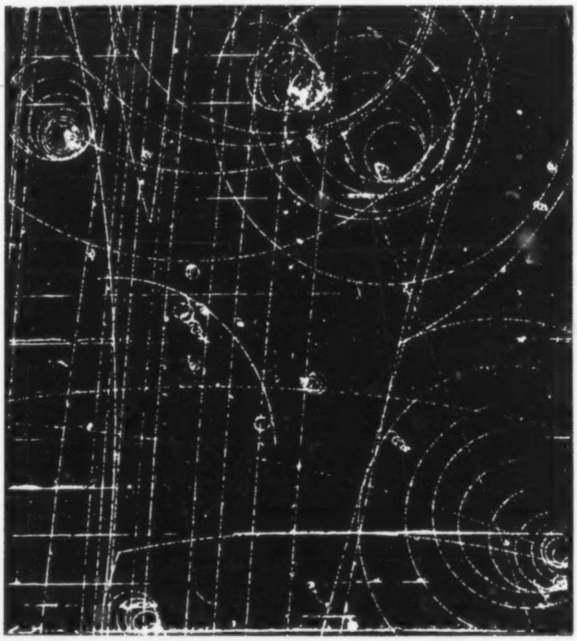
Fossilized trilobites



The history of the universe has been traced back as far into the past as a hundred million times the length of a man's life.

man's life x 100,000,000

Particle tracks in a bubble chamber



Events have been recorded that last only a millionth of a millionth of a millionth of a millionth of a man's heartbeat.

man's heartbeat x 0.000,000,000,000,000,000,000,001

It is hard to resist the temptation to say more about these intriguing extremes; however, this is not where physics started. Physics started with the man-sized world—the world of horse-drawn chariots, of falling rain, and of flying arrows. It is with this man-sized world that we shall begin.

Chapter 1 The Language of Motion

Section		Page
1.1	The motion of things	9
1.2	A motion experiment that does not quite work	11
1.3	A better motion experiment	12
1.4	Leslie's "50" and the meaning of average speed	15
1.5	Graphing motion	19
1.6	Time out for a warning	23
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1.8	Acceleration—by comparison	29

If your students already understand average speed, you can go through Sections 1.1 – 1.4 very quickly. Test them with a couple of problems on average speed, like Study Guide 1.6

Summary 1.1

1. Motions we observe daily are complex.

2. Historically, the study of motion proved to be a fruitful beginning for the study of nature.

There is a very old maxim: "To be ignorant of motion is to be ignorant of Nature."

Of changes that occur in nature the most common and easily measured is a change of position. Change of position is motion.

1.1 The motion of things. Man crawls, walks, runs, jumps, dances. To move himself faster, farther, higher, deeper, he invents things like sleds, bicycles, submarines, rocket ships. As human beings we are caught up in motion and fascinated by it. Perhaps this is why so many artists try to portray movement. It is one reason why scientists investigate motion. The world is filled with things in motion: things as small as dust, as large as stars, and as common as ourselves; motion fast and slow, motion smooth, rhythmic, and erratic. We cannot investigate all of these at once. So from this swirling, whirling world of ours let us choose just one moving object for attention, something interesting and typical, and, above all, something manageable.

•Ask the class this question.

But where shall we start?• We might start our investigation by looking at a modern machine—the Saturn rocket, say, or a supercharged dragster, or an automatic washing machine. But as you know, things such as these, though made and controlled by man, move in very complicated ways. We really ought to start with something easier. Then how about the bird in flight? Or a leaf falling from a tree?

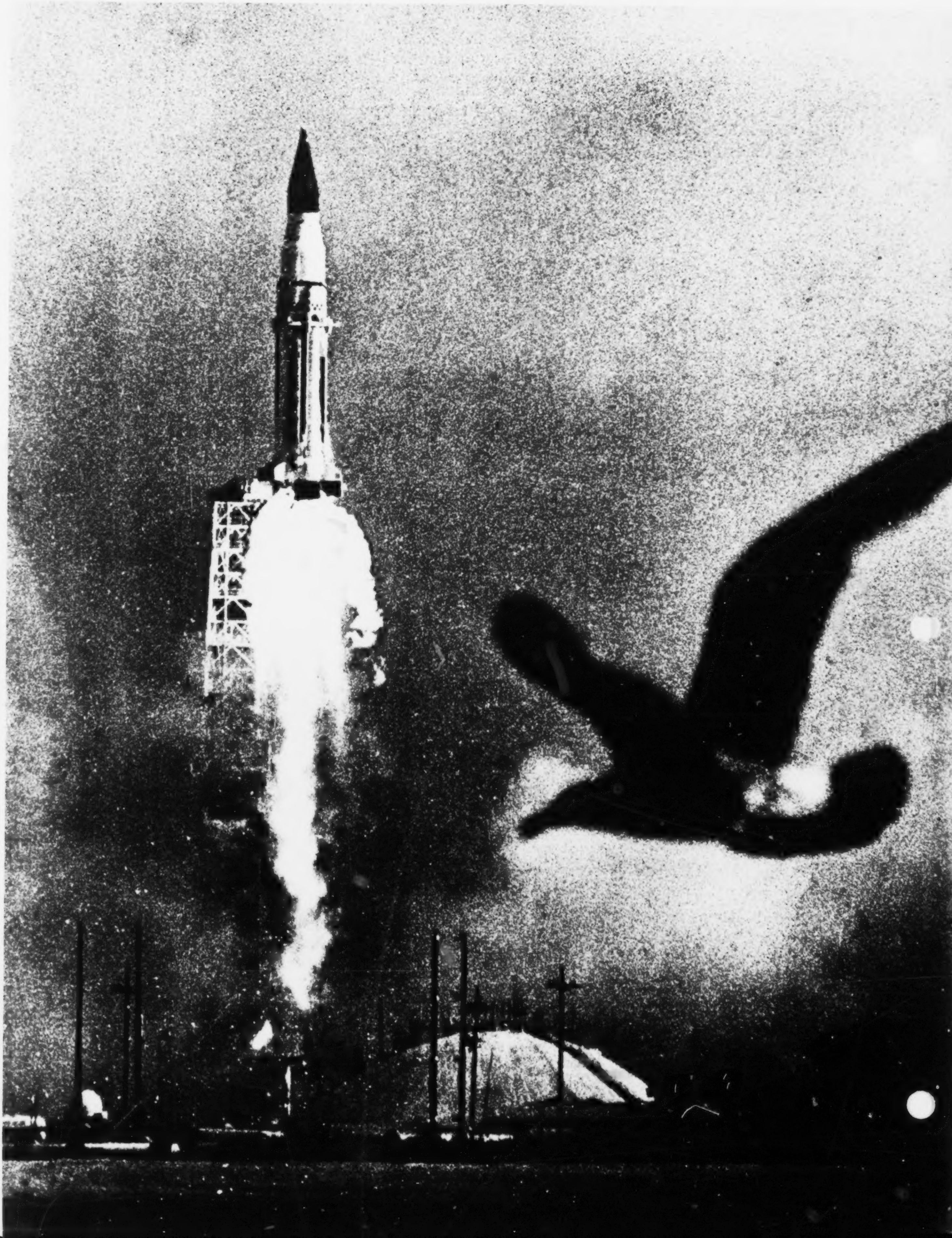
Surely in all of nature there is no motion more ordinary than that of a leaf fluttering down from a branch. Can we describe how it falls or explain why it falls? As we think about it we quickly realize that, while the motion may be natural, it is very complicated: the leaf twists, turns, sails to the right and left, back and forth, as it floats down. Even a motion as ordinary as this may turn out, on closer examination, to be more complicated than that of machines. Although we might describe it in detail, what would we gain? No two leaves fall in quite the same way; therefore, each leaf would require its own detailed description. Indeed, this individuality is typical of many naturally occurring events on earth!

And so we are faced with a real dilemma. We want to describe motion, but the motions that excite and interest us appear to be hopelessly complex. What shall we do? We shall find a very simple motion and attempt to describe it. Those of us who have learned to play a musical instrument will appreciate the wisdom of starting with simple tasks. If our music teacher confronted us in lesson number one with a Beethoven piano sonata, we would in all probability have quickly forgone music in favor of a less taxing activity.

The place to start is in the laboratory, because there we can find the simple ingredients that make up complex motions.

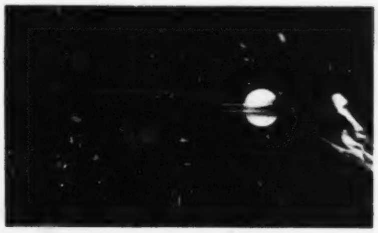
D1: Recognizing simple motions.

Study for "Dynamism of a Cyclist" (1913) by Umberto Boccioni. Courtesy Yale University Art Gallery.

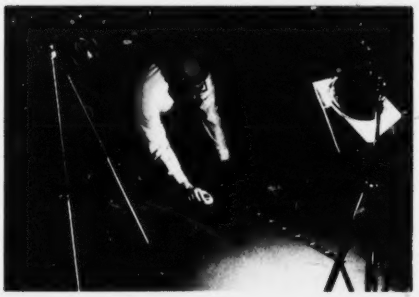


This section takes the student through the beginning steps of the laboratory method that will be used frequently in this course for the study of motion.

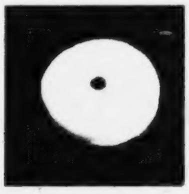
1.2 A motion experiment that does not quite work. A billiard ball hit squarely in the center speeds across the table in a straight line. Unfortunately, physics laboratories are not usually equipped with billiard tables. But never mind. Even better for our purposes is a disc of what is called dry ice (really frozen carbon dioxide) moving on the floor. The dry ice disc was placed on the floor and given a gentle push. It floated slowly across the floor in front of the camera. While the disc was moving, the shutter of the camera was kept open. The resultant time exposure shows the path taken by the dry ice disc.



See Study Guide 1.1 (page 32)



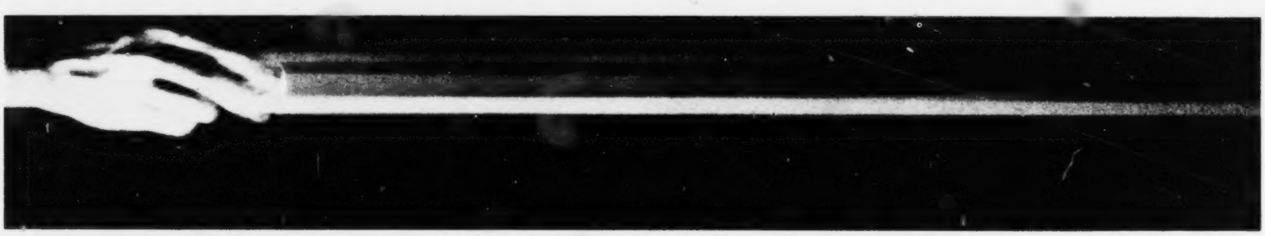
Laboratory setup



Close-up of a dry ice disc

If students repeat this demonstration, caution them about the real danger of frozen fingers.

TI : Stroboscopic measurements
A : Making frictionless pucks
A : Electronic stroboscope
A : Magnetic timer



Time exposure of the disc in motion

What can we learn about the disc's motion by examining the photographic record? Was the path a straight line? Did the disc slow down?

The question of path is easy enough to answer: as nearly as we can judge by placing a ruler on the photograph, the disc moved in a straight line. But did it slow down? From the photograph we cannot tell. Let us improve our experiment. Before we do so, however, we ought to be clear on just how we might expect to measure speed.

Why not use something like an automobile speedometer? All of us know how to read that most popular of all meters even though we may not have a clear notion of how it works. A speedometer tells us directly the speed at which the car is moving at any time. Very convenient. Furthermore, such

Summary 1.2
1. A time exposure photograph of the motion of a dry-ice puck shows only the direction of the motion.

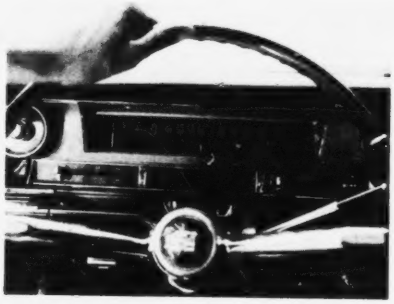
2. The familiar units in which we express speed suggest what we must measure to find the speed of an object.

We are assuming here that you already know what speed is, namely how fast an object moves from one place to another. A more formal way to say the same thing is: Speed is the time rate of change of position.

readings are independent of the path of our car. A given speedometer reading specifies the same speed, whether the car is moving uphill or down, or is traveling along a straight road or a curved one.

But, alas, there is at least one practical trouble with having to rely on a speedometer to measure speed: it is not easy to put a speedometer on a disc of dry ice, or on a bullet, or on many other objects whose speed you may wish to measure. However, the speedometer provides us with a good clue. Remember how we express speedometer readings? We say our car is moving 60 miles per hour. Translation: at the instant the reading was taken, the car was traveling fast enough to move a distance of 60 miles in a time interval of 1.0 hour, or 120 miles in 2.0 hours, or 6.0 miles in 1/10 hour—or any distance and corresponding time interval for which the ratio of distance to time is 60 miles per hour. To find speed we measure a distance moved, measure the time it took to move that distance, and then divide distance by time.

With this reminder of how to measure speed (without a speedometer), we can now return to the experiment with the dry ice disc. Our task now is to redesign the experiment so that we can find the speed of the disc as it moves along its straight-line path.



Summary 1.3

1. The ideas behind the stroboscope are developed in a sequence of steps.

2. Speed = $\frac{\Delta s}{\Delta t}$.

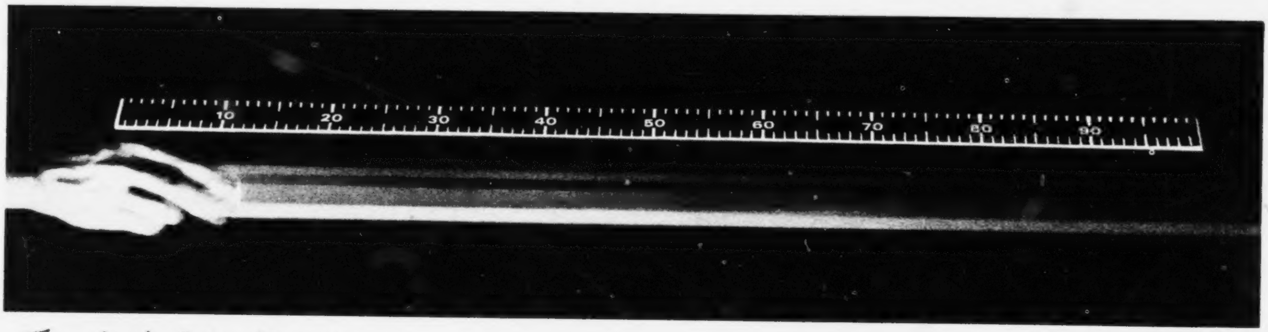
3. We study nature in the laboratory because it allows us to

a) select and deal with simple rather than complex phenomena that interest us;

b) isolate these instances from many (but not all) external interferences; and

c) repeat events as often as we wish.

1.3 A better motion experiment. To find speed we need to be able to measure both distance and time. So let's repeat the experiment with the dry ice disc after first placing a meter stick on the table parallel to the expected path of the disc. This is the photograph we obtain:



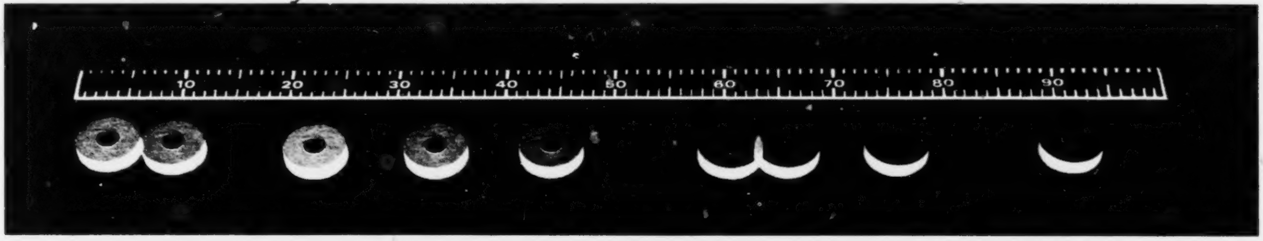
The streak fades toward the right. Is it possible to conclude from the photo that the puck is speeding up? Actually it is due to the lighting. Compare with the other photos. The right-hand side has received less light in each case.

Now the total distance traveled by the disc during the exposure can be measured. However, we still need to measure the time required for the disc to move through a particular distance. However, even if we could measure both the distance and the time we would still have too little information about the motion of the disc. Specifically, to find out whether or not the disc is slowing down—and, if so, by how much—we must be able to find its speed at different places. To do this, we must somehow obtain distance and time information for different places along the path. Knowing only the total distance and total time is not enough.

So let's try another modification. Instead of leaving the camera shutter open, we can open and close it rapidly. The result will be the multiple-exposure photograph shown below.

There are several other low friction devices - balloon puck, dry-ice puck, plastic beads and the air track or air table.

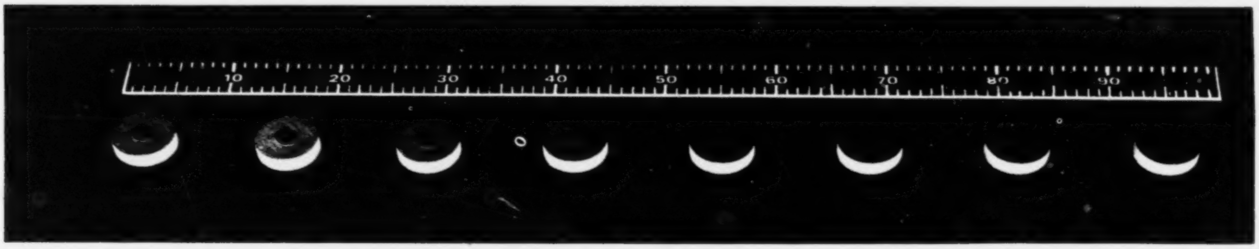
not a straight line this time!



Although we now have a variety of distances to measure, we still need to know the elapsed time between each exposure. With such information we could analyze the motion in detail, obtaining the distance-to-time ratio (speed) for various segments of the trip. One final change in the apparatus makes this possible.

L8: Analysis of a hurdle race Part 1.
D2: Uniform motion using accelerometer and dynamics cart.

The camera shutter is again kept open and everything else is the same as before, except that the only source of light in a darkened room comes from a stroboscopic lamp. This lamp flashes very brightly at a constant rate. Since each pulse or flash of light lasts for only about one-millionth of a second, we get a series of separate sharp exposures rather than a continuous, blurred one. The photograph below was made using such a stroboscopic lamp flashing 10 times a second.



Now we're finally getting somewhere. Our experiment enables us to record accurately many positions of a moving object. The meter stick measures the distance the disc moved between successive light flashes. The time elapsed between images is determined by the stroboscopic lamp flashes.

TO: Interpreting strobe pictures

How much did the disc slow down? We can find out by determining its speed at the two ends of its path. The front edge of the first clear image of the disc at the left is 6.0 cm from the zero mark on the meter stick. The front edge of the second image from the left is at the position 19.0 cm. The distance traveled during that interval of time is the difference between those two positions, or 13.0 cm. The corresponding time interval is 0.10 sec. Therefore, the speed at the start must have been 13.0 cm/0.10 sec, or 130 cm/sec.

Turning now to the two images farthest to the right in the photograph, we find that the distance traveled during 0.1 sec was 13.0 cm. Thus, the speed at the end was 13.0 cm/0.1 sec, or 130 cm/sec.

The disc did not slow down at all! The disc's speed was 130 cm/sec at the beginning of the path—and 130 cm/sec at the end of the path. As nearly as we can tell from this experiment, the speed was constant.

That result is hard to believe. Perhaps you are thinking that the disc might have changed speed several times as it moved from left to right but just happened to have identical speeds over the two intervals selected for measurement. That would be a strange coincidence but certainly not an impossible one. You can easily check this possibility for yourself. Since the time intervals between images are equal in all cases, the speeds will be equal only if the distance intervals are equal to each other. Is the scale distance between images always 13.0 cm?

Or perhaps you are thinking, "It was rigged!" or, if you are less skeptical you may think it was just a rare event and it would not happen again. All right then, you try it. Most school physics laboratories have cameras, strobe lamps (or mechanical strobes, which work just as well), and low-friction discs of one sort or another. Repeat the experiment several times at different initial speeds, and then compare your results with ours.

You may have even a more serious reservation about the experiment. If you ask, "How do you know that the disc didn't slow down an amount too small to be detected by your measurements?", we can only answer that we don't. All measurements are approximations. If we had measured distances to the nearest 0.001 cm (instead of to the nearest 0.1 cm) we might have detected some slowing down. But if we again found no change in speed, you could still raise the same objection.

There is no way out of this. We must simply admit that no physical measurements are ever exact or infinitely precise. Thus it is fair to question any set of measurements and the findings based on them. Not only fair, but expected.

See the articles "Motion in Words" and "Representation of Movement" in Project Physics Reader 1.

Before proceeding further in our study of motion, let us briefly review the results of our experiment. We devised a way to measure the successive positions of a moving dry ice disc at known time intervals. From this we calculate first the distance intervals and then the speed between selected positions. We discovered that the speed did not change. Objects that move in such a manner are said to have uniform speed. What about nonuniform speed? That is our next concern.

Some practice problems dealing with constant speed are given in Study Guide 1.2 (a,b,c and d).

Summary 1.4
1. A fifty-yard swim is analyzed as an example of "real-life" motion.
2. The problem of non-uniform motion and its analysis.
3. Average speed is the only kind of speed that can be measured experimentally.

1.4 Leslie's "50" and the meaning of average speed. Consider the situation at a swimming meet. As a spectator, you want to see who are the fastest swimmers in each event. At the end of each race, the name of the winner is announced, and his total time given. Speeds as such are usually not announced, but since in a given race—say the 100-yard backstroke—every swimmer goes the same distance, the swimmer with the shortest time is necessarily the one having the highest average speed. We can define average speed as follows:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{elapsed time}}$$

So the speeds calculated on page 14 are all average speeds.

What information does a knowledge of the average speed convey? We shall answer this question by studying a real example.

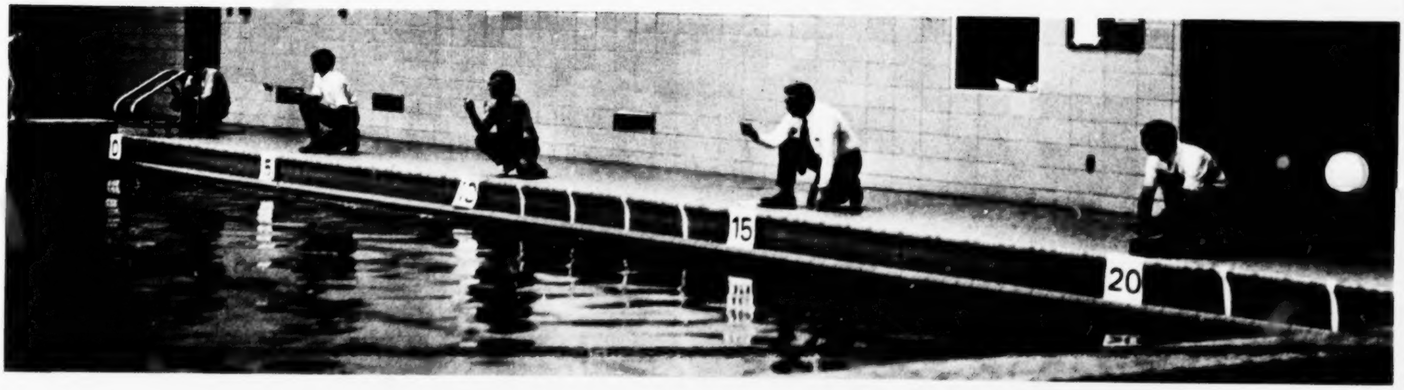
Leslie is not the fastest girl freestyle swimmer in the world, but Olympic speed is not necessary for our purposes. One day after school, Leslie was timed over two lengths of the Cambridge High School pool. The pool is 25 yards long, and it took her 56.1 seconds to swim the two lengths. Thus her average speed over the 50 yards was

$$\frac{50.0\ \text{yd}}{56.1\ \text{sec}} = 0.89\ \text{yd/sec}.$$

This is the equivalent of 1.8 mph. Some speed! A sailfish can do over 40 mph, and a fin-back whale can do 20 mph. But then man is a land animal. For short distances he can run better than 20 mph. But cheetahs have been clocked at 70 mph and ostriches at 50 mph.

Did Leslie swim with uniform speed? If not, which length did she cover more quickly? What was her greatest speed? Her least? How fast was she moving when she passed the 10, or 18, or 45-yard mark? Because we do not have the answer to any of these questions, we must admit that average speed does not tell us much. All we know is that Leslie swam the 50 yards in 56.1 seconds. The number 0.89 yd/sec probably comes closer than any other one number to describing the

L9: Analysis of a hurdle race - Part II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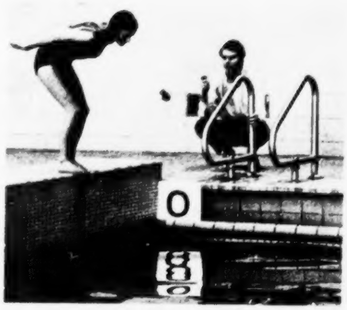
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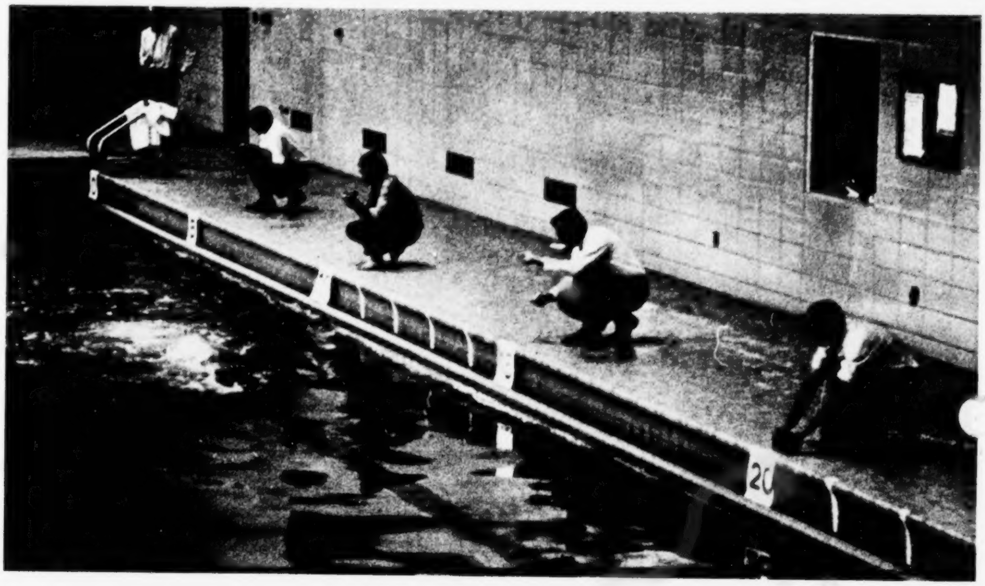
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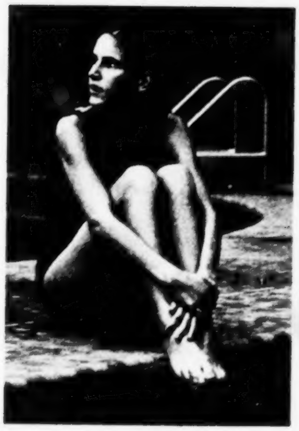


whole event. Such a number is useful and there is no denying that it is easy to compute.

But those questions about the details of Leslie's swim still nag us. To answer them, more data are necessary. That is why we arranged the event as shown on the opposite page.

Details of the speed at different parts of a race can help athletes improve their over-all showing.

Observers stationed at 5-yard intervals from the 0 mark to the 25-yard mark started their stop watches when the starting signal was given. Each observer had two watches, one which he stopped as Leslie passed his mark going down the pool, and the other which he stopped as she passed on her return trip. The data are tabulated below.

Position (yards)	0	5	10	15	20	25	30	35	40	45	50
Time (seconds)	0.0	2.5	6.0	11.0	16.0	22.0	26.5	32.0	39.5	47.5	56.0

From these data we can determine Leslie's average speed for the first 25 yards and for the last 25 yards.

1) $$\text{Average speed for first 25 yards} = \frac{\text{distance traveled}}{\text{elapsed time}} = \frac{25 \text{ yards}}{22 \text{ seconds}} = 1.1 \text{ yds/sec.}$$

2) $$\text{Average speed for last 25 yards} = \frac{\text{distance traveled}}{\text{elapsed time}} = \frac{25 \text{ yards}}{56 \text{ sec} - 22 \text{ sec}} = \frac{25 \text{ yds}}{34 \text{ sec}} = .74 \text{ yd/sec.}$$

It is clear that Leslie did not swim with uniform speed. She swam the first length much faster (1.1 yds/sec) than the second length (0.74 yd/sec). Notice that the overall average speed (0.89 yd/sec) does not describe either lap very well. If we wish to describe Leslie's performance in more detail, it will be advantageous to modify our data table.

Before we continue our analysis of Leslie's swim, however, we shall introduce some shorthand notation. In this shorthand notation the definition of average speed can be simplified from

$$\text{average speed} = \frac{\text{distance traveled}}{\text{elapsed time}}$$

to the concise statement

$$v_{av} = \frac{\Delta d}{\Delta t} .$$

The same concepts we are here developing to discuss this everyday type of motion will be needed to discuss the motion of planets, atoms, and so forth.

In this equation v_{av} is the symbol for average speed, d is the symbol for distance, and t is the symbol for time. The symbol Δ is the fourth letter in the Greek alphabet. It is called delta. When Δ precedes another symbol, it means "the change in...." Thus, Δd does not mean that Δ multiplies d, but rather "the change in d" or "distance interval." Likewise, Δt stands for "change in t" or "time interval."

We can now proceed with our analysis. Suppose as a next step we calculate the average speed for each 5-yard interval. This calculation is easily done; especially when our data are organized as they are in the table below. The results of this calculation for the first lap are entered in the right-hand column.

Data Table for Leslie's 50-yard Swim

Distance (yds)	Time (sec)	Δd (yds)	Δt (sec)	Δd/Δt (yd/sec)
0	0.0	5	2.5	2.0
5	2.5	5	3.5	1.4
10	6.0	5	5.0	1.0
15	11.0	5	5.0	1.0
20	16.0	5	6.0	.8
25	22.0	5	4.5	
30	26.5	5	5.5	
35	32.0	5		
40	39.5	5		
45	47.5	5		
50	56.1			

(The second-lap computations are left to you.)

Looking at the speed column, we discover that Leslie had her greatest speed right at the beginning. During the middle part of the first length she swam at a fairly steady rate, and she slowed down coming into the turn. You can use your own figures to see what happens after the turn.

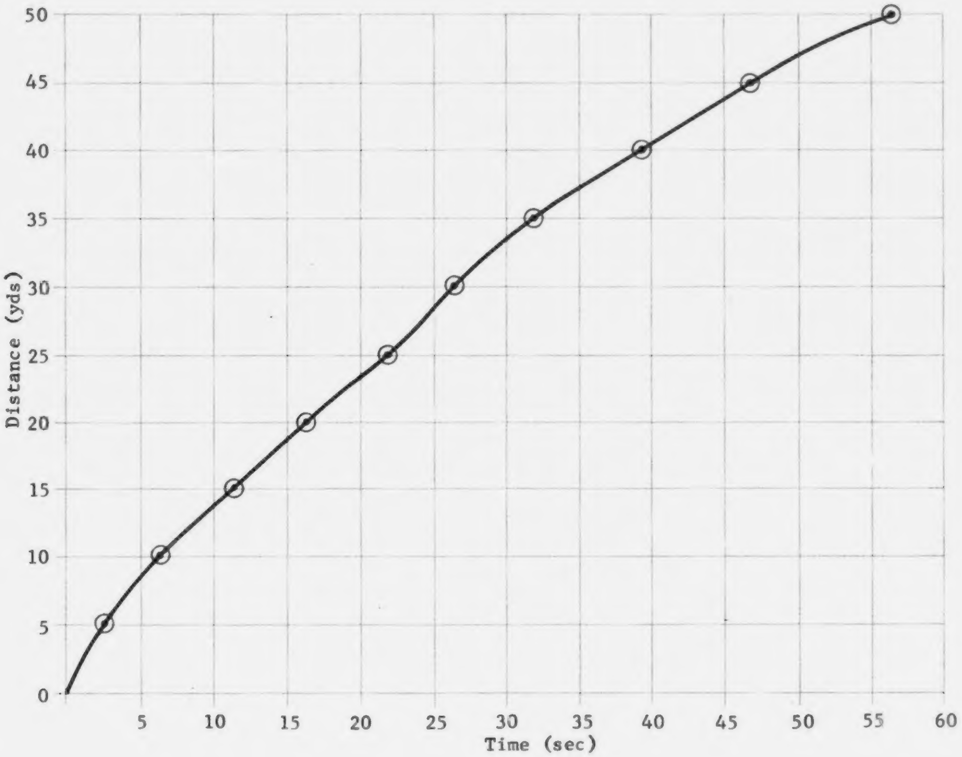
Now we have described Leslie's 50-yard swim in greater detail than when we gave a single, average speed for both lengths. But one point must be clear: although we have determined the speeds at various intervals along the path, we are still dealing with average speeds. The intervals are smaller—the time required to swim 5 yards rather than the entire 50—but we do not know the details of what happened within any of the intervals. Thus, Leslie's average speed between the 15 and 20-yard marks was 1.0 yd/sec, but her speed at the very instant she was 18 yards from the start is still uncertain. Even so, the average speed computed over the 15 to 20-yard interval is probably a better estimate of her speed at the 18-yard mark than the average speed computed over the whole 50 yards, or over either length. We shall come back to this problem of the determination of speed at a particular point in Sec. 1.7.

Practice problems on average speed can be found in Study Guide 1.2 (e, f, g and h). Study Guide 1.3, 1.4, 1.5 and 1.6 offer somewhat more challenging problems. Some suggestions for average speeds to measure are listed in Study Guide 1.7 and 1.8. Questions about the speedometer as a measure of speed are raised in Study Guide 1.9 and 1.10.

1.5 Graphing motion. What can we learn about motion by graphing data rather than just tabulating them? Let us find out by preparing a distance-versus-time graph using the data from Leslie's 50-yard swim. It is shown below. (We assumed there were no abrupt changes in her motion and so joined the data points with a smooth curve.)

T1: Stroboscopic measurements
T2: Graphs of various motions
L8, L9: Analysis of a hurdle race parts I and II
P1: Review of graphs
E4: Uniform motion

Now let us "read" the graph. If you will accept the idea that the steepness of the graph in any region indicates something about the speed (the steeper the faster) you will have no trouble seeing how Leslie's speed changed throughout the trial. It will be proven to you a little later that the speed can be calculated by measuring the steepness of the graph. Notice that the graph is steepest at the start and

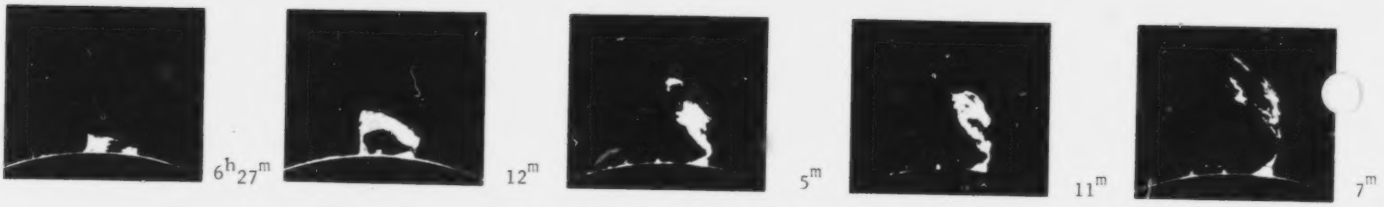


Summary 1.5
1. The distance-time graph for objects moving with uniform speed is a straight line.

2. The slope of a straight line is given by the ratio $\frac{\Delta y}{\Delta x}$.

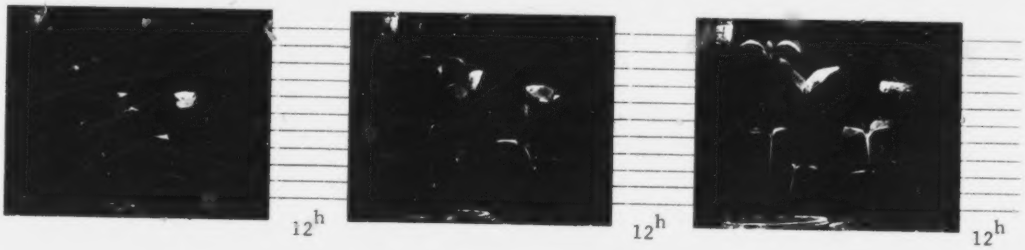
3. The slope of a line in a distance-time graph is a measure of speed.

4. The speed-time graph of an object moving with uniform speed is a straight horizontal line.



The scale for the solar flare sequence is provided by the curvature of the sun.

These photographs show a stormy outburst at the edge of the sun, a river of ice, and a developing sunflower plant. From these pictures and the included time intervals you can determine the average speeds (1) of the solar flare with respect to the sun's surface (radius of sun is about 432,000 mi.), (2) of the glacier with respect to the "river's bank," and (3) of the sunflower plant with respect to the flower pot.

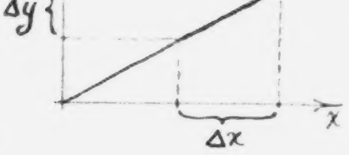


Multiply your measurements on these photographs by 8 to get actual plant sizes.

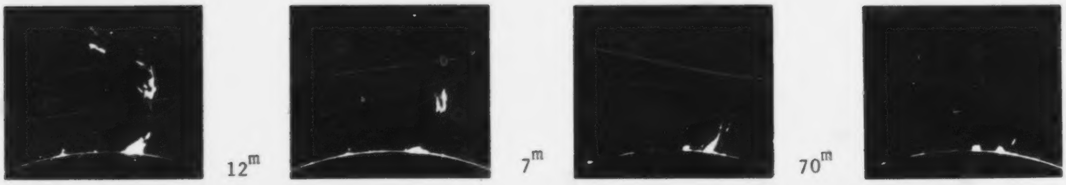
decreases slightly up to the 10-yard mark. From 10 yards to 20 yards the graph appears to be a straight line becoming neither more nor less steep. This means that her speed in this stretch neither increased nor decreased but was uniform. Reading the graph further, we see that she slowed down somewhat before she reached the 25-yard mark but gained some speed at the turn. The steepness decreases gradually from the 30-yard mark to the finish indicating that Leslie was slowing down. (She could barely drag herself out of the pool after the trial.)

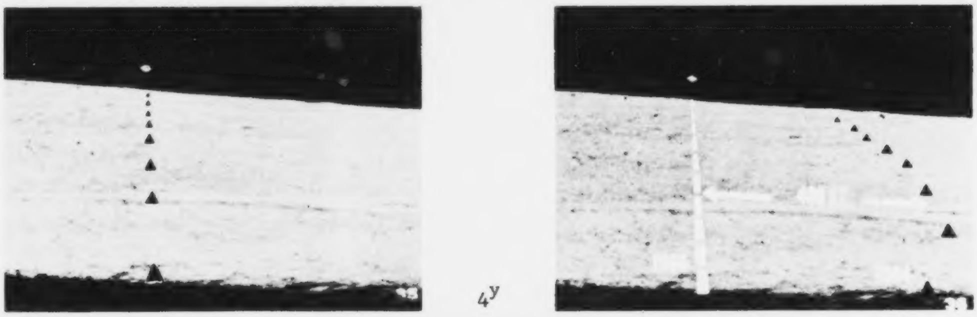
Looked at in this way, a graph provides us with a picture or visual representation of motion. But our interpretation of it was merely qualitative. If we want to know just how fast or slow Leslie was swimming at various times, we need a quantitative method of expressing the steepness. The way to indicate the steepness of a graph quantitatively is by means of the "slope."

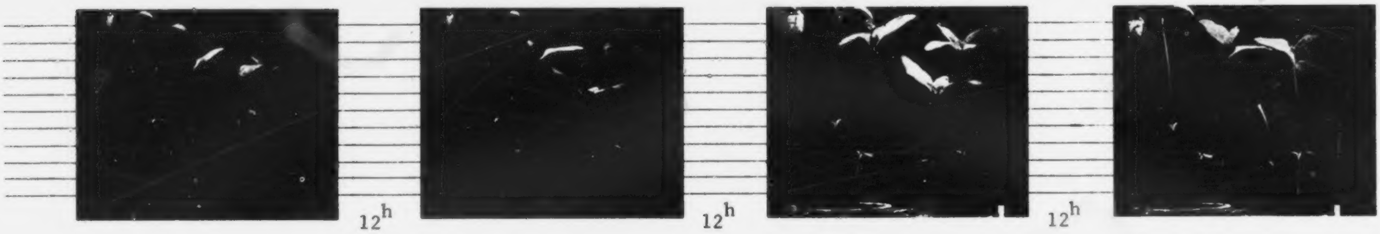
Slope is a widely used mathematical concept, and can be used to indicate the steepness in any graph. If, in accordance with custom, we call the vertical axis of any graph the y-axis and the horizontal axis the x-axis, then by definition,



$$\text{slope} = \frac{\Delta y}{\Delta x}.$$

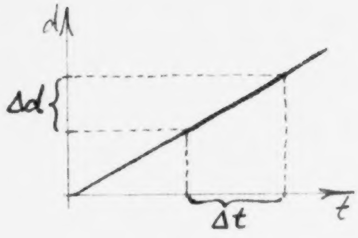






In a distance-time graph, distance is usually plotted on the vertical axis (d replaces y) and time on the horizontal axis (t replaces x). Therefore, in such a graph,

$$\text{slope} = \frac{\Delta d}{\Delta t} .$$



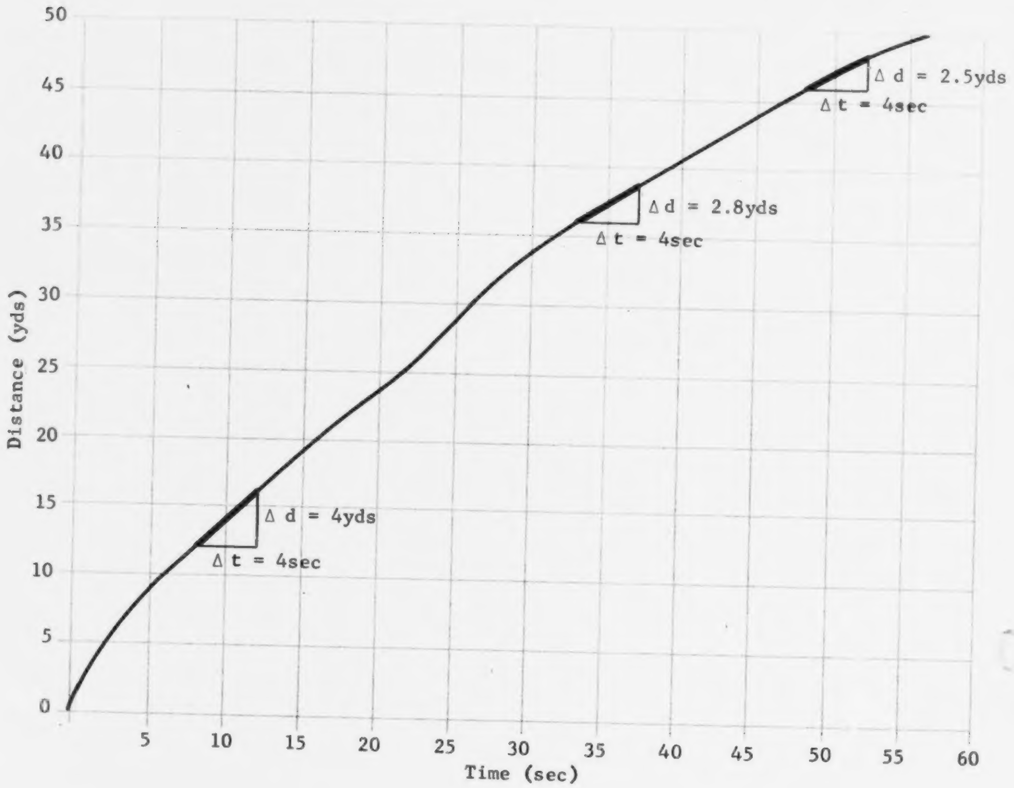
But this is just the definition of average speed. In other words, the slope of any part of a graph of distance versus time gives a measure of the average speed of the object during that interval.

There is really nothing mysterious about slope or its measurement. Highway engineers specify the steepness of a road by the slope. They simply measure the rise in the road and divide that rise by the horizontal distance one must go in order to achieve that rise. If you have never encountered the mathematical concept of slope before, or if you wish to review it, you might find it helpful to turn to Study Guide 1.11 before continuing here.

We can now ask, "What was Leslie's speed at the 14 or 47-yard marks, or at 35 seconds after the start"? In fact, by determining the slope, Leslie's speed can be estimated at any position or time by taking the slope of a small region on the distance-time graph of her motion that includes the particular instant or spot of interest. The answers to the above question are worked out on the graph below.

Time (sec)	Position (yds)	Speed (yds/sec)
10	14	1
20		
35		
50		

Determine Leslie's speed at these times using the graph.



The plausibility of the results can be checked by comparing them with Leslie's average speeds near those regions. For example, her average speed during the last 10 yards (from d = 40 to d = 50) was

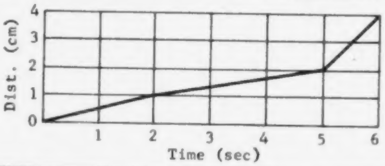
The data used here were taken from the table on page 18.

$$\frac{10 \text{ yards}}{56.1 \text{ sec} - 39.5 \text{ sec}} = .60 \text{ yards/sec.}$$

Similarly from the graph we determined that Leslie's speed was .62 yards/sec at the 47-yard mark.

Q1 Find the speeds at different points for a moving object from the following distance-time graph:

Q2 What was the average speed for the first 6 seconds?



(The end-of-section questions are to help you check your understanding of the section. If your answers don't agree reasonably well with those given on pp. 127-128, you should read the section again.)

Summary 1.6
1. Estimating values between (interpolating) and beyond (extrapolating) the data in a graph is risky, and to be undertaken with care.

1.6 Time out for a warning. Graphs are useful—but they can also be misleading. You must always be aware of the limitations of any graph you use. The only certain pieces of information in a graph are the data points, and even they are certain only to within the accuracy limits of the measurements. Furthermore, we often lessen the accuracy when we place the points on a graph.

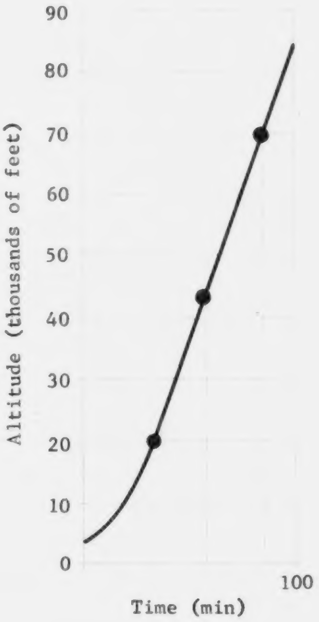
Turn back to p. 13 and in the margin draw a distance-time graph for the motion of the dry ice disc.

The line drawn through the points depends on personal judgment and interpretation. The process of estimating values between data points is called interpolation. That is essentially what you are doing when you draw a line between data points. Even more risky than interpolation is extrapolation, where the graph line is extended to estimate values beyond the known data.

An example of a high-altitude balloon experiment carried out in Lexington, Massachusetts, will nicely illustrate the danger of extrapolation. A cluster of gas-filled balloons carried some cosmic ray detectors high above the earth's surface, and from time to time a measurement was made of the height of the cluster. The adjoining graph shows the results for the first hour and a half. As the straight line drawn through the points suggests, the assumption is that the balloons are rising with uniform speed. Thus the speed can be calculated from the slope:

$$\text{speed of ascent} = \frac{\Delta h}{\Delta t} = \frac{27{,}000 \text{ ft}}{30 \text{ min}} = 900 \text{ ft/min.}$$

If you were asked how high the balloons would be at the end of the experiment (500 min), you might extrapolate, obtaining the result 500 min × 900 ft/min = 450,000 ft, which is over 90 miles high! Would you be right? Turn to Study Guide 1.13 to see for yourself.

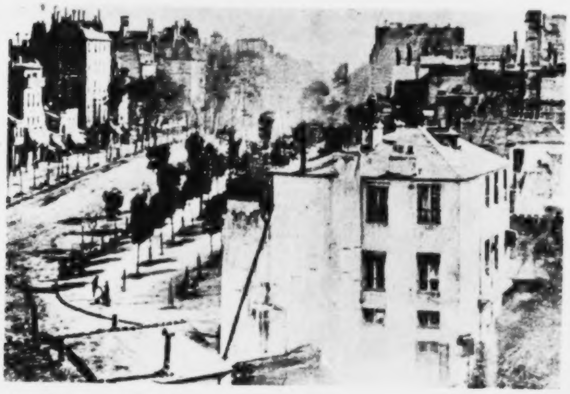


Q3 What is the difference between extrapolation and interpolation?

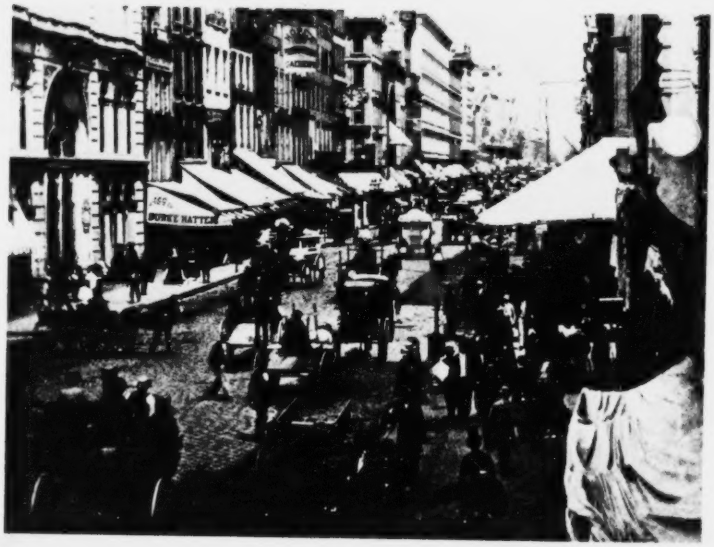
1.7 Instantaneous speed. Now back to Leslie. In Sec. 1.5 we saw that distance-time graphs could be extremely helpful in describing motion. When we reached the end of the section, we were speaking of specific speeds at particular points along the path (e.g., "the 14-yard mark") and at particular instants of time (e.g., "the 35-second instant"). You might have been bothered by this, for earlier we had gone out of

Summary 1.7
1. Instantaneous speed is defined as the average speed taken over a time interval so small that the value of $\Delta s/\Delta t$ does not change as Δt is made still smaller.
2. If a distance-time graph is made of the motion of an object, the value of the instantaneous speed at any point will be given by the slope of the tangent to the curve at that point.
3. Average speed: $v_{av} = \Delta s/\Delta t$. Instantaneous speed as Δt approaches zero: $v = \Delta s/\Delta t$.

See LIFE magazine for Dec. 23, 1966. The entire issue is devoted to photography.



1 Paris street scene, 1839



2 American street scene, 1859

Photography 1839 to the Present

Photography has an important role in our analysis of motion. These pages illustrate some of the significant advances in technique over the last century.

3 Boys on skateboards

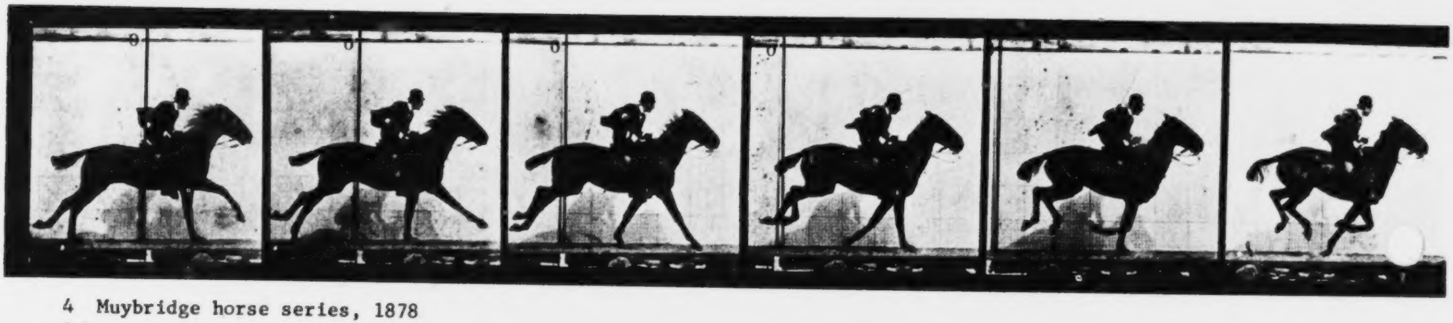
1 Note the lone figure in the otherwise empty street. He was getting his shoes shined. The other pedestrians did not remain in one place long enough to have their images recorded. With exposure times several minutes long the outlook for the possibility of portraiture was gloomy.

2 However, by 1859, due to improvements in photographic emulsions, illumination and lenses, it was not only possible to photograph a person at rest, but one could capture a bustling crowd of people, horses and carriages. Note the slight blur of the jaywalker's legs.

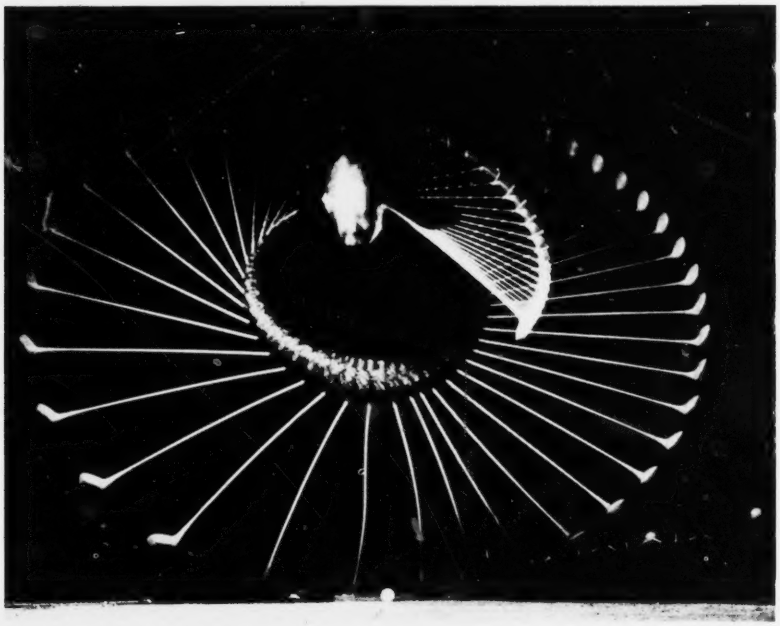
3 Today, even with an ordinary camera one can "stop" action.

4 A new medium—the motion picture. In 1873 a group of California sportsmen called in the photographer Eadweard Muybridge to settle the question, "Does a trotting horse ever have all four feet off the ground at once?" Five years later he answered the question with these photos. The six pictures were taken with six cameras lined up along the track, each camera being triggered when the horse broke a string which tripped the shutter. The motion of the horse can be reconstituted by making a flip pad of the pictures.

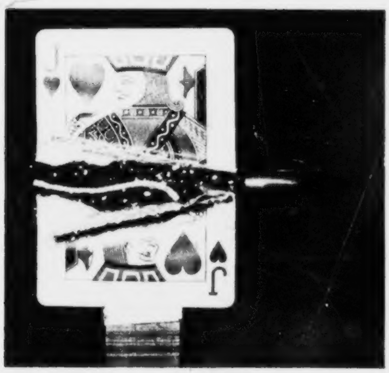
With the perfection of flexible film, only one camera was needed to take many pictures in rapid succession. By 1895, there were motion picture parlors throughout the United States. Twenty-four frames each second were sufficient to give the viewer the illusion of motion.



4 Muybridge horse series, 1878



6 Stroboscopic photo of golfer's swing, Harold Edgerton (See the article "The Dynamics of a Golf Club" in Project Physics Reader 1.)



5 Bullet cutting jack of hearts, Harold Edgerton

5 It took another ninety years after the time the crowded street was photographed before a bullet in flight could be "stopped." This remarkable picture was made by Professor Harold Edgerton of MIT, using a brilliant electric spark which lasted for about one millionth of a second.

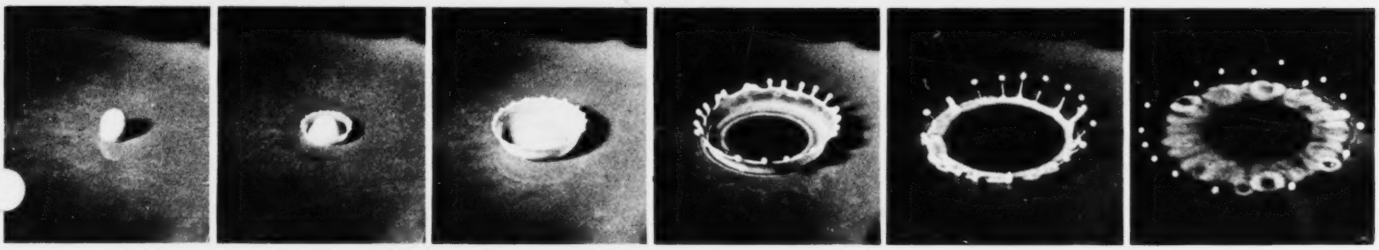
6 A light can be flashed successfully at a controlled rate and a multiple exposure (similar to the strobe photos in the book) can be made. In this photo of the golfer, the light flashed 100 times each second.

7 One does not need to have a flashing light to take multiple exposures. You can take them accidentally by forgetting to advance your film after each shot or you can do it purposely by snapping the camera shutter rapidly in succession.

8 An interesting offshoot of motion pictures is the high-speed motion picture. In the frames of the milk drop shown below, 1,000 pictures were taken each second. The film was whipped past the open camera shutter while the milk was illuminated with a flashing light (similar to the one used in photographing the golfer) synchronized with the film. When the film is projected at 24 frames each second, action which took place in 1 second is spread out over 42 seconds.

It is clear that the eye alone could not have seen the elegant details of this somewhat mundane event.

7 Girl rising

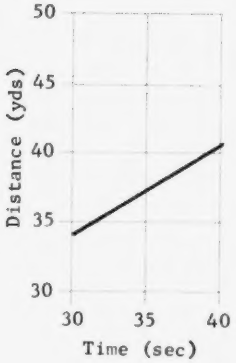


8 Action shown in high speed film of milk drop. Harold Edgerton

T3 Instantaneous speed
T4 Instantaneous rate of change
D3 Instantaneous speed using strobe photos of mass on spring

our way to assert that the only kind of speed we can measure is average speed. To find average speed we need a ratio of distance and time intervals; a particular point along the path does not define an interval. Nevertheless, there are grounds for stating the speed at a point. We will see what they are.

You remember that our answer to the question, "How fast was Leslie swimming at time t = 35 sec?" was "0.70 yd/sec." That answer was obtained by finding the slope of a small portion of the curve encompassing the point t = 35 sec. That section of the curve has been reproduced in the margin here. Notice that the part of the curve we used is seemingly a straight line. Thus, as the table under the graph shows, the value of the slope does not change as we decrease the time interval Δt. Now imagine that we closed in on the point where t = 35 sec until the amount of curve remaining became vanishingly small. Could we not safely assume that the slope of that infinitesimal part of the curve would be the same as that on the straight line of which it seems to be a part? We think so. That is why we took the slope of the straight line lying along the graph from t = 33.0 sec to t = 37.0 and called it the speed at t = 35.0 sec.



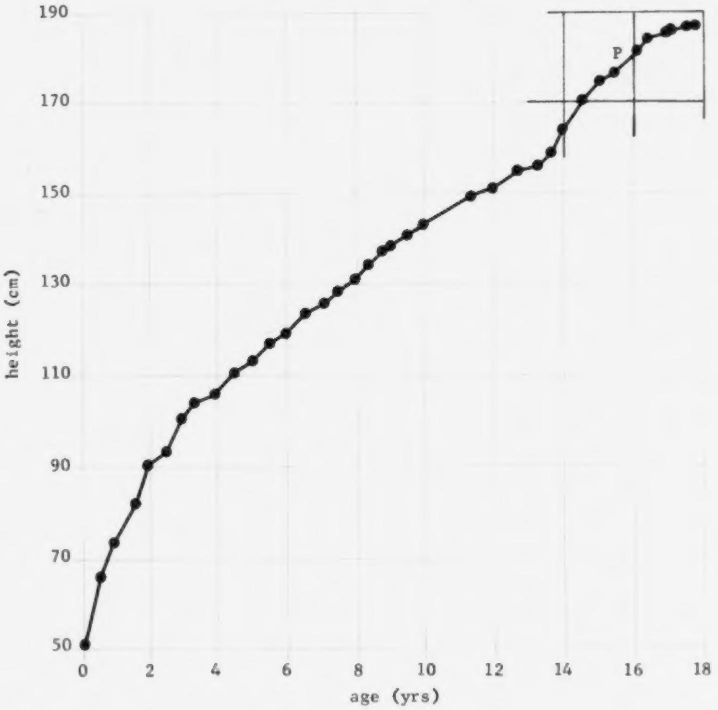
Time interval (sec)	Distance interval (yds)	$\frac{\Delta d}{\Delta t}$ (yds/sec)
33 37	36 38.8	.70
34 36	36.7 38.1	.70
34.5 35.5	37.05 37.75	.70
34.75 35.25	37.225 37.575	.70

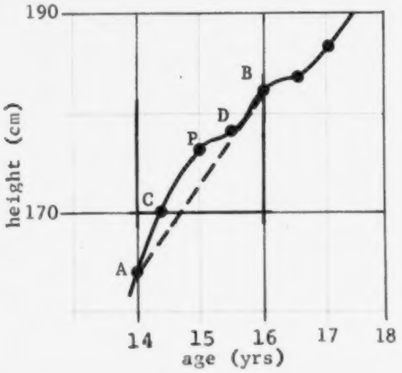
We hope you noticed that in estimating a value for Leslie's instantaneous speed at a particular time, we actually measured the average speed over a 4.0-sec interval. Conceptually, we have made a leap here. We have decided that the instantaneous speed at a particular time can be equated to an average speed Δd/Δt provided: 1) that the particular time is encompassed by the time interval, Δt, used to compute Δd/Δt and 2) that the ratio Δd/Δt does not change appreciably as we compute it over smaller and smaller time intervals.

A concrete example will help here. In the oldest known study of its kind, the French scientist de Montbeillard periodically recorded the height of his son during the period 1759-1777. A graph of height versus age is shown on the next page.

Finding speed at a point, or instantaneous speed, was a puzzle to mathematicians for centuries. Finaly the differential calculus provided a rigorous solution.

From the graph we can compute the boy's average growth rate over the entire 18-year interval or over any other time interval. Suppose, however, we wanted to know how fast the boy was growing on his fifteenth birthday. The answer becomes evident if we enlarge the graph in the vicinity of the fifteenth year. His height at age 15 is indicated as point P, and the other letters designate time intervals on either side of P. The boy's average growth rate over a two-year interval is given by the slope AB. Over a one-year interval this average growth rate is given in the slope DC. The slope of EF gives the average growth rate over six months,etc. The three lines are not quite parallel to each other and so their slopes will be different. In the enlarged sections below, lines have been drawn joining the end points of time intervals of 4 mo, 2 mo and 1 mo around the point t = 15 years.





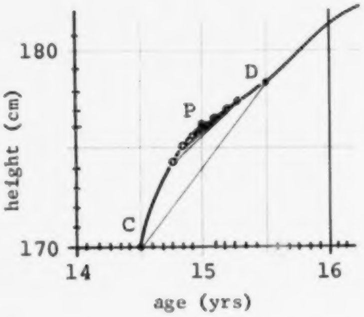
The graph above is an enlargement of the corner of the graph at the top. The graph below is a further enlargement of the middle of the enlargement.

Notice that for intervals less than t = 1 yr, the lines appear to be parallel to each other and gradually to merge into the curve, becoming nearly indistinguishable from it. You can approximate the tangent to this curve by placing a ruler along the line GH and extending it on both sides.

The values of the slopes have been computed for the several time intervals and are tabulated below.

Δt	Δd	v
8 yr	49.0 cm	6.1 cm/yr
2 yr	19.0 cm	9.5 cm/yr
1 yr	8.0 cm	8.0 cm/yr
6 mo	3.5 cm	7.0 cm/yr
4 mo	2.0 cm	6.0 cm/yr
2 mo	1.0 cm	6.0 cm/yr

We note that the values of v_{av} calculated for shorter and shorter time intervals approach closer and closer to 6.0 cm/yr. In fact, for any time interval less than 2 months, the average speed v_{av} will be 6.0 cm/yr within the limits of accuracy of the measurement of d and t. Thus, we can say that on young de Montbeillard's fifteenth birthday, he was growing at a rate of 6.0 cm/yr.



• The idea of $V = \lim_{\Delta t \to 0} \left(\frac{\Delta d}{\Delta t}\right)$ need not be elaborated mathematically but should be given some empirical support. Two good examples are described in Demonstration 3.

Average speed, we have said, is the ratio of distance traveled to elapsed time, or, in symbols,

$$v_{av} = \frac{\Delta d}{\Delta t} .$$

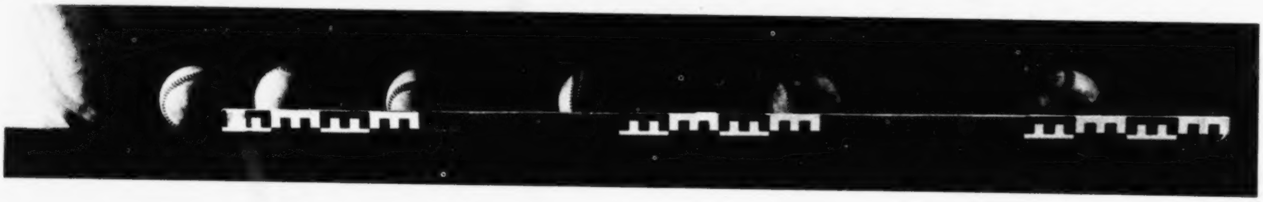
We now define instantaneous speed at a point in time as the limiting value approached by the average speeds in time-intervals including that point, as Δt is made smaller and smaller. • In almost all physical situations such a limiting value can be accurately estimated by the method described on the previous page. From now on we will use the letter v, without any subscript, to mean the instantaneous speed defined in this way. (For further discussion, see the article "Speed" in Project Physics Reader 1.)

Why this definition of instantaneous speed? We can, of course, define it any way we please; whether the definition is a wise one is a matter of how useful it turns out to be in analyzing motion. In chapter 3 we will find that change of instantaneous speed, defined in this way, is related in a beautifully simple way to force.

You may be wondering why we have used the letter "v" instead of "s" for speed. The word "velocity" is often used to mean the same thing as speed. In physics it is useful to reserve "velocity" for the concept of speed in a specified direction, and denote it by the symbol $\vec{v}$. When the direction is not specified, we remove the arrow and just use the letter v, calling it speed. This distinction between v and $\vec{v}$ will be discussed in more detail in Section 3.2.

Q4 Explain the difference between average speed and instantaneous speed.

Q5 The baseball shown in the figure below is presented here for your analysis. You might tabulate your measurements and construct a distance-time graph. From the distance-time graph, you can determine the instantaneous speed at several times and construct a speed-time graph. The time interval between successive flashes is 0.5 sec. You can check your results by referring to the answer page at the back of this unit.



1.8 Acceleration—by comparison.• The baseball in the problem above was changing speed—accelerating. You could tell that its speed was changing without having to take measurements and plot graphs. But how would you describe how fast the ball was changing speed?

To answer this question you have really only one new thing to learn—the definition of acceleration. Actually, the definition is simple, so the problem is not so much for you to learn it as it is to learn how to use it in situations like the one above. For the time being we will define the time-rate of change of speed as acceleration. Later, this definition will have to be modified somewhat when we encounter motion in which change in direction▲becomes an important factor. But for now, as long as we are dealing only with straight line motion, we can equate the time-rate of change of speed with acceleration.

Many of the effects of acceleration are well known to us. It is acceleration, not speed, that we feel when an elevator starts up or slows down. The sudden flutter in our stomachs comes only during the speeding up and slowing down portions of the trip, and not during most of the ride when the elevator is moving at a steady speed. Likewise, much of the excitement of the roller coaster and other rides at amusement parks is directly related to their unexpected accelerations. How do you know it is really not speed that causes these sensations? Simply stated, you always detect speed by reference to objects outside yourself. You can only tell you are moving at a high speed in an automobile by watching the scenery as it whizzes past you, or by listening to the sounds of air rushing against the car or the whine of the tires on the pavement. In contrast, you "feel" accelerations and do not need to look out your car window to realize the driver has stepped on the accelerator or slammed on the brakes.

Now let us compare acceleration and speed:

The rate of change of position is speed.

The rate of change of speed is acceleration.•

This similarity of form will enable us to use our previous work on the concept of speed as a guide for making use of the concept of acceleration. The techniques which you have already learned for analyzing motion in terms of speed can be used to study motion in terms of acceleration. For example you have learned that the slope of a distance-time graph at a point is the instantaneous speed. What would the slope (i.e., $\Delta v/\Delta t$) of a speed-time graph indicate?

Summary 1.8

1. The time rate of change of speed is acceleration.

2. For uniform acceleration, $a = \Delta v/\Delta t$.

3. Average acceleration and instantaneous acceleration can be defined in ways entirely analogous to the definitions of average speed and instantaneous speed.

• The development of acceleration is parallel to the development of speed and can serve as a review of the chapter.

▲ Emphasize that direction is important but is not considered yet in order to simplify the development of the ideas.

D4: Uniform acceleration using liquid accelerometer
T1: Stroboscopic measurements
T2: Graphs of various motions
F1: Straight line kinematics

• This statement is incomplete, but correct in this context.

The remainder of this section is made up of a list of statements about motion along a straight line. The list has two purposes: 1) to help you review some of the main ideas about speed presented in this chapter, and 2) to present the corresponding ideas about acceleration so you may take advantage of your knowledge of speed. For this reason, each statement about speed is immediately followed by a parallel statement about acceleration.

1. Speed is the rate of change of position. Acceleration is the rate of change of speed.•

•Acceleration is, of course, the rate of change of velocity. The statement here is valid only in the very limited context of rectilinear motion. In the next chapter this simplified idea is extended to vectors.

2. Speed is expressed in the units distance/time. Acceleration is expressed in the units speed/time.

For example, if speed changes from 4 m/sec to 5 m/sec during an interval of 1 second, average acceleration is 1 (m/sec)/sec. This is usually written more briefly as 1 m/sec^2.

3. Average speed over any interval is the ratio of the corresponding distance and time intervals:

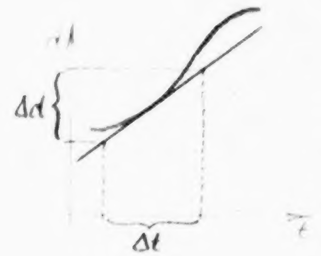
$$v_{av} = \frac{\Delta d}{\Delta t} .$$

Average acceleration over any interval is the ratio of the corresponding speed and time intervals:

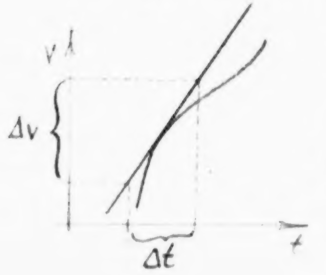
$$a_{av} = \frac{\Delta v}{\Delta t} .$$

An airplane changes its speed from 350 mi/hr to 470 mi/hr in 6.0 min. Its average acceleration is 20 (mi/hr)/min—whether or not the acceleration is uniform.

4. Instantaneous speed is the value approached by the average speed as Δt is made smaller and smaller. Instantaneous acceleration is the value approached by the average acceleration as Δt is made smaller and smaller.



$$V = \frac{\Delta d}{\Delta t}$$



$$a = \frac{\Delta v}{\Delta t}$$

5. If a distance-time graph is made of the motion of an object, the instantaneous speed at any position will be given by the slope of the tangent to the curve at the point of interest. If a speed-time graph is made of the motion of an object, the instantaneous acceleration at any position will be given by the slope of the tangent to the curve at the point of interest.

In this listing of statements about speed and acceleration, the concepts of average and instantaneous acceleration have been included for the sake of completeness. However, it will be helpful to remember that when the acceleration is uniform, it can be found by using the relationship

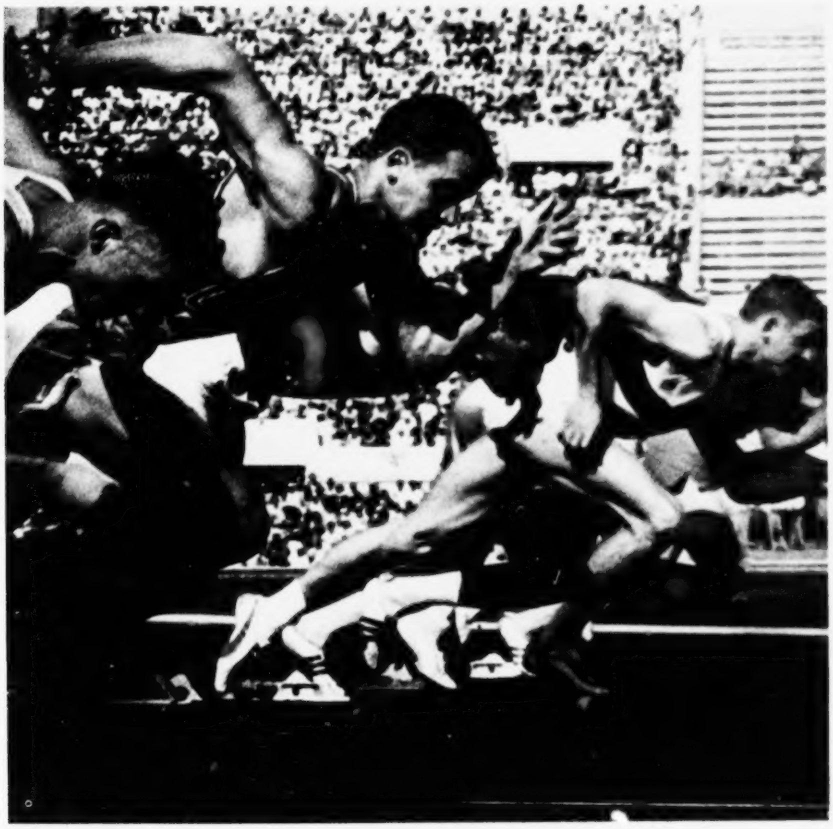
$$a = \frac{\Delta v}{\Delta t}$$

for any interval whatever. That is, instantaneous and average acceleration have the same numerical value for constant acceleration—which will be the most usual case of motion we shall encounter.

Until the work of Galileo in the seventeenth century, acceleration proved to be a particularly difficult concept. In the next chapter, we will examine Galileo's contribution to our understanding of the nature of accelerated motion. His work provides a good example of how scientific theory and actual measurements are combined to develop physical concepts.

Q6 What is the average acceleration of a sports car which goes from 0 to 60 mph in 5 seconds?

Q7 What is your average acceleration if you change your speed from 4 miles per hour to 2 miles per hour in an interval of 15 minutes?



Discuss briefly the Boccioni on page 8 in context with Chapter 1. Boccioni in his medium and the scientist with his equations and graphs are both portraying motion.

1.1 This book is probably different in many ways from textbooks you have had in other courses. Therefore we feel it might help to make a few suggestions concerning how to use it.

1. Unless you are told otherwise by your teacher you should feel free to write in the book. Indeed we encourage you to do so. You will note that there are wide margins. One of the purposes of leaving that much space is to enable you to write down questions or statements as they occur to you as you are studying the material. Mark passages that you do not understand so that you can seek help from your teacher. You also notice that from time to time tables are left incomplete or problems appear in the text or margin. Complete such tables and write your answers to problems right in the text at the point where they are raised.

2. You will find answers to all of the end-of-section review questions on page 127, and brief answers to some of the Study Guide Questions on page 129. Always try to do the problems yourself first and then check your answers. If your answer agrees with the one in the book, then it is a good sign that you understand the material (although it is true, of course, that you can sometimes get the right answer for the wrong reason).

3. There are many different kinds of items in the Study Guide at the end of each chapter. It is not intended that you should do everything there. Sometimes we put into the Study Guide material which we think will interest some students but not enough students to merit putting into the main part of the text. Notice also that there are several kinds of problems. Some are intended to give practice and help the student in learning a particular concept whereas others are designed to help you bring together several related concepts. Still other problems are intended to challenge those students who like numerical problems.

4. Activities and experiments which you can carry out at home or outside the laboratory are described. We do not suppose that you want to do all of these but we do want you to take them seriously. If you do you will find that you are able to do quite a bit of science without having to have an elaborate laboratory.

5. The Project Physics course includes many other materials in addition to this book, such as film loops, programmed instruction booklets, and transparencies. Be sure to familiarize yourself with the Student Handbook, which describes further outside activities as well as laboratory experiments, and the Reader, which contains interesting articles related to physics.

1.2 Some practice problems:

	Situation	Find	Do work
a	Speed uniform, distance = 72 cm, time = 12 sec	Speed	$V = 6.0 \frac{cm}{sec}$
b	Speed uniform at 45 mph	Distance traveled in 20 min	$d = 15$ mi
c	Speed uniform at 36 ft/min	Time to move 9.0 ft	$t = 0.25$ min
d	$d_1 = 0, s_2 = 15$m, $d_3 = 30$ m $t_1 = 0$, $t_2 = 5.0$ sec, $t_3 = 10$ sec	Speed and position at 8.0 sec	$V = 3$ m/sec
e	You drive 240 mi in 6.0 hr	Average speed	$\bar{V} = 40 \frac{mi}{hr}$
f	Same	Speed and position after 3.0 hr	$V = 40 \frac{mi}{hr}$
g	Average speed = 76 cm/sec computed over a distance of 418 cm	Time taken	$\Delta t = 5.5$ sec
h	Average speed = 44 m/sec computed over time interval of 0.20 sec	Distance moved	$\Delta d = 8.8$ m

1.3 If you traveled one mile at a speed of 1000 miles per hour and another mile at a speed of 1 mile per hour your average speed would not be $\frac{1000 + 1}{2}$ mph or 500.5 mph. What would be your average speed?

$\bar{V} = 1.998$ mi/hr

1.4 A tsunami (incorrectly called "tidal wave") caused by an earthquake occurring near Alaska in 1946 consisted of several sea waves which traveled at the average speed of 490 miles/hour. The first of the waves reached Hawaii four hours and 34 minutes after the earthquake occurred. From these data, calculate how far the origin of the tsunami was from Hawaii. $d = 2.2 \times 10^3$ mi

1.5 Light and radio waves travel through a vacuum in a straight line at a speed of nearly 3×10^8 m/sec. The nearest star, Alpha Centauri, is 4.06×10^{16} m distant from us. If this star possesses planets on which highly intelligent beings live, how soon could we expect to receive a reply after sending them a radio or light signal strong enough to be received there? $t = 8.5$ yr

1.6 What is your average speed in the following cases:

a) You run 100 m at a speed of 5.0 m/sec and then you walk 100 m at a speed of 1.0 m/sec. $\bar{v} = 1.7$ m/sec
b) You run for 100 sec at a speed of 5.0 m/sec and then you walk for 100 sec at a speed of 1.0 m/sec? $\bar{v} = 3.0$ m/sec

1.7 Design some experiments which will enable you to make estimates of the average speeds for some of the following objects in motion.

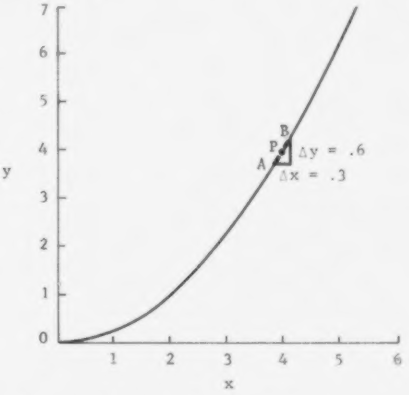
a) Baseball heaved from outfield to home plate
b) The wind
c) A cloud
d) A raindrop (do all drops have different speeds?)
e) Hand moving back and forth as fast as possible
f) The tip of a baseball bat
g) Walking on level ground, up stairs, down stairs
h) A bird flying
i) An ant walking
j) A camera shutter opening and closing Discussion

1.8 What problems arise when you attempt to measure the speed of light? Can you design an experiment to measure the speed of light? Discussion

1.9 Sometime when you are a passenger in an automobile compare the speed as read from the speedometer to the speed calculated from $\Delta s/\Delta t$. Explain any differences. Discussion

1.10 An automobile speedometer is a small current generator driven by a flexible cable run off the drive shaft. The current produced increases with the rate at which the generator is turned by the rear axle. The speedometer needle indicates the current. Until the speedometer is calibrated it can only indicate changes in speed, but not actual speeds in miles per hour. How would you calibrate the speedometer in your car if the company had forgotten to do the job? If you replaced your 24" diameter rear wheels with 28" diameter wheels, what would your actual speed be if your speedometer read 50 mph? Would your speedometer read too high or too low if you loaded down the rear end of your car and had the tire pressure too low? What effect does the speedometer have on the speed of the car? Can you invent a speedometer that has no effect on the motion of the car? Discussion

1.11 Take a look at the graph of y versus x shown below:



Notice that in this graph the steepness increases as x increases. One way to indicate the steepness of the graph at a point is by means of the "slope." The numerical value of the slope at a point P is obtained by the following procedure, which is diagramed above. Move a short distance along the graph from point A to point B, which are on the curve and lie on either side of point P. Measure the change in y, (Δy) in going from A to B. In this example $\Delta y = .6$. Measure the corresponding change in x, (Δx) in going from A to B. Δx here is .3. The slope is defined as the ratio of Δy to Δx.

$$\text{Slope} = \frac{\Delta y}{\Delta x}\,.$$

In the example

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{.6}{.3} = 2.$$

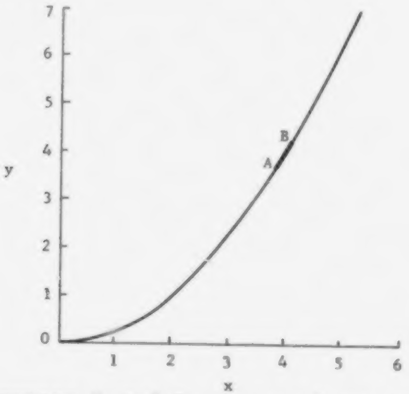
Now there are three important questions concerned with slopes that we must answer.

Q. What are the dimensions or units for the slope?

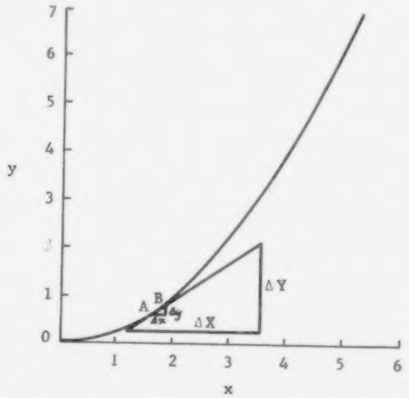
A. The dimensions are just those of y/x. For example, if y represents a distance in meters and x represents a time in seconds then the units for slope will be meters/seconds or meters per second.

Q. In practice how close do A and B have to be to point P? (Close is not a very precise adjective. New York is close to Philadelphia if you are traveling by jet. If you are walking it is not close.)

A. Choose A and B near enough to point P so that the line connecting A and B lies along the curve at point P. For example:



Q. Suppose A and B are so close together that you cannot read Δx or Δy from your graph. What does one do to calculate the slope?

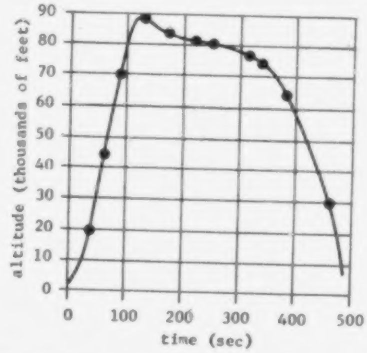


A. Extend line AB as it is shown in the figure and compute its slope. Notice that the small triangle is similar to the large triangle and that $\frac{\Delta Y}{\Delta X} = \frac{\Delta y}{\Delta x}$.

Determine the slopes of this graph of distance versus time at t = 1, 2, 3 and 4 seconds. 0.5, 1.0, 1.5, 2.0

1.12 The electron beam in a TV set sweeps out a complete picture in 1/30th of a second and each picture is composed of 525 lines. If the width of the screen is 20 inches, what is the speed of that beam over the surface of the screen? v = 315,000 in/sec

1.13 (Answer to question in text, page 23.) Indeed the prediction based upon the first 1½ hour was vastly wrong. Such a prediction, based on a drastic extrapolation from the first 1½ hour's observation, neglects all the factors which limit the maximum height obtainable by such a cluster of balloons, such as the bursting of some of the balloons, the change in air pressure and density with height, etc. In fact, at the end of 500 minutes, the cluster was not 450,000 feet high, but had come down again, as the distance-time graph for the entire experiment shows. For another extrapolation problem, see Study Guide 1.14.



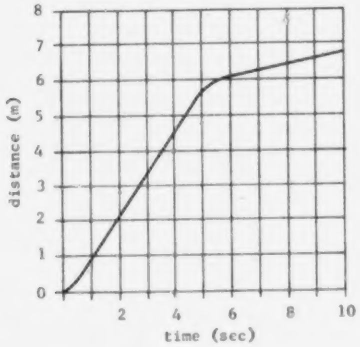
The altitude of a cosmic ray detector carried aloft by a cluster of balloons.

1.14 World's 400-meter swimming records for men and women. Ages are in parentheses:

1926	4:57.0	Weissmuller (18)
	5:53.2	Gertrude Ederle (17)
1936	4:46.4	Syozo Makino (17)
	5:28.5	Helene Madison (18)
1946	4:46.4	Makino (17)
	5:00.1	Hveger (18)
1956	4:33.3	Hironoshin Furuhashi (23)
	4:47.2	Crapp (18)
1966	4:11.1	Frank Weigand (23)
	4:38.0	Martha Randall (18)

By about how many meters would Martha Randall have beaten Johnny Weissmuller if they had raced each other? Could you predict the 1976 world's record for the 400-meter race by extrapolating the graph of world records vs. dates up to the year 1976? d = 25.6 m

1.15



Detailed analysis of a stroboscopic photograph of a rolling ball yielded information which was plotted on the graph above. By placing your ruler tangent to the curve at appropriate points estimate the following:

a) At what moment or interval was the speed greatest? What was the value of the speed at that time?— from 1 to 4.5 sec

b) At what moment or interval was the speed least? What was it at that time? from 6 to 10 sec

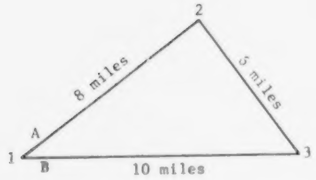
c) What was the speed at time 5.0 sec? $v = 0.74$ m/sec

d) What was the speed at time 0.5 sec? $v = 0.79$ m/sec

e) How far did the ball move from time 7.0 sec to 9.5 sec? $d = 0.42$ m

1.16 Suppose you must measure the instantaneous speed of a bullet as it leaves the barrel of a rifle. Explain how you would do this. Discussion

1.17



Car A and car B leave point 1 simultaneously and both travel at the same speed. Car A moves from 1 to 2 to 3 while car B moves from 1 to 3 directly. If B arrives at point 3 six minutes before A arrives, what was the speed of either car? $v = 40$ mi/hr

1.18 The data below show the instantaneous speed in a test run of a Corvette car, starting from rest. Plot the speed-versus-time graph, and derive and plot the acceleration-time graph.

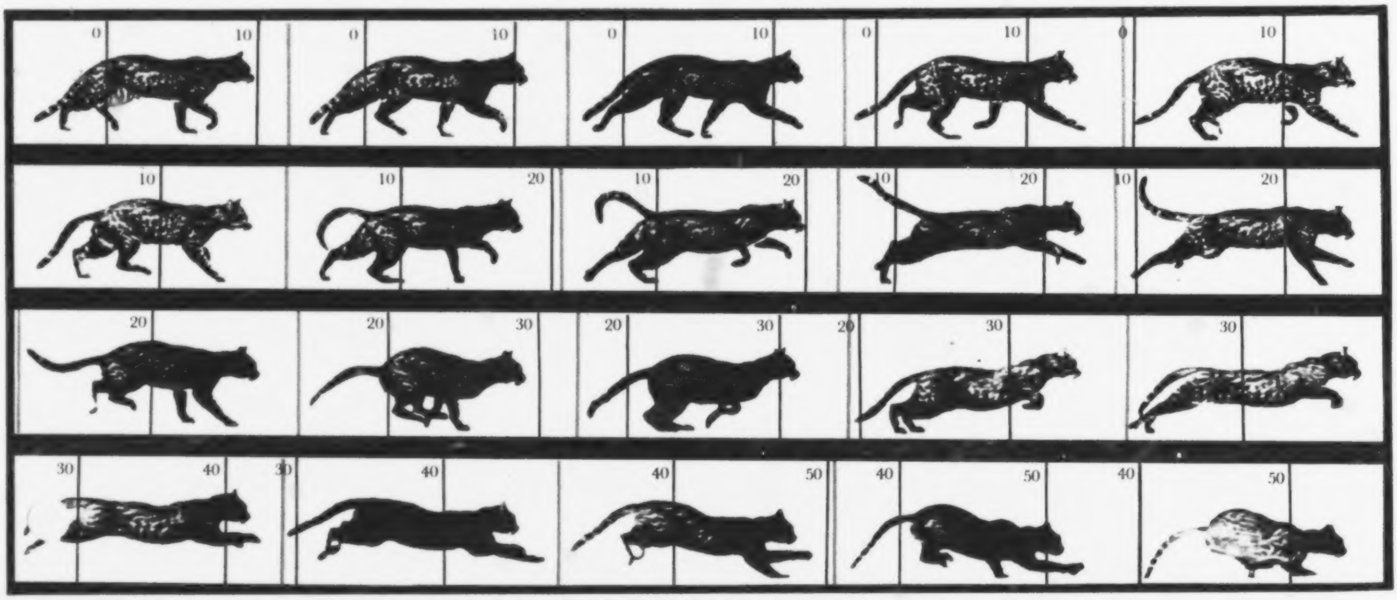
a) What is the speed at t = 2.5 sec?

b) What is the maximum acceleration?

Time (sec)	Speed (m/sec)
0.0	0.0
1.0	6.3
2.0	11.6
3.0	16.5
4.0	20.5
5.0	24.1
6.0	27.3
7.0	29.5
8.0	31.3
9.0	33.1
10.0	34.9

Discussion

1.19 Discuss the motion of the cat in the following photographs. Discussion



The numbers on each photograph indicate the number of inches measured from the line marked "0"

Chapter 2 Free Fall-Galileo Describes Motion

Summary 2.1

1. Medieval physical science was based on the writings of Aristotle.

2. In the Aristotelian scheme, there was a sharp dividing line between the objects on earth and those in the sky: earthly objects moved only to return to their "natural places"; celestial objects, on the other hand, moved in endless circles.

3. Aristotle's theory of motion survived for centuries in spite of certain known weaknesses because it was part of a larger accepted conceptual scheme, because it was consistent with common-sense ideas, because no strong alternative theories were available, and because the mathematical and quantitative methods which were to prove so significant had not yet been sufficiently developed.

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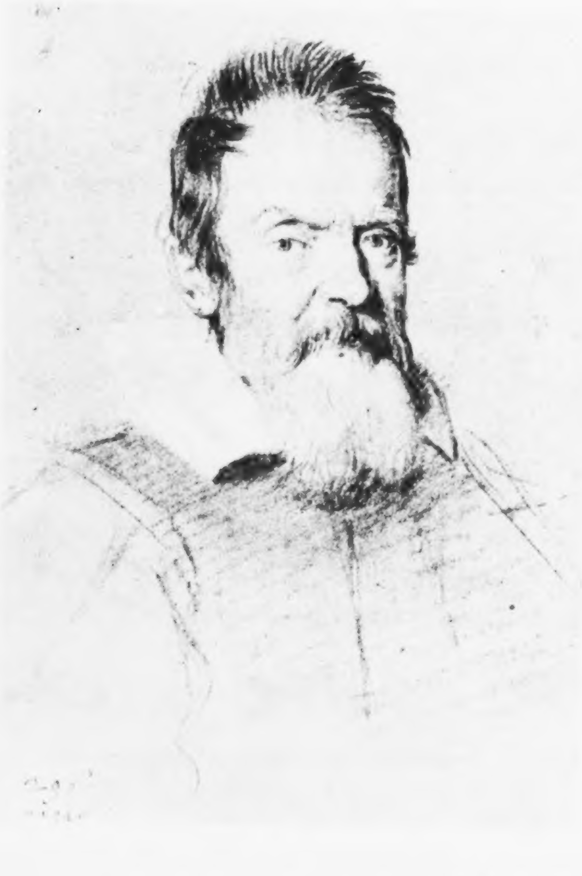


Portrait of Galileo in crayon by Ottavio Leoni, a contemporary of Galileo.

Because only a limited context can be provided here, there is a danger that students will be led to ridicule Aristotle's physics. Take care to point out that it was part of a grand and brilliant philosophical system, at once consistent with a common sense view and philosophically impressive enough to guide the thoughts of scholars for almost two thousand years. The Greek view is dealt with more extensively at the beginning of Unit 5.

2.1 The Aristotelian theory of motion. In this chapter we shall take a look at an important piece of research: Galileo's study of freely falling bodies. While the physical problem of free fall is fascinating in itself, our emphasis will be on Galileo as one of the first modern scientists. Thus Galileo's view of the world, his way of thinking, his use of mathematics and his reliance upon experimental tests are as important to us as the actual results of his investigation.

To understand the nature of Galileo's work and to appreciate its significance, we must first examine the differences between Galileo's new science of physics and the medieval system of physical thought that it eventually replaced. By comparing the new with the old, we can see how Galileo helped change our way of thinking about the world.

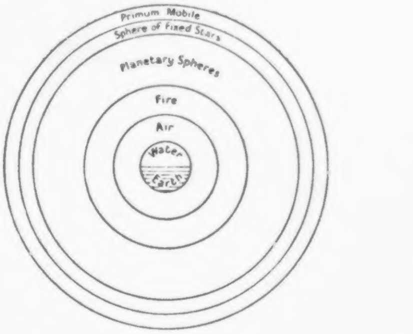


Diagram of medieval concept of the world structure.

In medieval physical science, as Galileo learned it at the University of Pisa, there was a sharp distinction between the objects on the earth and those in the sky. All terrestrial matter, the matter within our physical reach, was believed to be a mixture of four "elements"—Earth, Water, Air and Fire. Each of these four elements was thought to have a natural place in the terrestrial region. The highest place was allotted to Fire. Beneath Fire was Air, then Water and, finally, in the lowest position, Earth. Each was thought to seek its own place. Thus, Fire would tend to rise through Air, and Air through Water, whereas Earth would tend to fall through both Air and Water. The actual movement of any real object depended on the particular mixture of these four elements making it up and where it was in relation to its natural place.

A good deal of commonsense experience supports this view. For example, Water bubbles up through Earth at springs. When sufficient Fire is added to ordinary Water, by heating it, the resulting mixture of elements (what we call steam) rises through the air. Can you think of other examples?

The medieval thinkers also believed that the stars, planets and other celestial bodies moved in a far simpler manner than those objects on, or near, the earth. The celestial bodies were believed to contain none of the four ordinary elements, but instead consisted solely of a fifth element, the quintessence. The natural motion of objects composed of this element was neither rising nor falling, but endless revolution in circles around the center of the universe. The center of the universe was considered to be identical with the center of the Earth. Heavenly bodies, although moving, were thus at all times in their natural places. They were thus set apart from terrestrial objects which displayed natural motion only as they returned to their natural places from which they were displaced.

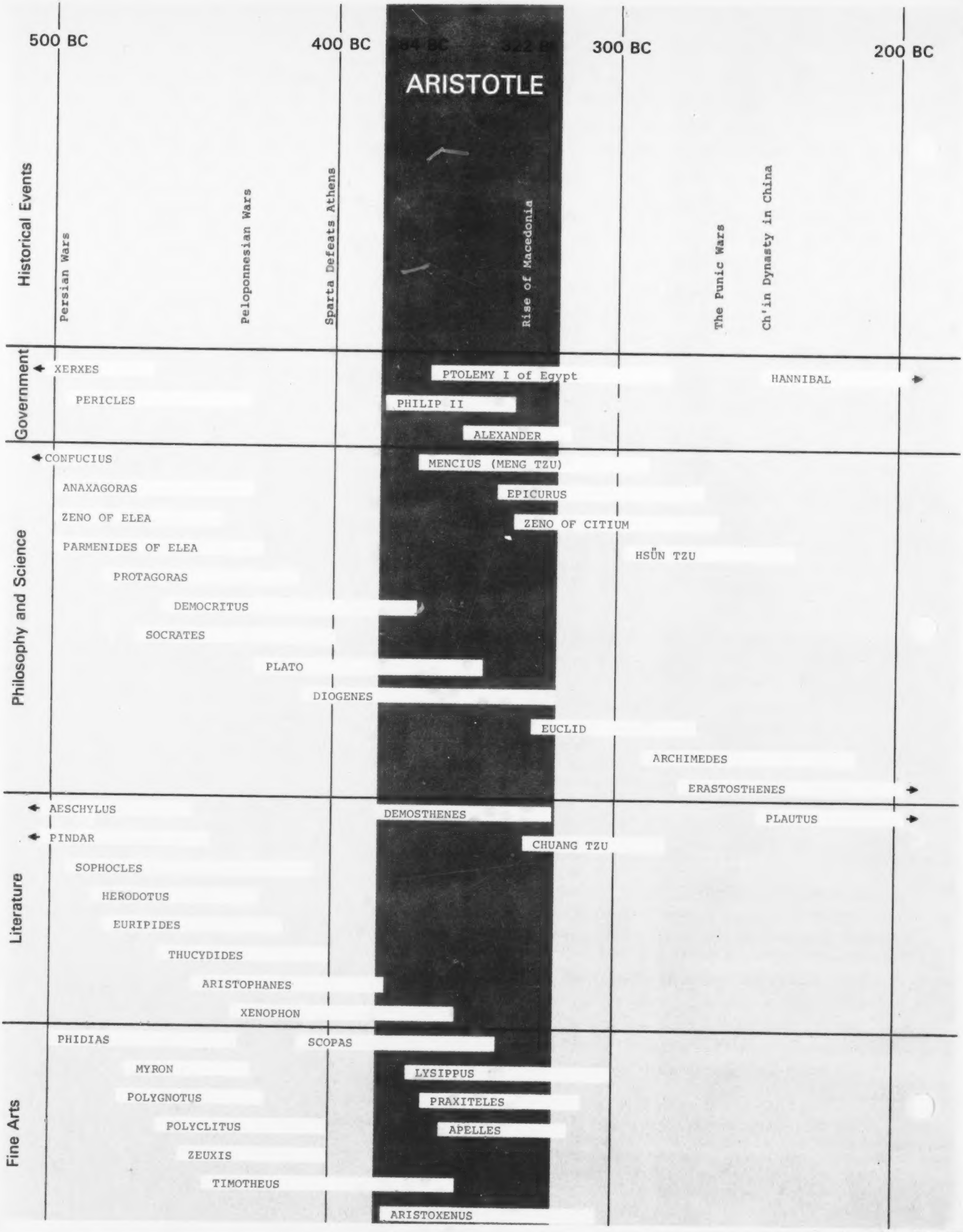
D5 Comparative fall rates of light and heavy objects

A When is air resistance important?

From quinta essentia meaning fifth essence. In earlier Greek writings the term for it was ether.

E.g., a rock falling freely to the earth is in natural motion because the rock is moving toward its natural place.

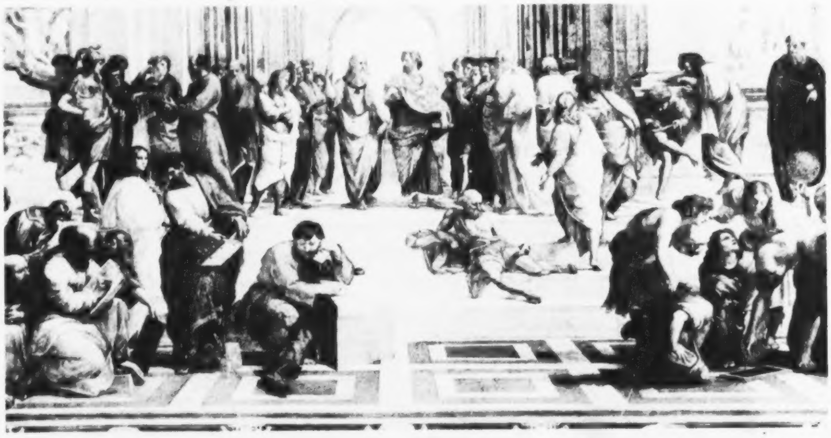
This theory, so widely held in Galileo's time, originated in the fourth century B.C.; we find it mainly in the writings


500 BC
400 BC
384 BC
322 BC
300 BC
200 BC
ARISTOTLE
Historical Events
Persian Wars
Peloponnesian Wars
Sparta Defeats Athens
Rise of Macedonia
The Punic Wars
Ch'in Dynasty in China
Government
XERXES
PTOLEMY I of Egypt
HANNIBAL
PERICLES
PHILIP II
ALEXANDER
Philosophy and Science
CONFUCIUS
MENCIUS (MENG TZU)
ANAXAGORAS
EPICURUS
ZENO OF ELEA
ZENO OF CITIUM
PARMENIDES OF ELEA
HSÜN TZU
PROTAGORAS
DEMOCRITUS
SOCRATES
PLATO
DIOGENES
EUCLID
ARCHIMEDES
ERASTOSTHENES
Literature
AESCHYLUS
DEMOSTHENES
PLAUTUS
PINDAR
CHUANG TZU
SOPHOCLES
HERODOTUS
EURIPIDES
THUCYDIDES
ARISTOPHANES
XENOPHON
Fine Arts
PHIDIAS
SCOPAS
MYRON
LYSIPPUS
POLYGNOTUS
PRAXITELES
POLYCLITUS
APELLES
ZEUXIS
TIMOTHEUS
ARISTOXENUS

of the Greek philosopher Aristotle. A physical science of order, rank and place, it fits well many facts of everyday observation. Moreover, these conceptions of matter and motion were part of an all-embracing scheme or "cosmology" by which Aristotle sought to relate ideas which are nowadays discussed separately under the headings of science, poetry, politics, ethics and theology.

Aristotle was born in 384 B.C. in Stageira, a city in the Greek province of Macedonia. His father was the physician to Amyntas II, the king of Macedonia, and so Aristotle's early childhood was spent in an environment of court life. At the age of 17 he was sent to Athens to complete his education. He spent 20 years there, first as a student and then as a colleague of Plato. When Plato died, Aristotle left Athens and later returned to Macedonia to become the private tutor of Alexander the Great (356-323 B.C.). In 335 B.C., Aristotle came back to Athens and founded the Lyceum, a school and center of research. Little is known of his physical appearance and little biographical information has survived. Fortunately, 50 volumes of his writings (out of perhaps 400 in all) did survive. These works of Aristotle remained unknown in Western Europe for 1500 years after the decline of the ancient Greek civilization, until they were rediscovered in the thirteenth century A.D. and incorporated into Christian theology. Aristotle became such a dominant influence in the late Middle Ages that he was referred to simply as "The Philosopher."

Because of his habit of lecturing in the walking place (peripatos, in Greek) of the Lyceum, Aristotle's company of philosophers came to be known as the "Peripatetics."

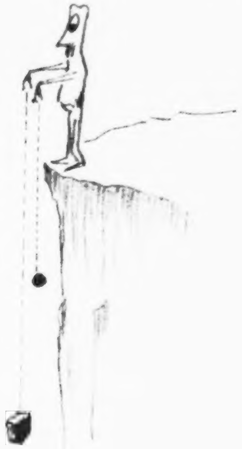


This painting titled "School of Athens" was done by Raphael in the beginning of the sixteenth century. The painting clearly reflects one aspect of the Renaissance, a rebirth of interest in classical Greek culture. The central figures are Plato (on left) and Aristotle. Raphael used Leonardo da Vinci as his model for Plato.

The works of Aristotle constitute an encyclopedia of ancient Greek thought—some of it summarized from the work of others, but much of it created by Aristotle himself. Today it seems incredible that one man could have written so intelligently and knowledgeably on such different subjects as logic, philosophy, theology, physics, astronomy, biology, psychology, politics and literature. Some scholars doubt it was all the work of one man.

Unfortunately, Aristotle's physical theories had limitations which became evident much later, and we will devote part of this chapter to showing where these limitations lie in some specific cases. But this should not detract from Aristotle's great achievements in other fields.

Perhaps in 3500 AD Einstein's views may seem a little naive.



Aristotle: rate of fall is proportional to weight divided by resistance.

According to Aristotle, the fall of a heavy object toward the center of the earth is a natural motion. What factors determine the rate of fall? A rock falls faster than a leaf; therefore, he reasoned, weight must be a factor. An object falls faster in air than in water, so the resistance of the medium must also be a factor. Other factors, such as the color and temperature of the object, could conceivably affect the rate of fall, but to Aristotle these were evidently of little importance. He assumed that the rate of fall must therefore increase in proportion to the weight of the object, and decrease in proportion to the resisting force of the medium. The actual rate of fall in any particular case would be determined by dividing the weight by the resistance. In his book On the Heavens, Aristotle makes the following statement about natural motion (such as falling):

> A given weight moves a given distance in a given time; a weight which is heavier moves the same distance in less time, the time being inversely proportional to the weights. For instance, if one weight is twice another, it will take half as long over a given distance.

Aristotle also discussed "violent" motion—that is, motion of an object which is not toward its natural place. Such motion, he argued, must always be caused by a force, and the speed of the motion will increase as the force increases. When the force is removed, the motion must stop. This theory agrees with our common experience in pushing desks or tables across the floor. It doesn't seem to work quite so well for objects thrown through the air, since they keep moving for a while even after we have stopped exerting a force on them. To account for this kind of motion, Aristotle assumed that the air itself somehow exerts a force that continues to propel an object moving through it.

See Study Guide 2.13.

Later scientists proposed some modifications in Aristotle's theory of motion. For example, John Philoponus of Alexandria, in the fifth century A.D., argued that the speed of an object in natural motion should be found by subtracting the resistance of the medium from the weight of the object, rather than dividing by the resistance. Philoponus claimed that he had actually done experiments to support his theory, though he did not report all the details; he simply said that he dropped two weights, one of which was twice as heavy as the other, and observed that the heavy one did not reach the ground in half the time taken by the light one.

John Philoponus: rate of fall is proportional to weight minus resistance.

There were still other difficulties with Aristotle's theory of motion. However, the realization that his teachings concerning motion had their limitations did little to modify the important position given to them in the universities of France and Italy during the fifteenth and sixteenth centuries. In any case, the study of motion through space was of major interest to only a few scholars and, indeed, it had been only a very small part of Aristotle's own work. Nevertheless, Aristotle's theory of motion fitted much of human experience in a general—if qualitative—way.

Two further influences stood in the way of radical changes in the theory of motion. First, Aristotle had believed that mathematics was of little value in describing change.• Second, he had put great emphasis upon qualitative observation as the basis for all theorizing. Simple qualitative observation was very successful in Aristotle's biological studies. But progress in physics began only when careful measurements were made under controlled conditions.

•Greeks did not know algebra - they knew only geometry (and even that without numbers). Since geometric proofs of even the simplest algebraic propositions are very complicated, we should not be surprized at Aristotle's prejudice against mathematics. The mathematics that would have made it possible to deal with quantitative observations was not developed until the fifth to tenth centuries A.D. It was not actually used extensively in science until the seventeenth century.

It would not be at all rash to suggest that when, over 19 centuries after Aristotle, Galileo turned his eyes away from all the complicated motions of things in the outside world and fixed them on the curiously artificial motion of a polished brass ball rolling down an inclined plane, his eyes made one of the most important turns in history. And when he succeeded in describing the motion of that ball mathematically he not only paved the way for other men to describe and explain the motions of everything from planets to pebbles but did in fact begin the intellectual revolution which led to what we now call modern science.

Devmo et Affmo Serre
Galileo Galilej

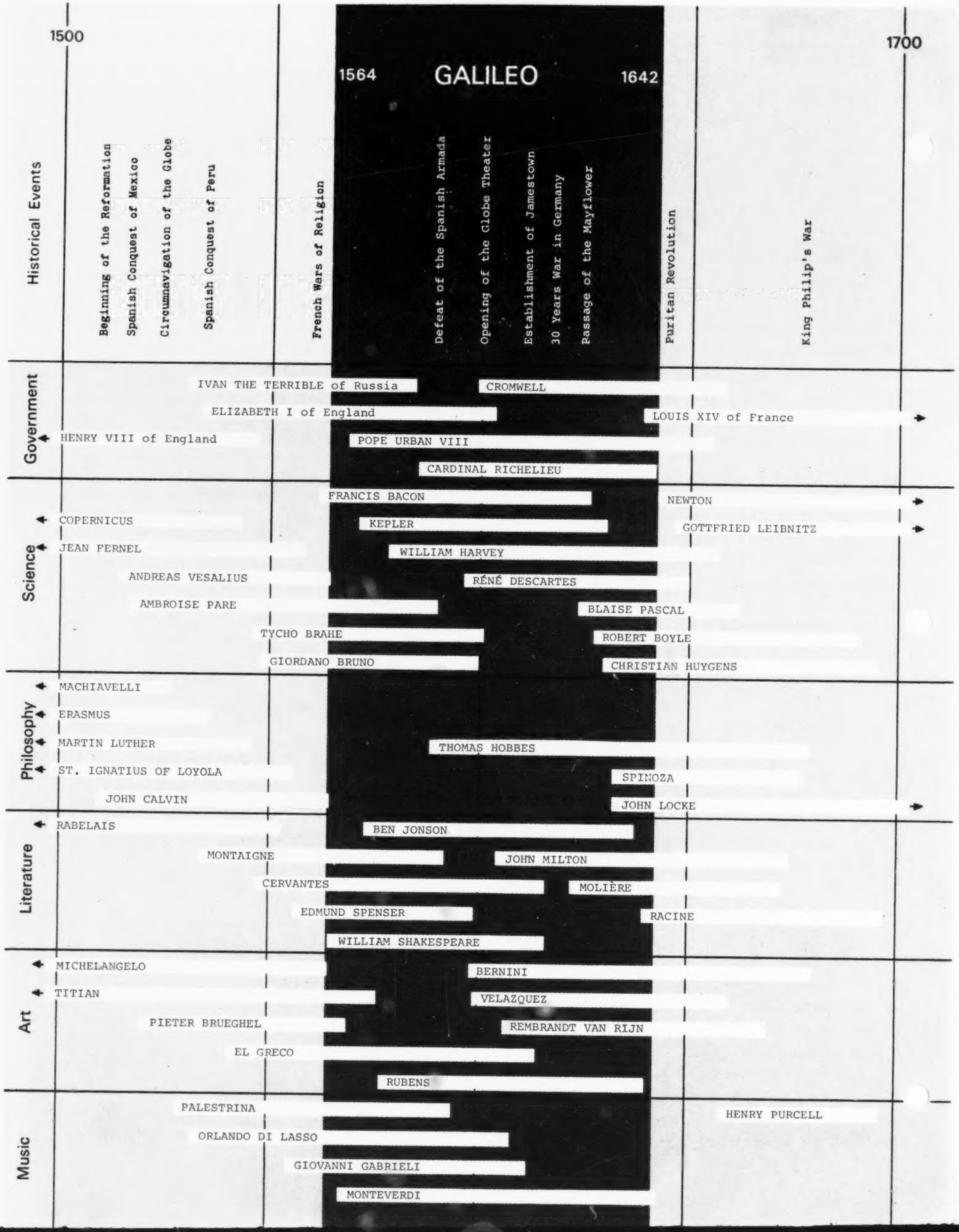
2.2 Galileo and his times. The new developments in both physics and astronomy came to focus in the writings of Galileo Galilei. This great scientist was born at Pisa in 1564—the year of Michelangelo's death and Shakespeare's birth. Galileo was the son of a nobleman from Florence and he acquired his father's active interest in poetry, music, and the classics. His scientific inventiveness also began to show itself early. For example, as a young medical student at the University of Pisa, he constructed a simple pendulum-type timing device for the accurate measurement of pulse rates.

Republic of Venice
Venice
Padua
Pisa
Florence
Province of Tuscany
Rome

Map of Italy at the time of Galileo.

Lured from medicine to physical science by reading Euclid and Archimedes, Galileo quickly became known for his unusual ability. At the age of 26, he was appointed Professor of Mathematics at Pisa. There he showed an independence of spirit unmellowed by tact or patience. Soon after his appointment, he began to challenge the opinions of his older

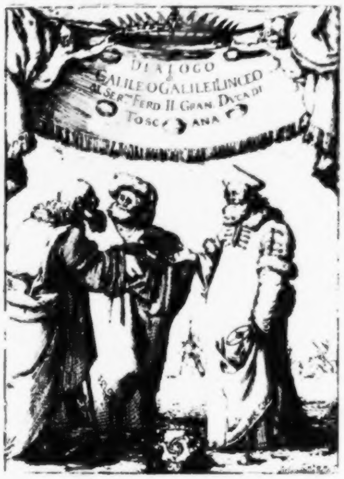
Summary 2.2 1. Galileo lived a long, thoughtful and productive life. 2. His great overall contribution to science (in addition to his direct scientific discoveries) was in establishing mathematical and quantitative procedures as a valid part of science.


1500
1700
1564
GALILEO
1642
Historical Events
Beginning of the Reformation
Spanish Conquest of Mexico
Circumnavigation of the Globe
Spanish Conquest of Peru
French Wars of Religion
Defeat of the Spanish Armada
Opening of the Globe Theater
Establishment of Jamestown
30 Years War in Germany
Passage of the Mayflower
Puritan Revolution
King Philip's War
Government
IVAN THE TERRIBLE of Russia
CROMWELL
ELIZABETH I of England
LOUIS XIV of France
HENRY VIII of England
POPE URBAN VIII
CARDINAL RICHELIEU
Science
FRANCIS BACON
NEWTON
COPERNICUS
KEPLER
GOTTFRIED LEIBNITZ
JEAN FERNEL
WILLIAM HARVEY
ANDREAS VESALIUS
RÉNÉ DESCARTES
AMBROISE PARE
BLAISE PASCAL
TYCHO BRAHE
ROBERT BOYLE
GIORDANO BRUNO
CHRISTIAN HUYGENS
Philosophy
MACHIAVELLI
ERASMUS
MARTIN LUTHER
THOMAS HOBBES
ST. IGNATIUS OF LOYOLA
SPINOZA
JOHN CALVIN
JOHN LOCKE
Literature
RABELAIS
BEN JONSON
MONTAIGNE
JOHN MILTON
CERVANTES
MOLIÈRE
EDMUND SPENSER
RACINE
WILLIAM SHAKESPEARE
Art
MICHELANGELO
BERNINI
TITIAN
VELAZQUEZ
PIETER BRUEGHEL
REMBRANDT VAN RIJN
EL GRECO
RUBENS
Music
PALESTRINA
HENRY PURCELL
ORLANDO DI LASSO
GIOVANNI GABRIELI
MONTEVERDI

colleagues, many of whom became his enemies. Indeed, he left Pisa before his term was completed, apparently forced out by financial difficulties and by his enraged opponents. Later, at Padua in the Republic of Venice, he began his work in astronomy. His support of the sun-centered theory of the universe eventually brought him additional enemies, but it also brought him immortal fame. You will read more about this in Unit 2.

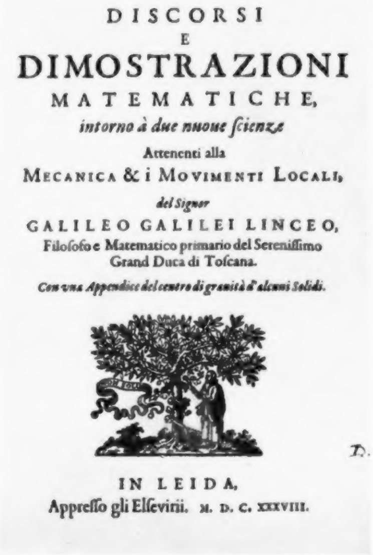
Drawn back to his native province of Tuscany in 1610 by a generous offer of the Grand Duke, Galileo became the Court Mathematician and Philosopher, a title which he chose himself. From then until his death at 78 in 1642, he produced much of his excellent work. Despite illness, family troubles, occasional brushes with poverty, and quarrels with his enemies, he continued his research, teaching and writing.

Galileo gave us a new mathematical orientation toward the natural world. His philosophy of science had its roots in the ancient Greek tradition of Pythagoras, Plato and Archimedes, but it was in conflict with the qualitative approach characteristic of Aristotle. Unlike most of his predecessors, however, Galileo respected the test of truth provided by quantitative observation and experiment.



Title page of Dialogue on Two Great World Systems (1632).

2.3 Galileo's "Two New Sciences." Galileo's early writings on mechanics (the study of the behavior of matter under the influence of forces)were in the tradition of the standard medieval theories of physics. Although he was keenly aware of the short-comings of those theories, his chief interest during his mature years was in astronomy. However, when his important astronomical work, Dialogue on the Two Great World Systems (1632), was condemned by the Roman Catholic Inquisition and he was forbidden to teach the "new" astronomy, Galileo decided to concentrate on mechanics. This work led to his book, Discourses and Mathematical Demonstrations Concerning Two New Sciences Pertaining to Mechanics and Local Motion, usually referred to as the Two New Sciences. The new approach to the science of motion described in the Two New Sciences signaled the beginning of the end not only of the medieval theory of mechanics, but also of the entire Aristotelian cosmology.

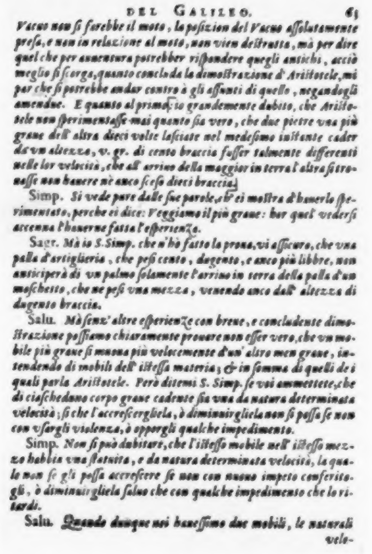

DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue ſcienze
Attenenti alla
MECANICA & i MOVIMENTI LOCALI,
del Signor
GALILEO GALILEI LINCEO,
Filoſofo e Matematico primario del Sereniſſimo
Grand Duca di Toſcana.
Con vna Appendice del centro di granità d'alcuni Solidi.

IN LEIDA,
Appreſſo gli Elſevirii. M. D. C. XXXVIII.

Title page of Discourses and Mathematical Demonstrations Concerning Two New Sciences Pertaining to Mechanics and Local Motion (1638).

Galileo was old, sick and nearly blind at the time he wrote Two New Sciences, yet his style in it is spritely and delightful. He used the dialogue form to allow a lively conversation between three "speakers": Simplicio, who rep-

Summary 2.3 1. Galileo presented his theories of motion in the book Two New Sciences. In this work he set out his own views in opposition to those of Aristotelians. 2. In interpreting observations care must be exercised to separate significant results from trivial ones. (As has been said, however, one man's trivial errors are another man's Nobel Prize.)



DEL GALILEO. 65

Vacuo non ſi farebbe il moto, la poſizion del Vacuo aſſolutamente preſa, e non in relazione al moto, non vien deſtrutta, mà per dire quel che per auuentura potrebber riſpondere quegli antichi, acciò meglio ſi ſcorga, quanto concluda la dimoſtrazione d' Ariſtotele, mi par che ſi potrebbe andar contro à gli aſſunti di quello, negandogli amendue. E quanto al primo, io grandemente dubito, che Ariſtotele non ſperimentaſſe mai quanto ſia vero, che due pietre una più graue dell' altra dieci volte laſciate nel medeſimo inſtante cader da vn altezza, v. gr. di cento braccia fuſſer talmente differenti nelle lor velocità, che all' arriuo della maggior in terra l' altra ſi trouaſſe non hauere nè anco ſceſo dieci braccia.

Simp. Si vede pure dalle ſue parole, ch' ei moſtra d' hauerlo ſperimentato, perche ei dice: Veggiamo il più graue: hor quel vederſi accenna l' hauerne fatta l' eſperienza.

Sagr. Mà io S. Simp. che n' hò fatto la proua, vi aſſicuro, che vna palla d' artiglieria, che peſi cento, dugento, e anco più libbre, non anticiperà di vn palmo ſolamente l' arriuo in terra della palla d' un moſchetto, che ne peſi vna mezza, venendo anco dall' altezza di dugento braccia.

Salu. Mà ſenz' altre eſperienze con breue, e concludente dimoſtrazione poſſiamo chiaramente prouare non eſſer vero, che vn mobile più graue ſi muoua più velocemente d' un' altro men graue, intendendo di mobili dell' iſteſſa materia; & in ſomma di quelli de i quali parla Ariſtotele. Però ditemi S. Simp. ſe voi ammettete, che di ciaſchedun corpo graue cadente ſia vna da natura determinata velocità; ſi che l' accreſcergliela, ò diminuirgliela non ſi poſſa ſe non con vſargli violenza, ò opporgli qualche impedimento.

Simp. Non ſi può dubitare, che l' iſteſſo mobile nell' iſteſſo mezzo habbia vna ſtatuita, e da natura determinata velocità, la quale non ſe gli poſſa accreſcere ſe non con nuouo impeto conferitogli, ò diminuirgliela ſaluo che con qualche impedimento che lo ritardi.

Salu. Quando dunque noi haueſſimo due mobili, le naturali

vele-

A page from the original Italian edition of the Two New Sciences, showing Salviati's statement about Aristotle (see translation in text).

D6 Coin and Feather

resents the Aristotelian view; Salviati, who presents the new views of Galileo; and Sagredo, the uncommitted man of good will and open mind, eager to learn. To no one's surprise, Salviati leads his companions to Galileo's views. Let us listen to Galileo's three speakers as they discuss the problem of free fall:

> Salviati: I greatly doubt that Aristotle ever tested by experiment whether it is true that two stones, one weighing ten times as much as the other, if allowed to fall at the same instant, from a height of, say, 100 cubits, would so differ in speed that when the heavier had reached the ground, the other would not have fallen more than 10 cubits. [A "cubit" is equivalent to about 20 inches.]
>
> Simplicio: His language would seem to indicate that he had tried the experiment, because he says: We see the heavier; now the word see shows that he had made the experiment.
>
> Sagredo: But, I, Simplicio, who have made the test can assure you that a cannon ball weighing one or two hundred pounds, or even more, will not reach the ground by as much as a span ahead of a musket ball weighing only half a pound, provided both are dropped from a height of 200 cubits.

Here, perhaps, one might have expected to find a detailed report on an experiment done by Galileo or one of his colleagues. Instead, Galileo presents us with a "thought experiment"—an analysis of what would happen in an imaginary experiment, in which Galileo ironically uses Aristotle's own method of logical reasoning to attack Aristotle's theory of motion:

> Salviati: But, even without further experiment, it is possible to prove clearly, by means of a short and conclusive argument, that a heavier body does not move more rapidly than a lighter one provided both bodies are of the same material and in short such as those mentioned by Aristotle. But tell me, Simplicio, whether you admit that each falling body acquires a definite speed fixed by nature, a velocity which cannot be increased or diminished except by the use of violence or resistance?
>
> Simplicio: There can be no doubt but that one and the same body moving in a single medium has a fixed velocity which is determined by nature and which cannot be increased except by the addition of impetus or diminished except by some resistance which retards it.•

•Salviati and Simplicio both agree on the premise.

> Salviati: If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will be partly retarded by the slower, and the slower will be somewhat hastened by the swifter. Do you not agree with me in this opinion?
>
> Simplicio: You are unquestionably right.•

•Also they agree on the consequences of the premise.

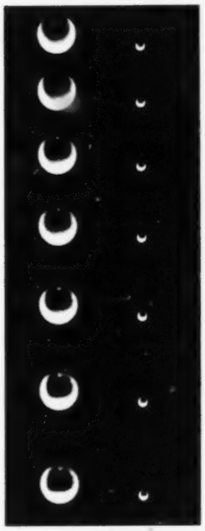
> Salviati: But if this is true, and if a large stone moves with a speed of, say, eight while a smaller moves with a speed of four, then when they are united, the system will move with a speed less than eight, but the two stones when tied together make a stone larger than that which before moved with a speed of eight. Hence the heavier body moves with less speed than the lighter; an effect which is contrary to your supposition. Thus you see how, from your assumption that the heavier body moves more rapidly than the lighter one, I infer that the heavier body moves more slowly.
>
> Simplicio: I am all at sea....This is, indeed, quite beyond my comprehension....

Galileo's argument, voiced here by Salviati, is splendid. Salviati shows that a consequence of the premise is illogical. Help your students follow the line of argument.

See Study Guide 2.5 and 2.14.

As Simplicio retreats in confusion, Salviati presses forward with the argument, showing that it is self-contradictory to assume that an object would fall faster if its weight were increased by a small amount. Simplicio cannot refute Galileo's logic, but on the other hand his own eyes tell him that a heavy object does fall faster than a light object:

> Simplicio: Your discussion is really admirable; yet I do not find it easy to believe that a bird-shot falls as swiftly as a cannon ball.
>
> Salviati: Why not say a grain of sand as rapidly as a grindstone? But, Simplicio, I trust you will not follow the example of many others who divert the discussion from its main intent and fasten upon some statement of mine that lacks a hairsbreadth of the truth, and under this hair hide the fault of another that is as big as a ship's cable. Aristotle says that "an iron ball of one hundred pounds falling from a height of 100 cubits reaches the ground before a one-pound ball has fallen a single cubit." I say that they arrive at the same time. You find, on making the experiment, that the larger outstrips the smaller by two fingerbreadths....Now you would not hide behind these two fingers the 99 cubits of Aristotle, nor would you mention my small error and at the same time pass over in silence his very large one.



A stroboscopic photograph of two freely falling balls of unequal weight. The balls were released simultaneously. The time interval between images is 1/30 sec.

This is a clear statement of an important principle: in careful observation of a common natural event the observer's attention may be distracted from a fundamental regularity unless he considers the possibility that small, separately explainable, variations will be associated with the event. Different bodies falling in air from the same height do not reach the ground at exactly the same time. However, the important point is not that the times of arrival are slightly different, but that they are very nearly the same! The failure of the bodies to arrive at exactly the same time is seen to be a minor matter which can be explained by a deeper understanding of motion in free fall. Galileo himself attributed the observed results to the resistance of the air. A few years after Galileo's death, the invention of the air pump

American Journal of Physics
June '64 "Fall of bodies near the earth" (equation for "drag" on a sphere).

allowed others to show that Galileo was right. When a feather and a heavy gold coin are dropped from the same height at the same time inside an evacuated container, they fall at the same rate and strike the bottom of the container at the same instant.

Another argument against the possibility of a vacuum could be deduced from Aristotle's theory: if the rate of fall is equal to the weight divided by the resistance and the resistance of a vacuum is zero, then the rate of fall of all bodies must be infinite in a vacuum. But that is absurd. Hence, a vacuum is impossible!

We might say that learning what to ignore has been almost as important in the growth of science as learning what to take into account. In this particular case, Galileo's explanation depended on his being able to imagine how an object would fall if there were no air resistance. This may be easy for us who know of vacuum pumps. But in Galileo's time it was an explanation unlikely to be accepted because of the basic beliefs held by most educated people. For them, as for Aristotle, common sense said that air resistance is always present in nature. Thus, a feather and a coin could never fall at the same rate. Why should one talk about hypothetical motions in a vacuum, when a vacuum does not exist? Physics, said Aristotle and his followers, should describe the real world as we observe it, not some imaginary world which can never be found. Aristotle's physics had dominated Europe since the thirteenth century, not merely because of the authority of the Catholic Church, as is sometimes said, but also because many intelligent scientists were convinced that it offered the most rational method for describing natural phenomena. To overthrow such a firmly established doctrine required much more than writing reasonable arguments or simply dropping heavy and light objects from a tall building, as Galileo is supposed to have done in his legendary experiment on the Leaning Tower of Pisa. It demanded Galileo's unusual combination of mathematical talent, experimental skill, literary style, and tireless campaigning to defeat Aristotle's theories and to get on the path to modern physics.

By Aristotelian cosmology is meant the whole interlocking set of ideas about the structure of the physical universe and the behavior of all the objects in it. This was briefly and incompletely outlined in Sec. 2.1. Other aspects of it will be presented in Unit 2.

2.4 Why study the motion of freely falling bodies? To attack the Aristotelian cosmology, Galileo gathered concepts, methods of calculation, and techniques of measurement in order to describe the motion of objects in a rigorous, mathematical form. Few details of his work were actually new, but together his findings provided the first coherent presentation of the science of motion. He realized that free-fall motion, now seemingly so trite, was the key to the understanding of all motions of all bodies.

Galileo also provides an example of a superb scientist. He was an investigator whose skill in discovery and eloquence in argument produced a deep and lasting impression on his listeners. His approach to the problems of motion will

Summary 2.4
Galileo's study of free fall is worth careful examination because it presents a case study of an outstanding early scientist using what was to develop into the modern mode of scientific thought and action.

provide us with an opportunity for discussion of strategies of inquiry that are used in science. We shall see a new mode of scientific reasoning emerge, to become, eventually, an accepted pattern for scientific thought.

These are the reasons why we study in detail Galileo's attack on the problem of free fall. But perhaps Galileo himself should tell us why he studied motion:

> My purpose is to set forth a very new science dealing with a very ancient subject. There is, in nature, perhaps nothing older than motion, concerning which the books written by philosophers are neither few nor small; nevertheless, I have discovered some properties of it that are worth knowing and that have not hitherto been either observed or demonstrated. Some superficial observations have been made, as, for instance, that the natural motion of a heavy falling body is continuously accelerated; but to just what extent this acceleration occurs has not yet been announced....
>
> Other facts, not few in number or less worth knowing I have succeeded in proving; and, what I consider more important, there have been opened up to this vast and most excellent science, of which my work is merely the beginning, ways and means by which other minds more acute than mine will explore its remote corners.

He was wrong in this: more than mere "superficial observations" had been made long before Galileo set to work. For example, Nicolas Oresme and others at the University of Paris had by 1330 discovered the same distance-time relationship for falling bodies that Galileo was to announce with a flourish in the Two New Sciences.

2.5 Galileo chooses a definition of uniform acceleration. In studying the following excerpts from the Two New Sciences, which deal directly with the motion of freely falling bodies, we must be alert to his overall plan. First, Galileo discusses the mathematics of a possible, simple type of motion, namely, motion with uniform acceleration. Then he assumes that this is the type of motion that a heavy body undergoes during free fall. This assumption is his main hypothesis about free fall. Third, he deduces from this hypothesis some predictions that can be tested experimentally. Finally, he shows that these tests do indeed bear out the predictions.

Galileo is saying that a definition should correspond to what is observed in nature.

It will help you to have this plan clearly in mind as you progress through the rest of this chapter. As you study each succeeding section, ask yourself whether Galileo is
- presenting a definition
- stating an assumption
- deducing predictions from his hypothesis
- experimentally testing the predictions.

In the first part of Galileo's presentation there is a thorough discussion of motion with uniform speed similar to the one in our Chapter 1. The second part concerns "uniformly accelerated motion":

> We pass now to...naturally accelerated motion, such as that generally experienced by heavy falling bodies....
>
> And first of all it seems desirable to find and explain a definition best fitting natural phenomena. For anyone may invent an arbitrary type of motion and discuss its properties...we have decided to consider the phenomena of bodies falling with an acceleration such as actually occurs in nature and to make this definition of accelerated motion exhibit the essential features of observed accelerated motions.

Summary 2.5

1. Galileo arbitrarily defined uniform acceleration as motion in which the speed of a moving body increases by equal amounts in equal times.

2. He then claimed that this definition fits natural phenomena, that in fact objects falling freely at the earth's surface accelerate in accordance with that definition.

3. Accurate description (kinematics) must come before a search for causes (dynamics).

This is sometimes known as the rule of parsimony: unless you know otherwise, assume the simplest possible hypothesis to explain natural events.

> Finally, in the investigation of naturally accelerated motion we were led, by hand as it were, in following the habit and custom of nature herself, in all her various other processes, to employ only those means which are most common, simple and easy....
>
> When, therefore, I observe a stone initially at rest falling from an elevated position and continually acquiring new increments of speed, why should I not believe that such increases take place in a manner which is exceedingly simple and rather obvious to everybody? If now we examine the matter carefully we find no addition or increment more simple than that which repeats itself always in the same manner. This we readily understand when we consider the intimate relationship between time and motion; for just as uniformity of motion is defined by and conceived through equal times and equal spaces (thus we call a motion uniform when equal distances are traversed during equal time-intervals), so also we may, in a similar manner, through equal time-intervals, conceive additions of speed as taking place without complication....

Galileo is saying that just as we have defined uniform speed so that (to use our symbols, not his):

$$v = \frac{\Delta d}{\Delta t},$$

let us also define uniform acceleration so that:

$$a = \frac{\Delta v}{\Delta t}.$$

This is the same definition we used in Chapter 1. Since Galileo always deals with the case of objects falling from rest, this can be written in the form

$$a = \frac{v}{t}.$$

> Hence the definition of motion which we are about to discuss may be stated as follows:
>
> A motion is said to be uniformly accelerated, when starting from rest, it acquires during equal time-intervals, equal increments of speed.
>
> Sagredo: Although I can offer no rational objection to this or indeed to any other definition devised by any author whosoever, since all definitions are arbitrary, I may nevertheless without defense be allowed to doubt whether such a definition as the foregoing, established in an abstract manner, corresponds to and describes that kind of accelerated motion which we meet in nature in the case of freely falling bodies....

Here Sagredo, the challenger, questions whether Galileo's arbitrary definition of acceleration actually corresponds to the way real objects fall. Is acceleration, as defined, useful in describing their change of motion? Sagredo tries to divert the conversation:

> From these considerations perhaps we can obtain an answer to a question that has been argued by philosophers, namely, what is the cause of the acceleration of the natural motion of heavy bodies....

Salviati, the spokesman of Galileo, sternly turns away from this ancient concern for causes. It is premature, he declares, to ask about the cause of any motion until an accurate description of it exists: •

•Quoted at the beginning of Chapter 3.

Here Salviati refers to Aristotle's assumption that air propels an object moving through it (Sec. 2.1).

> Salviati: The present does not seem to be the proper time to investigate the cause of the acceleration of natural motion concerning which various opinions have been expressed by philosophers, some explaining it by attraction to the center, others by repulsion between the very small parts of the body, while still others attribute it to a certain stress in the surrounding medium which closes in behind the falling body and drives it from one of its positions to another. Now, all these fantasies, and others, too, ought to be examined; but

> it is not really worth while. At present it is the purpose of our Author* merely to investigate and to demonstrate some of the properties of accelerated motion, whatever the cause of this acceleration may be.

*Galileo himself

T5: Uniform velocity vs. uniform acceleration.

Galileo has now introduced two distinct suggestions, which we must take up in turn. 1) "Uniform acceleration" means equal increases in speed Δv in equal times Δt; and 2) things actually fall that way. Let us first look more closely at Galileo's proposed definition.

Is this the only possible way of defining acceleration? Is it obviously right? Not at all! As Galileo goes on to admit, he once believed that in uniform acceleration the speed increased in proportion to the distance traveled, Δd, rather than to the time Δt. In fact, both definitions had been discussed since early in the fourteenth century, and both met Galileo's first command: assume a simple relationship among the physical quantities concerned. Furthermore, both definitions seem to match our commonsense idea of acceleration.* For example, when we say that a body is "accelerating," we seem to imply "the farther it goes, the faster it goes," as well as "the longer it keeps moving, the faster it goes." And what, you might ask, is there to choose between these two ways of putting it?

*Later you might come back to this and ask your students to discuss the consequences of a definition where acceleration is $\frac{\Delta v}{\Delta d}$.

Acceleration could be defined either way. But which definition can be found useful in a description of nature? This is where experimentation is important. Galileo defined uniform acceleration so that change of speed is proportional to elapsed time,▲ and this definition led to fruitful consequences. Other scientists chose to define acceleration so that speed is proportional to distance traversed. Galileo's definition turned out to be the most useful so it was brought into the language of physics.

▲Before you proceed to Sec. 2.6 ask your students how they would test the definition of acceleration as adopted by Galileo.

2.6 Galileo cannot test his hypothesis directly. Galileo defined uniform acceleration so that it would match the way he believed freely falling objects behaved. The next task for Galileo was to show that the definition for uniform acceleration ($a = v/t =$ constant) was useful for describing observed facts.

This was not as easy as it seems. Suppose we drop a heavy object from several different heights—say, from windows on different floors of a building. In each case we observe the time of fall t and the speed v just before the object strikes the ground. Unfortunately, it would be very difficult to make direct measurements of the speed v just before striking the ground. Furthermore, the times of fall

Summary 2.6
Because velocity and time could not be measured accurately, Galileo could not directly test his hypothesis that in free fall from rest v/t is constant.

Summary 2.7 1. From $a = \frac{v}{t}$, Galileo derived the equivalent relationship $s = \frac{at^2}{2}$. 2. This allowed him to restate his hypothesis: for a body falling freely from the rest, s/t^2 is constant. 3. He still could not test this directly because of the short time intervals involved in free fall.

are smaller (less than 3 sec even from the top of a 10-story building) than Galileo could have measured accurately with the clocks available.

T6: Derivation of $d = v_0 t + \frac{1}{2} at^2$

A: Measuring your reaction time

2.7 Looking for logical consequences of Galileo's hypothesis.

The inability to make direct measurements to test his hypothesis that v/t is constant did not stop Galileo. He turned to mathematics to derive some other relationship that could be measured with the equipment available to him.

He wanted to answer the question: for an object moving with uniform acceleration what is the relationship between the distance traveled and the time elapsed?

Distance, of course, is easily determined, so Galileo set out to derive an equation for acceleration expressed in terms of distance and time rather than speed and time. We shall derive such an equation by using relationships familiar to us, rather than by following Galileo's derivation exactly. First, we recall the definition of average speed as the distance traversed divided by the elapsed time. In symbols we write

$$v_{av} = \frac{d}{t} .$$

This is a general definition and can be used to compute the average speed for any moving object.

As before,

$v_{initial}$ = initial speed

v_{final} = final speed

v_{av} = average speed.

For the special case of an object moving with uniform acceleration, we can express the average speed in another way—in terms of initial and final speed:

★ $$v_{av} = \frac{v_{initial} + v_{final}}{2} .$$

If this uniformly accelerating object starts from rest, that is $v_{initial} = 0$, we can write

★ $$v_{av} = \frac{v_{final}}{2} = \frac{1}{2} v_{final} .$$

See Study Guide 2.6.

In words we would say the average speed of any object starting from rest and accelerating uniformly is one-half the final speed.

We now have two equations which can be applied to the special case of uniformly accelerated motion. Since the average speed is given by both of these equations, we can eliminate v_{av}. Thus,

★ $$v_{av} = \frac{d}{t} \text{ or } d = v_{av}\, t.$$

So, substituting $\frac{1}{2} v_{final}$ for v_{av} we have

★ $$d = \frac{1}{2} v_{final}\, t.$$

Does this equation hold for cases of uniform acceleration only?

★ Emphasize that all these equations are for the special case of uniform acceleration: also, all except the first are for the special case of $v_{initial} = 0$.

We now have to take a final step. Somehow we need to get acceleration into the equation and speed out of it. Our starting place was:

$$★ \quad a = \frac{v_{final}}{t}$$

which, when we solve for v_{final}, becomes

$$★ \quad v_{final} = at.$$

If we now combine this with

$$d = \tfrac{1}{2} v_{final}\, t$$

we get

$$★ \quad d = \tfrac{1}{2}(at)t$$

or

$$d = \tfrac{1}{2}at^2.$$

• It was graphic rather than algebraic

What is the unwritten text behind this equation?

The unwritten text includes the implicit assumptions that were made in the derivation on p. 50.
1) a is constant
2) $v_0 = 0$
3) $d_0 = 0$ } at t_0

Galileo's own derivation was somewhat different from this. However, he reached the same conclusion: in uniformly accelerated motion the distance traveled in any time by an object starting from rest is equal to one-half the acceleration times the square of the time. Since we are dealing only with the special case in which acceleration is uniform and ½a is constant, we can state the conclusion as a proportion: in uniform acceleration the distance traveled is proportional to the square of the time elapsed. For example, if a uniformly accelerating cart moves 3 m in 2 sec, it would move 12 m in 4 sec.

Now let us see where we are with reference to Galileo's problem. Using the three expressions $a = \frac{v_{final}}{t}$, $v_{av} = \frac{v_{final}}{2}$ and $d = v_{av}t$, we found that $d = \tfrac{1}{2}at^2$. This simple relation, derived from Galileo's definition of acceleration, is the key to an experimental test which he proposed. The relation can be put into a form of more direct interest if we divide it by t^2:

$$\frac{d}{t^2} = \tfrac{1}{2}a\ .$$

Thus a logical result of the original definition of uniform acceleration is: whenever a is constant, the ratio, d/t^2, is constant. Therefore, any motion in which this ratio is constant for different distances and times must be a case of uniform acceleration as defined by Galileo. Of course, it was his hypothesis that freely falling bodies exhibited just such motion.

Galileo's hypothesis restated: for freely falling bodies the ratio d/t^2 is constant. How else could this be worded?

$d \propto t^2$
or $d = kt^2$

The derived relationship $d/t^2 = \tfrac{1}{2}a$ has one big advantage over the definition of uniform acceleration: it does not

See Study Guide questions 2.16 and 2.17.

contain the speed v which Galileo had no reliable way of measuring. Instead, it contains the distance d, which he could measure directly and easily. However, the measurement of the time of fall t remains as difficult as before. Hence, a direct test of his hypothesis still eluded Galileo.

Q1 Why did Galileo use the equation $d = \frac{1}{2}at^2$ rather than $a = \frac{v}{t}$ in testing his hypothesis?

Q2 If you simply combined the two equations $d = vt$ and $a = \frac{v}{t}$ you might expect to get the result $d = at^2$. Why is this wrong?

Summary 2.8
1. Galileo used the inclined plane as an instrument for lengthening the time of fall on the assumption that free fall is the extreme limiting case of motion along an inclined plane.

2.8 Galileo turns to an indirect test. Realizing that it was still impossible to carry out direct quantitative tests with freely falling bodies, Galileo next proposed a related hypothesis which could be tested much more easily. According to Galileo, the truth of his new hypothesis would be established when we find that the inferences from it correspond and agree exactly with experiment.

The new hypothesis is this: if a freely falling body has an acceleration that is constant, then a perfectly round ball rolling down a perfectly smooth inclined plane will also have a constant, though smaller, acceleration. Thus, Galileo claims that if $\frac{d}{t^2}$ is constant for a body falling freely from rest, this ratio will also be constant, although smaller, for a ball released from rest and rolling different distances down an inclined plane.

2. His hypothesis then became: if a freely falling body has an acceleration that is constant, then a perfectly round ball rolling down a perfectly smooth inclined plane will also have a constant (though smaller) acceleration.

Here is how Salviati described Galileo's own experimental test:

Note the careful description of the experimental apparatus. Today an experimenter would add to his verbal description any detailed drawings, schematic layouts, or photographs needed to make it possible for any other competent scientist to duplicate the experiment.

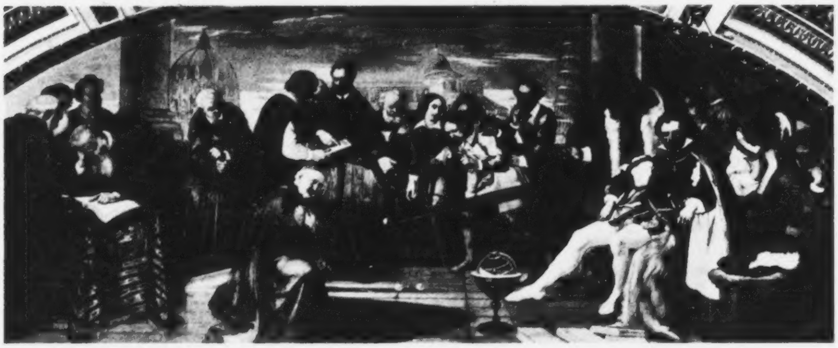
3. Experiment showed that for all angles tested, s/t^2 is constant.

Do you think measurements can actually be made to 1/10-pulse beat? Try it.

4. If s/t^2 is constant, then $\frac{v}{t}$ must be constant, and therefore, Galileo claimed, the definition of uniform acceleration ($a = \frac{v}{t}$) must fit actual free fall.

> A piece of wooden moulding or scantling, about 12 cubits long, half a cubit wide, and three finger-breadths thick, was taken; on its edge was cut a channel a little more than one finger in breadth; having made this groove very straight, smooth, and polished, and having lined it with parchment, also as smooth and polished as possible, we rolled along it a hard, smooth, and very round bronze ball. Having placed this board in a sloping position, by lifting one end some one or two cubits above the other, we rolled the ball, as I was just saying, along the channel, noting, in a manner presently to be described, the time required to make the descent. We repeated this experiment more than once in order to measure the time with an accuracy such that the deviation between two observations never exceeded one-tenth of a pulse-beat. Having performed this operation and having assured ourselves of its reliability, we now rolled the ball only one-quarter of the length of the channel; and having measured the time of its descent, we found it precisely one-half of the former. Next we tried other distances, comparing the time for the whole length with that for the half, or with that for two-thirds, or three-fourths, or indeed for any fraction; in such experiments,

> repeated a full hundred times, we always found that the spaces traversed were to each other as the squares of the times, and this was true for all inclinations of the...channel along which we rolled the ball....



• This picture, painted in 1841 by G. Bezzuoli, reconstructs for us an experiment Galileo is alleged to have made during his time as lecturer at Pisa. To the left and right are men of ill-will: the blasé Prince Giovanni de Medici (Galileo had shown a dredging-machine invented by the prince to be unusable), and Galileo's scientific opponents. These were leading men of the universities, who are bending over a sacrosanct book of Aristotle, where it is written in black and white that, according to the rules of gravity, bodies of unequal weight fall with different speeds. Galileo, the tallest figure left of center in the picture, is surrounded by a group of students.

• The painting is a romanticized and historically inaccurate representation. There is no evidence that Galileo performed this experiment in front of the nobles and men of distinction depicted in the painting.

Galileo has packed a great deal of information into these lines. He describes his procedures and apparatus clearly enough to allow other investigators to repeat the experiment for themselves if they wish; he gives an indication that consistent measurements can be made; and he restates the two experimental results which he believes support his free-fall hypothesis. Let us examine the results carefully.

First, he found that when a ball rolled down an incline at a fixed angle to the horizontal, the ratio of the distance covered to the square of the corresponding time was always the same. For example, if d_1, d_2, and d_3 represent distances from the starting point on the inclined plane, and t_1, t_2, and t_3 the corresponding times, then

$$\frac{d_1}{(t_1)^2} = \frac{d_2}{(t_2)^2} = \frac{d_3}{(t_3)^2}$$

and in general (for a given angle of incline),

$$\frac{d}{t^2} = \text{constant}.$$

Galileo did not present his experimental data in detail, for that had not yet become the custom. However, his experiment has been repeated by others, and they have obtained results which paralleled his. For example, one experimenter obtained the results shown in Table 2.1. But this is an experiment which you can perform yourself with the help of one or two other students. The students seen conducting this experiment recorded the findings in their notebook shown on the next page.

Galileo's second experimental finding relates to what happens when the angle of inclination of the plane is changed. He found when the angle changed, the ratio $\frac{d}{t^2}$ also changed, although it was constant for any one angle.

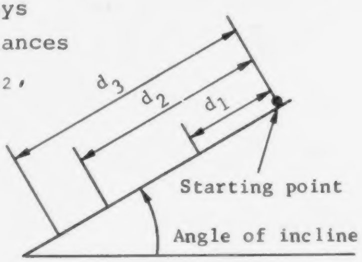
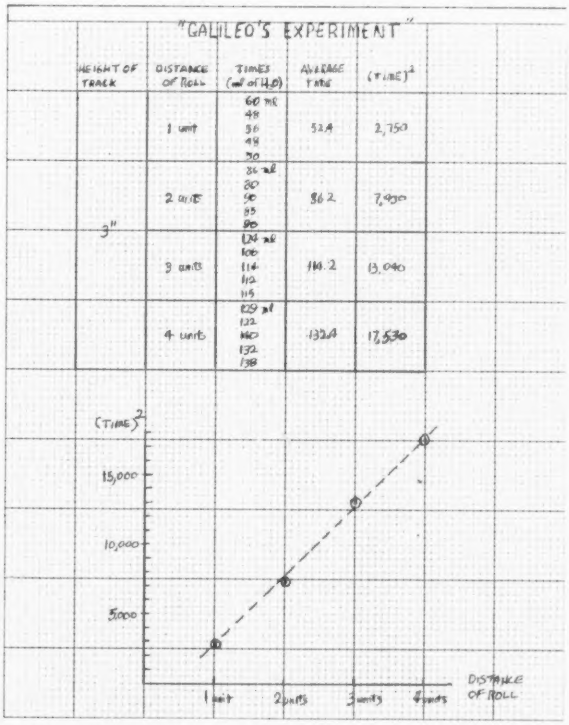


Table 2.1. Results from an experiment of Thomas Settle in which the angle of inclination was 3° 44' (See Science, 133, 19-23, June 6, 1961).

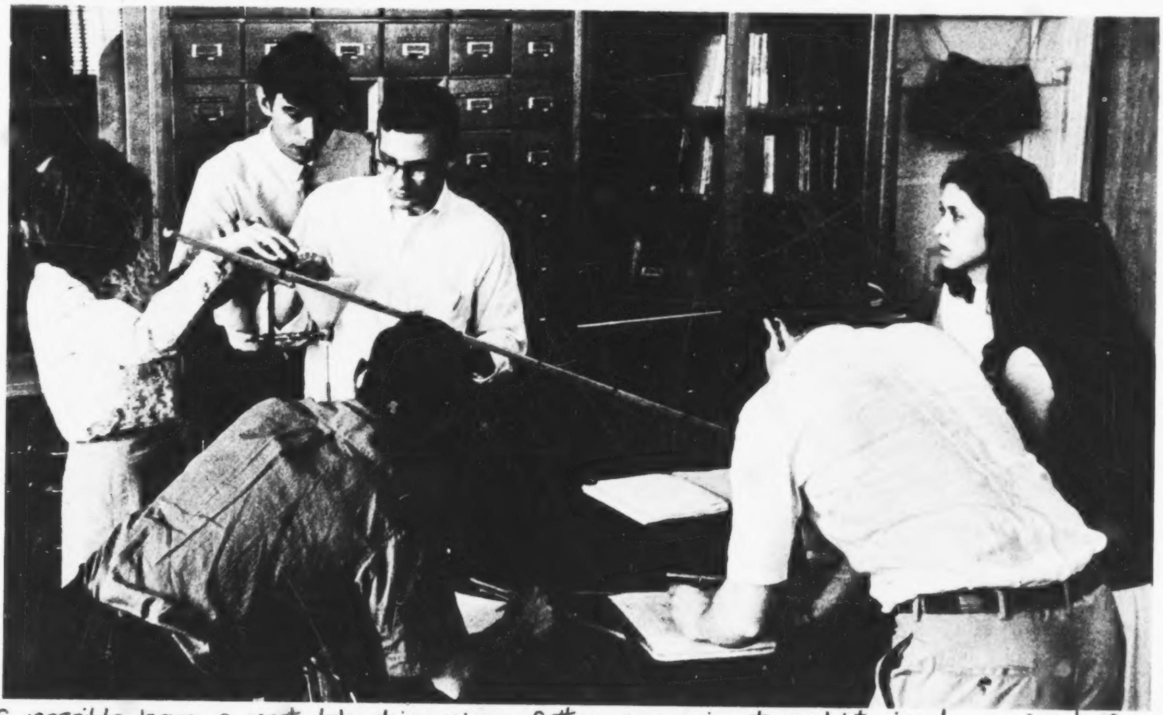
Distance	Time (ml of water)	d/t^2
15 ft	90	.00185
13	84	.00183
10	72	.00192
7	62	.00182
5	52	.00185
3	40	.00187
1	23.5	.00182

E5: A seventeenth-century experiment or E6: A twentieth-century version of Galileo's experiment
A: $d \propto t^2$ falling weights - magnetic timer



"GALILEO'S EXPERIMENT"

HEIGHT OF TRACK	DISTANCE OF ROLL	TIMES (ml of H_2O)	AVERAGE TIME	$(\text{TIME})^2$
3"	1 unit	60 ml 48 56 48 50	52.4	2,750
	2 units	86 ml 80 90 85 90	86.2	7,430
	3 units	124 ml 106 114 112 115	114.2	13,040
	4 units	129 ml 122 140 132 138	132.4	17,530

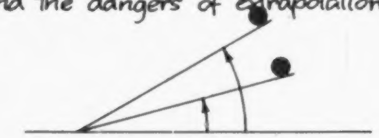


If possible, have a post-lab discussion of the experiment and its inadequacies before this section is assigned.

The inclined plane experiment and sample data in a Project Physics class.

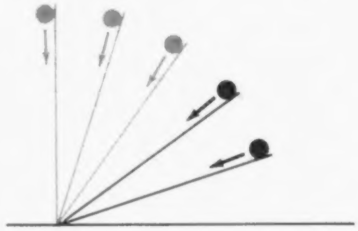
This was confirmed by repeating the experiment "a full hundred times" for each of many different angles. After finding that the ratio $\frac{d}{t^2}$ was constant for each angle of inclination for which measurements of t could be carried out conveniently, Galileo was willing to extrapolate.●

● Remind students of Section 1.6 and the dangers of extrapolation.



For each angle, the acceleration is found to be a constant.

He reasoned that the ratio $\frac{d}{t^2}$ is a constant even for larger angles where the motion of the ball is too fast for accurate measurements of t to be made. Further, he reasoned that if the ratio $\frac{d}{t^2}$ is constant when the angle of inclination is 90°, then $\frac{d}{t^2}$ is also a constant for a falling object. Thus, by combining experimentation and reason, Galileo was able to make a convincing argument that for a falling object the ratio $\frac{d}{t^2}$ is a constant.



Spheres rolling down planes of increasingly steep inclination. At 90° the inclined plane situation matches free fall. (Actually, the ball will start slipping long before the angle has become that large.)

Now let us review the steps we have taken. By mathematics we showed that $\frac{d}{t^2}$ = constant is a logical consequence of $\frac{v}{t}$ = constant. In other words, if the statement

$$\frac{v}{t} = \text{constant}$$

is true, then the statement

$$\frac{d}{t^2} = \text{constant}$$

is also true.

Next, Galileo proceeded to prove that $\frac{d}{t^2}$ is a constant for a falling object. By reversing the previous mathematics you can show that if the statement

$$\frac{d}{t^2} = \text{constant}$$

is true, then the statement

$$\frac{v}{t} = \text{constant}$$

must also be true.

But $\frac{v}{t}$ = constant matches Galileo's definition of uniform acceleration, namely

$$a = \frac{v}{t} .$$

Therefore, his hypothesis that falling objects move downward with uniform acceleration appears to be correct.

See Study Guide questions 2.1, 2.2, 2.3, 2.4.

Q3 Galileo's verification of his hypothesis that free fall is uniformly accelerated motion depends on the assumption that

(a) d/t^2 is constant.

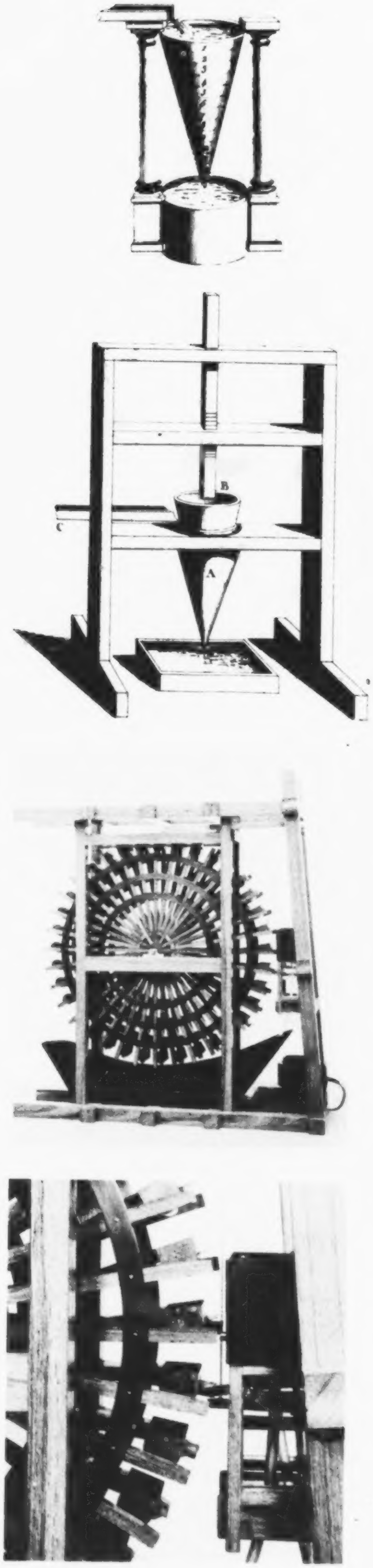
(b) the angle of inclination of the plane does not change.

(c) the results for small angles of inclination can be extrapolated to large angles.

(d) the speed of the ball is constant as it rolls.

(e) the acceleration of the rolling ball is the same as the acceleration in free fall.

Early Water Clocks.



Summary 2.9
1. The water clock was a sufficiently accurate instrument for measuring time intervals in Galileo's inclined plane experiment.

2.9 How valid was Galileo's procedure? Some doubts arise concerning this whole process of reasoning and experimentation. First, was Galileo's measurement of time accurate enough to establish the constancy of $\frac{d}{t^2}$ even for the earlier case of a slowly rolling object? Galileo tries to reassure possible critics by providing a detailed description of his experimental arrangement (thereby inviting any skeptics to try it for themselves!):

> For the measurement of time, we employed a large vessel of water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water, which we collected in a small cup during the time of each descent, whether for the whole length of the channel or for a part of its length; the water thus collected was weighed on a very accurate balance; the differences and ratios of these weights gave us the differences and ratios of the time intervals, and this with such accuracy that, although the operation was repeated many, many times, there was no appreciable discrepancy in the results.

The water clock described by Galileo was not invented by him. Indeed, there are references to water clocks in China as early as the sixth century B.C., and they were probably used in Babylonia and India even earlier. In Galileo's time, the water clock was the most accurate of the world's time measuring instruments, and it remained so until shortly after his death when the work of Christian Huygens and others resulted in the pendulum clock. Although Galileo's own water clock was not the most precise available at the time, it was, nevertheless, good enough for a convincing verification that $\frac{d}{t^2}$ is constant.

Another reason for questioning Galileo's results is related to the large extrapolation involved. Galileo does not report what angles he used in his experiment. However, as you may have found out from doing a similar experiment, the angles must be kept rather small. Naturally, as the angle increases, the speed of the ball soon becomes so great that it is difficult to measure the times involved. The largest angle reported by Settle in his modern repetition of Galileo's experiment was only 6°. It is unlikely that Galileo worked with much larger angles. This means that Galileo's extrapolation was a large one, perhaps much too large for a cautious person—or for one not already convinced of the truth of Galileo's hypothesis.

Still another reason for questioning Galileo's results is the observation that, as the angle of incline is increased, there comes a point where the ball starts to slide as well as roll. This change in behavior could mean that the same

2. Galileo's extrapolation from rolling at small angles to free fall is very dubious. Not only is the extrapolation large, but rolling changes to sliding. (Nonetheless, d/t^2 is constant for free fall.)

3. Galileo tested the logical consequences of this hypothesis rather than the hypothesis itself. This is a common practice in science.

general law does not apply to both cases. Galileo does not answer this objection. It is surprising that he never repeated the experiment with blocks which would slide, rather than roll, down a smooth incline. If he had, he would have found that for both sliding and rolling the ratio $\frac{d}{t^2}$ is a constant although it is a different constant for the two cases.

L1: Acceleration due to gravity
L2: Acceleration due to gravity-Method II

He probably felt that he could not eliminate friction well enough to do a valid experiment.

Q4 The main reason why we might doubt the validity of Galileo's procedure is

(a) his measurement of time was not sufficiently accurate.

( ) he used too large an angle of inclination.

(c) it is not clear that his results apply to the case when the ball can slide as well as roll.

(d) in Galileo's experiment the ball was actually sliding rather than rolling, and therefore his results cannot be extrapolated to the case of free fall.

(e) d/t^2 would not be constant for a sliding object.

2.10 The consequences of Galileo's work on motion. As was pointed out at the end of the previous section, one can not get the correct value for the acceleration of a body in free fall simply by extrapolating the results for larger and larger angles of inclination. In fact Galileo did not even attempt to calculate a numerical value for the acceleration of freely falling bodies. Galileo's purpose could be well served without knowing the value of the acceleration for free fall; it was enough that he showed the acceleration to be constant. This is the first consequence of Galileo's work.

We now know by measurement that the magnitude of the acceleration of gravity, symbol a_g or simply g, is about 9.8 m/sec^2 or 32 ft/sec^2 at the earth's surface (see Study Guide 3.17). The Student Handbook contains five experiments for getting a_g.

Second, if spheres of different weights are allowed to roll down an inclined plane, they have the same acceleration. We do not know how much experimental evidence Galileo himself had for this conclusion. At any rate, later work confirmed his "thought experiment" on the rate of fall of bodies of different weights (Sec. 2.3). The fact that bodies of different weights all fall at the same rate (aside from the understandable effects of air resistance) is a decisive refutation of Aristotle's theory of motion.

Third, Galileo developed a mathematical theory of accelerated motion from which other predictions about motion could be derived. We will mention just one example here, which will turn out to be very useful in Unit 3. Recall that Galileo chose to define acceleration not as the rate of change of speed in a given space, but rather as the rate of change of speed in a given time. He then found by experiment that accelerated bodies in nature actually do experience equal changes of speed in equal times. But one might also ask: if speed does not change by equal amounts in equal distances, is there anything else that does change by equal amounts in equal distances, for a uniformly accelerated motion? The answer is yes: the square of the

Summary 2.10

1. Galileo did not give the acceleration of freely falling bodies a numerical value.

2. He brought about a decisive refutation of Aristotle's theory of motion.

3. He developed a mathematical theory of accelerated motion from which other predictions about motion could be derived.

4. His work led to a new method of doing scientific research.

E7: Measuring acceleration due to gravity.
T6: Derivation of $d = v_0 t + \frac{1}{2} a t^2$
T7: Free fall analysis

Can you derive this equation? (Hint: start from equations for d and v and eliminate t.)

$v = at \qquad d = \frac{1}{2}at^2$

$t = \frac{v}{a}$

See Study Guide 2.21 and 2.23.

$d = \frac{1}{2}a\frac{v^2}{a^2}$

$d = \frac{1}{2}\frac{v^2}{a}$

$2ad = v^2$

speed changes by equal amounts in equal distances. There is a mathematical equation which expresses this result:

$$v^2 = 2ad$$

In words: if an object starts from rest and moves with uniform acceleration a through a distance d, then the square of its speed will be equal to twice the product of its acceleration and the distance it has moved. We shall see the importance of $v^2 = 2ad$ in Unit 3.

These consequences of Galileo's work, important as they are to modern physics, would scarcely have been enough to bring about a revolution in science by themselves. No sensible person in the seventeenth century would have given up his belief in the Aristotelian cosmology simply because its predictions had been refuted in the case of falling bodies. The significance of Galileo's work is that it prepared the way for the development of a new kind of physics, and indeed a new cosmology.

The more vexing scientific problem during Galileo's lifetime was not the motion of accelerated bodies, but the structure of the universe. For example, is the earth or the sun the center of the universe? Galileo supported the theory that the earth and other planets revolve around the sun. To accept such a theory meant, ultimately, to reject the Aristotelian cosmology; but in order to do this a physical theory of the motion of the earth would have to be developed. Galileo's theory of motion turned out to be just what was needed for this purpose, but only after it had been combined with further assumptions about the relation between forces and motion by the English scientist Isaac Newton. We shall return to the story of this revolution in science in Unit 2.

•Ask students to re-read and comment on the last paragraph of sec. 2.1.

There is another significant aspect of Galileo's work on motion: it led to a new way of doing scientific research• The heart of this approach is the cycle, repeated as often as necessary: general observation → hypothesis → mathematical analysis → experimental test → modification of hypothesis as necessary in light of test, and so forth. But while the steps in the mathematical analysis are determined by "cold logic," this is not the case for the other elements. Thus a variety of paths can lead to the hypothesis in the first place: an inspired hunch based on general knowledge of the experimental facts, a desire for simple and pleasing foundations, a change of a previous hypothesis that failed. Moreover, there are no general rules about how well the experimental data have to agree with the theoretical predictions. In some areas of science, a theory is expected to be accurate to better than one 1/1000th of a percent; in other areas, scientists would be delighted to find a theory that could make predictions with as little as 50 percent error.

The process of proposing and testing hypotheses, so skillfully demonstrated by Galileo in the seventeenth century, is widely used by scientists today. It is perhaps the most significant thing that distinguishes modern science from ancient and medieval science. The method is used not out of respect for Galileo as a towering figure in the history of science, but because it works so well so much of the time.

Galileo himself was aware of the value of both the results and the methods of his pioneering work. He concluded his treatment of accelerated motion by putting the following words into the mouths of the commentators on his book:

> Sagredo: I think we may concede to our Academician, without flattery, his claim that in the principle laid down in this treatise he has established a new science dealing with a very old subject. Observing with what ease and clearness he deduces from a single principle the proofs of so many theorems, I wonder not a little how such a question escaped the attention of Archimedes, Appolonius, Euclid and so many other mathematicians and illustrious philosophers, especially since so many ponderous tomes have been devoted to the subject of motion.
>
> Salviati: ...we may say the door is now opened, for the first time, to a new method fraught with numerous and wonderful results which in future years will command the attention of other minds.
>
> Sagredo: I really believe that...the principles which are set forth in this little treatise will, when taken up by speculative minds, lead to another more remarkable result; and it is to be believed that it will be so on account of the nobility of the subject, which is superior to any other in nature.
>
> During this long and laborious day, I have enjoyed these simple theorems more than their proofs, many of which, for their complete comprehension, would require more than an hour each; this study, if you will be good enough to leave the book in my hands, is one which I mean to take up at my leisure after we have read the remaining portion which deals with the motion of projectiles; and this if agreeable to you we shall take up tomorrow.
>
> Salviati: I shall not fail to be with you.

The "Academician" is the author of the treatise being discussed in the dialogues—that is, Galileo himself.

Projectile motion will be taken up in Chapter 4.

Q5 Which of the following was not a consequence of Galileo's work on motion?

(a) The correct numerical value of the acceleration in free fall was obtained by extrapolating the results for larger and larger angles of inclination.

(b) If an object starts from rest and moves with uniform acceleration a through a distance d, then the square of its speed will be proportional to a and also proportional to d.

(c) Bodies moving on a smooth inclined plane are uniformly accelerated (according to Galileo's definition of acceleration).

2.1 List the steps by which Galileo progressed from his first definition of uniformly accelerated motion to his final confirmation that this definition is useful in describing the motion of a freely falling body. Identify each step as a hypothesis, deduction, observation, or computation, etc. What limitations and idealizations appear in the argument? Discussion

2.2 Which of the following statements best summarizes the work of Galileo on free fall? (Be prepared to defend your choice.) Galileo: Discussion

a) proved that all objects fall at exactly the same speed regardless of their weight.

b) proved that for any one freely falling object, the ratio: $\frac{d}{t^2}$ is constant for any distance.

c) demonstrated conclusively that an object rolling down a smooth incline accelerates in the same way as (although more slowly than) the same object falling freely.

d) used logic and experimentation to verify indirectly his assertion that the speed of a freely falling object at any point depends only upon, and is proportional to, the time elapsed.

e) made it clear that until a vacuum could be produced, it would not be possible to settle the free-fall question once and for all.

2.3 Write a short statement (not more than two or three sentences) summarizing Galileo's work on free fall better than any of those in 2.2 above. Discussion

2.4 As Director of Research in your class, you receive the following research proposals from physics students wishing to improve upon Galileo's free-fall experiment. Would you recommend support for any of them? If you reject a proposal, you should make it clear why you do so.

a) Historians believe that Galileo never dropped objects from the Leaning Tower of Pisa. Too bad! Such an experiment is more direct and more fun than inclined plane experiments, and of course, now that accurate stopwatches are available, it can be carried out much better than in Galileo's time. The experiment involves dropping, one by one, different size spheres made of copper, steel, and glass from the top of the Leaning Tower and finding how long it takes each one to reach the ground. Knowing d (the height of the tower) and time of fall t, I will substitute in the equation $d = \frac{1}{2}at^2$ to see if the acceleration a has the same value for each sphere.

b) A shotput will be dropped from the roof of a 4-story building. As the shotput falls, it passes a window at each story. At each window there will be a student who starts his stopwatch upon hearing a signal that the shot has been released, and stops the watch as the shot passes his window. Also, each student records the speed of the shot. From his own data, each student will compute the ratio v/t. All four students should obtain the same numerical value of the ratio.

c) Galileo's inclined planes "dilute" motion all right, but the trouble is that there is no reason to suppose that a ball rolling down a board is behaving like a ball falling straight downward. A better way to accomplish this is to use light, fluffy, cotton balls. These will not drop as rapidly as metal spheres, and therefore it would be possible to measure the time of the fall t for different distances d. The ratio d/t^2 could be determined for different distances to see if it remained constant. The compactness of the cotton ball could then be changed to see if a different value was obtained for the ratio. Discussion

2.5 Consider Aristotle's statement "A given weight moves [falls] a given distance in a given time; a weight which is as great and more moves the same distance in less time, the times being in inverse proportion to the weights. For instance, if one weight is twice another, it will take half as long over a given movement." [De Caelo] Discussion

Indicate what Simplicio and Salviati each would predict for the rest of the falling motion in these cases:

a) A two-pound rock falls from a cliff and, while dropping, breaks into two equal pieces.

b) A hundred-pound rock is dropped at the same time as one hundred one-pound pieces of the same type of rock.

c) A hundred one-pound pieces of rock, falling from a height, drop into a loosely held sack which pulls loose and falls.

All the rocks are in the sack and continue falling while contained by the sack.

2.6 A good deal of work preceded tnat of Galileo on the topic of motion. In the period (1280-1340) mathematicians at Merton College, Oxford, carefully considered quantities that change with the passage of time. One result that had profound influence was a general theorem known as the "Merton Theorem" or "Mean Speed Rule." Proof

This theorem might be restated in our language and applied to uniform acceleration as follows: the distance an object goes during some time while its speed is changing uniformly is the same distance it would go if it went at the average speed the whole time.

Using a graph, and techniques of algebra and geometry, construct a proof of the "Merton Rule."

2.7 In the Two New Sciences Galileo states, "...for so far as I know, no one has yet pointed out that the distances traversed, during equal interval of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity (namely 1:3:5:7...)...."

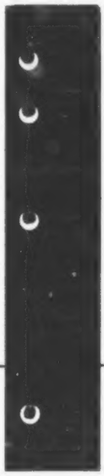
The area beneath a speed-time graph represents the distance traveled during some time interval. Using that idea, give a proof that the distances an object falls in successive equal time intervals will be in such a ratio.

Proof

Note sneaky introduction of integration idea.

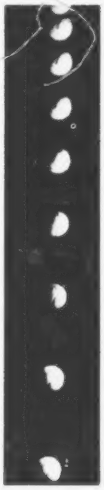
2.8 Indicate whether the following statements are true or false when applied to the strobe photo at the right:

true a) The speed of the ball is greater at the bottom than at the top.
true b) The direction of the acceleration is vertically downward.
false c) This could be a freely falling object.
false d) This could be a ball thrown straight upward.



2.9 Apply the same statements to the photo at the right, once again indicating whether each statement is true or false.

a) true
b) true
c) true
d) true



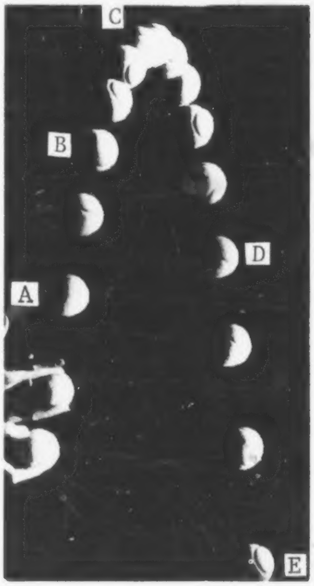
Table

2.10 These last two questions raise the issue of direction. The photograph in the figure below is of a ball thrown upward, yet its acceleration is downward. The acceleration due to gravity may appear as the slowing down of an upward moving object, or as the speeding up of a downward moving one. To keep these matters straight, a plus and minus sign convention is adopted. Such a convention is merely an arbitrary but consistent set of rules.

The main rule we adopt is: up is the positive direction. It follows that the acceleration due to free fall g always takes the negative sign; distances above the point of release are positive, those below it negative; and the speed of an object moving upward is positive, downward negative.

The figure below is a photo of the path that a ball might take if you threw it up and then let it fall to the ground rather than catching it when it reached your hand again. To assure yourself that you understand the sign convention stated above, complete the table below.

Physics Teacher
April '63.
"Demonstration of freely falling bodies"
October '64
"Falling bodies"



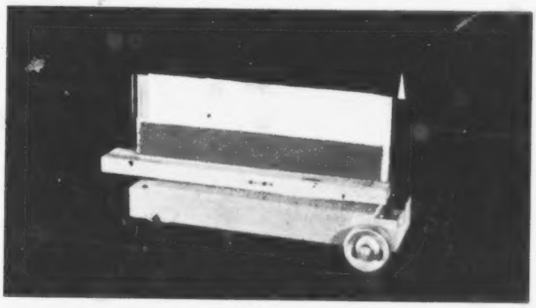
Stroboscopic photograph of a ball thrown into the air.

position	d	v	a
A	+	+	−
B	+	+	−
C	+	0	−
D	+	−	−
E	−	−	−

2.11 Draw a set of points (as in a strobe photo) to show the successive positions of an object that had a positive acceleration upward. Can you think of any way to produce such an event physically? Discussion

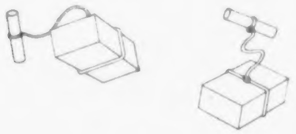
2.12 The instrument shown below on a cart is called a liquid surface accelerometer. Whenever the accelerometer experiences an acceleration in a direction parallel to its long dimension, the surface of the liquid tilts in the direction of the acceleration. Design a demonstration in which acceleration remains constant but speed and direction change.

Discussion



2.13 Drop sheets of paper with various degrees of "crumpling." Can you crumple a sheet of paper tight enough that it will fall at the same rate as a tennis ball? Discussion

2.14 Tie two objects (of greatly different weight) together with a piece of string. Drop the combination with different orientations of objects. Watch the string. In a few sentences summarize your results. Discussion



2.15 In these first two chapters we have been concerned with motion in a straight line. We have dealt with distance, time, speed and acceleration, and with the relationships between them. Surprisingly, most of the results of our discussion can be summarized in the three equations listed below.

$$v_{av} = \frac{\Delta d}{\Delta t} \qquad a_{av} = \frac{\Delta v}{\Delta t} \qquad d = \tfrac{1}{2}at^2$$

The last of these equations applies only to those cases where the acceleration is constant. Because these three equations are so useful, they are worth remembering.

a) State each of the three equations in words.

b) Which of the equations can be applied only to objects starting from rest?

c) Make up a simple problem to demonstrate the use of each equation. For example: How long will it take a jet plane to travel 3200 miles if it averages 400 mi/hr? Also work out the solution just to be sure the problem can be solved. Discussion

2.16 Memorizing equations will not save you from having to think your way through a problem. You must decide if, when and how to use equations. This means analyzing the problem to make certain you understand what information is given and what is to be found. Test yourself on the following problem. Assume that the acceleration due to gravity is 10 m/sec^2.

Problem: A stone is dropped from rest from the top of a high cliff.

a) How far has it fallen after 1 second? $d = -5.0$ m

b) What is the stone's speed after 1 second of fall? $V = -10$ m/sec

c) How far does the stone fall during the second second? (That is, from the end of the first second to the end of the second second.) $d = -15$ m

2.17 Think you have it now? Test yourself once more. If you have no trouble with this, you may wish to try problem 2.18, 2.19, or 2.20.

Problem: An object is thrown straight upward with an initial velocity of 20 m/sec.

a) What is its speed after 1.0 sec? $V =$ 10m/sec

b) How far did it go in this first second? $d = 15$m

c) How long did the object take to reach its maximum height? $t = 2$ sec

d) How high is this maximum height? $S = 20$m

e) What is its final speed just before impact? $V = -20$ m/sec

2.18 A batter hits a pop fly that travels straight upwards. The ball leaves his bat with an initial speed of 40 m/sec.

a) What is the speed of the ball at the end of 2 seconds? $V_f = 20$ m/sec

b) What is its speed at the end of 6 seconds? $V_f = -20$ m/sec

c) When does the ball reach its highest point? $t = 4$ sec

d) How high is this highest point? $S = 80$m

e) What is the speed of the ball at the end of 10 seconds? (Graph this series of speeds.) $V_f = 0$ m/sec

f) What is its speed when caught by the catcher? $V_f = -40$ m/sec

2.19 A ball starts up an inclined plane with a speed of 4 m/sec, and comes to a halt after 2 seconds.

a) What acceleration does the ball experience? $a = -2\ m/sec^2$

b) What is the average speed of the ball during this interval? $\bar{v} = 2\ m/sec$

c) What is the ball's speed after 1 second? $v_f = 2\ m/sec$

d) How far up the slope will the ball travel? $d = 4\ m$

e) What will be the speed of the ball 3 seconds after starting up the slope? $v_f = -2\ m/sec$

f) What is the total time for a round trip to the top and back to the start? $t = 4\ sec$

2.20 Lt. Col. John L. Stapp achieved a speed of 632 mph (284 m/sec) in an experimental rocket sled at the Holloman Air Base Development Center, Alamogordo, New Mexico, on March 19, 1954. Running on rails and propelled by nine rockets, the sled reached its top speed within 5 seconds. Stapp survived a maximum acceleration of 22 g's in slowing to rest during a time interval of 1½ seconds. (22 g's means 22 × a_g.)

a) Find the average acceleration in reaching maximum speed. $\bar{a} = 57\ m/sec^2$

b) How far did the sled travel before attaining maximum speed? $s = 710 m$

c) Find the average acceleration while stopping. $\bar{a} = -190\ m/sec^2$

2.21 Sometimes it is helpful to have a special equation relating certain variables. For example, initial and final speed, distance, and acceleration are related by the equation

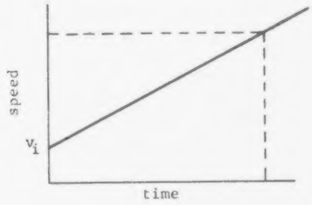
$$v_f^2 = v_i^2 + 2ad.$$

Try to derive this equation from some others you are familiar with. Proof

2.22 Use the graph below, and the idea that the area under a curve in a speed-time graph gives a value for the distance traveled, to derive the equation

$$d = v_i t + \tfrac{1}{2}at^2.$$

Proof



2.23 A student on the planet Arret in another solar system dropped an object in order to determine the acceleration due to gravity. The following data are recorded (in local units):

Time (in surgs)	Distance (in welfs)
0.0	0.00
0.5	0.54
1.0	2.15
1.5	4.84
2.0	8.60
2.2	10.41
2.4	12.39
2.6	14.54
2.8	16.86
3.0	19.33

a) What is the acceleration due to gravity on the planet Arret, expressed in welfs/surg^2?

b) A visitor from Earth finds that one welf is equal to about 6.33 cm and that one surg is equivalent to 0.167 sec. What would this tell us about Arret?

a) $a = 4.8\ \frac{welfs}{surg^2}$

b) $a = 11\ m/sec^2$

Chapter 3 The Birth of Dynamics - Newton Explains Motion

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3.1 The concepts of mass and force. Galileo investigated many topics in mechanics with insight, ingenuity and gusto. The most valuable part of that work dealt with special types of motion, such as free fall. In a clear and consistent way, he developed useful schemes for describing how objects move. Kinematics is the study of how objects move.

When Isaac Newton began his studies of motion in the second half of the seventeenth century, Galileo's earlier insistence that "the present does not seem to be the proper time to investigate the cause of the acceleration of natural motion...." was no longer valid. Indeed, largely because Galileo had been so effective in describing motion, Newton could turn his attention to dynamics; that is, to the question of why objects move the way they do.

How does dynamics differ from kinematics? ● As we have seen in the two earlier chapters, kinematics deals with the description of motion. Dynamics goes beyond kinematics by taking into account the cause of the motion. For example, in describing the motion of a stone dropped from a cliff, we might include the height from which the stone is dropped and the time the stone remains in its fall. With this information we could compute the stone's average speed and its acceleration. But, when we have completed our description of the stone's motion, we are still not satisfied. Why, we might ask, does the stone accelerate rather than fall with a constant speed? Why does it accelerate uniformly? To answer these questions, we must add to our arsenal of concepts those of force and mass; and in answering, we are doing dynamics.

Fortunately, the concepts of force and mass are not exactly new. Our common sense idea of force is closely linked with our own muscular activity. We know a sustained effort is required to lift and support a heavy stone. When we push a lawnmower, row a boat, draw a bow, or pull a sled, our muscles let us know we are exerting a force. Perhaps you notice how naturally force and motion and muscular activity are united in our minds. In fact, when you think of moving or changing the motion of an object (e.g., hitting a baseball), you naturally think of the muscular sensation of exerting a force.

The idea of mass is a little more subtle. You have used the word mass, but common sense alone does not lead to a useful definition. Certainly it does not have to do only with size—a brick is more massive than a beachball. Think of a grapefruit and a shot put. Which has

Summary 3.1

1. While kinematics deals with the description of motion, dynamics is that branch of mechanics which attempts to explain motion.

2. Force and mass are dynamic concepts whose importance was first recognized by Newton.

● The topic of dynamics is introduced by contrasting it with kinematics. The distinction is mentioned in Chapter 2 (p. 47), when Salviati (i.e. Galileo) says that the time to talk about motion is after descriptions exist.

Kinematic Concepts

position
time
speed
acceleration

Dynamic Concepts

mass
force
momentum (Ch. 9)
kinetic energy (Ch. 10)

Summary 3.2

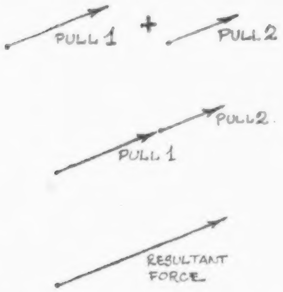
1. Quantities that involve direction (and that add according to the parallelogram law) are called vectors; non directional quantities are called scalars.

2. Displacement $\vec{d}$, velocity $\vec{v}$, and acceleration $\vec{a}$ are vector quantities.

3. Arrows are used to represent the direction and magnitude of vectors. An arrow points in the same direction as the vector it represents, and its length is proportional to the magnitude of the vector.

4. Acceleration is more precisely defined as $\vec{a}_{av} = \Delta\vec{v}/\Delta t$. If either the magnitude or the direction of $\vec{v}$ change, there is acceleration.

D7: Addition of vectors
D8: Direction of $\vec{v}$ and $\vec{a}$
D9: Direction of $\vec{v}$ and $\vec{a}$ (airtrack)
D10: Non-commutativity (rotations)



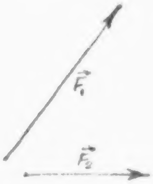
A vector is represented by an arrow-headed line segment whose length is drawn proportional to the magnitude of the vector and whose direction is the same as the vector.

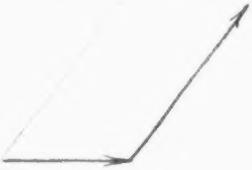
a greater mass? The shot put, you say. But why? Because it is heavier? Is mass merely a synonym for weight? No, because if an astronaut smuggled the shot put and the grapefruit aboard his space capsule, then once the vehicle is in a region where the gravitational pull is no longer felt, it will still be much more difficult to accelerate the shot put in throwing it forward (as on p. 64) than would be a grapefruit. Even in the absence of weight, mass remains, and one is felt to be more massive than the other. Probably we can agree that mass is a measure of the quantity of matter in any object; but even this does not solve our difficulty. The question still remaining is, "What is meant by 'quantity of matter'?"

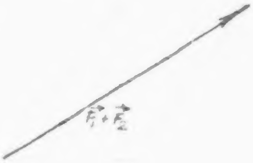
Newton did not "discover" the concepts of force and mass. What he did was this: first, he recognized that these concepts were basic for an understanding of motion and second, he clarified these concepts and defined them in a way that made them extremely useful. In the mind of Newton, the concepts of force and mass became more than fuzzy, qualitative notions—he found a way to attach numbers to them. This may not sound like much. But, by the end of this chapter, perhaps you will agree that Newton's contribution was indeed extraordinary. L3: Vector addition, velocity of a boat

3.2 About vectors. Force is a vector. If you are asked to push a piece of furniture from one part of a room to another, you size up the situation as follows. First, your experience suggests to you the magnitude of the force required. A force of greater magnitude is required to move a piano than to move a foot stool. Second, you determine the direction in which the force must be applied to make the desired move. Obviously, both the magnitude and the direction of the force are important.

We cannot define a vector until we understand how two vectors are added together. If two forces of equal magnitude, one directed due east and the other directed due north, are applied to a resting object free to move, it will take off in the northeast direction—the direction of the resultant force. The resultant force is the sum of the individual forces. The resultant force is found by application of the rule for vector addition—the parallelogram law. The parallelogram law is illustrated below.







• The vectors programs develop a "tip to tail" method of adding vectors. The addition law is, nonetheless, a "parallelogram law."

Now we can define a vector. Something which has both magnitude and direction, and which adds by the parallelogram law, is a vector. A surprising variety of things have both magnitude and direction and add together according to the parallelogram law. For example, displacement, velocity and acceleration are vector quantities. Concepts such as volume, distance, or speed do not require specifying a direction in space, and are called scalar quantities.

A vector is labeled by a letter with an arrow over it; for example, $\vec{F}$, $\vec{a}$, or $\vec{v}$.

Vectors
Part I

Vectors
Part II

Vectors
Part III

In Sec. 1.8 we hedged a bit on our definition of acceleration. There we defined acceleration as the rate of change in speed. That is correct, but it is incomplete. Now we want to consider the direction of motion as well. We shall define acceleration as the rate of change of velocity where velocity is a vector having both magnitude and direction. In symbols,

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$

where $\Delta\vec{v}$ is the change in velocity. Velocity can change in two ways: by changing its magnitude (speed) and by changing its direction. In other words, an object is accelerating when it speeds up, slows down, or changes direction.

We will use vectors frequently. To learn more about them, ask your teacher for the Project Physics program Vectors.

Q1 What is the difference between speed and velocity?

Q2 An object is moving with a velocity of 10 m/sec due north. Five seconds later, it is moving with a velocity of 10 m/sec due east.

(a) What is the change in the velocity $\Delta\vec{v}$?

(b) What is the average acceleration $\Delta\vec{v}/\Delta t$?

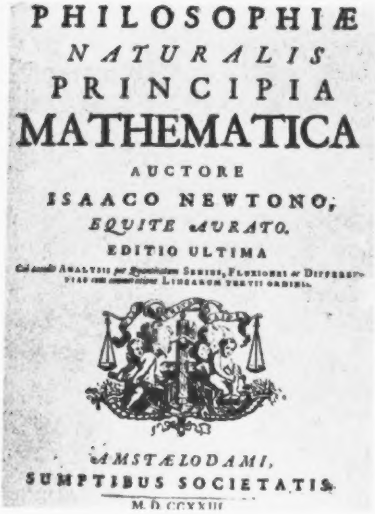
3.3 Explanation and the laws of motion. So far in our study of kinematics, we have encountered four situations: an object might

a) remain at rest
b) move uniformly in a straight line
c) speed up, and
d) slow down.

Because the last two of these are examples of acceleration the list could really be reduced to 1) rest, 2) uniform rectilinear motion, and 3) acceleration. These are the phenomena that we shall first try to explain.

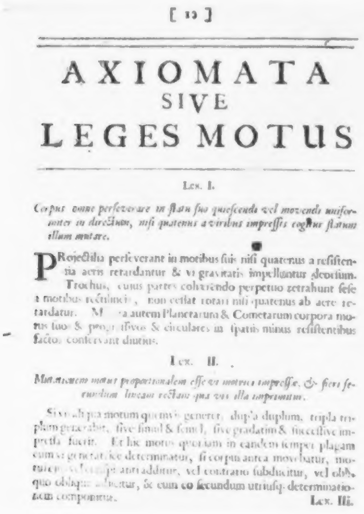
The words "explain" or "cause" have to be used with care. To the physicist, an event is "explained" when he can demonstrate that the event is a logical consequence of a law he assumes to be true. In other words, a physicist,

Summary 3.3
Newton's laws of motion "explain" almost all observations of motion in the sense that the observations are consistent with the descriptions given by the laws (when the conditions are known).



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA
AUCTORE
ISAACO NEWTONO,
EQUITE AURATO.
EDITIO ULTIMA

AMSTÆLODAMI,
SUMPTIBUS SOCIETATIS.
M.D.CCXXIII.



[12]

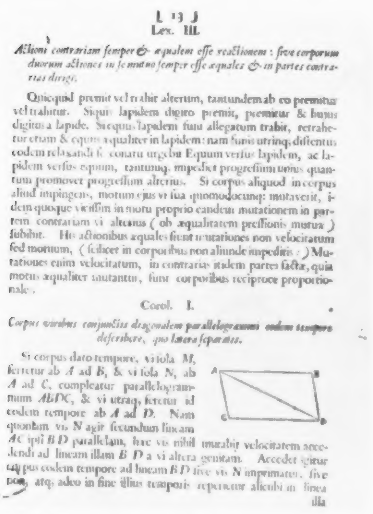
AXIOMATA
SIVE
LEGES MOTUS

Lex. I.

Corpus omne perſeverare in ſtatu ſuo quieſcendi vel movendi uniformiter in directum, niſi quatenus a viribus impreſſis cogitur ſtatum illum mutare.

Lex. II.

Mutationem motus proportionalem eſſe vi motrici impreſſæ, & fieri ſecundum lineam rectam qua vis illa imprimitur.



[13]

Lex. III.

Actioni contrariam ſemper & æqualem eſſe reactionem: ſive corporum duorum actiones in ſe mutuo ſemper eſſe æquales & in partes contrarias dirigi.

Corol. I.

Corpus viribus conjunctis diagonalem parallelogrammi eodem tempore deſcribere, quo latera ſeparatis.

with faith in a law, "explains" an observation by showing that it is consistent with the law. In a sense, the physicist's job is to show that the infinity of separate, different-looking occurrences are merely different manifestations of a few general rules on which the world is built.

To explain rest, uniform motion and acceleration, we must be able to answer such questions as: why does a vase placed on a table remain stationary? If a dry ice puck resting on a smooth, level surface is given a brief push, why does it move with uniform speed in a straight line, rather than slow down noticeably or curve to the right or left?

Answers to these (and almost all) specific questions about motion are contained either directly or indirectly in the three general laws of motion formulated by Newton. These laws appear in his famous book, Philosophiae Naturalis Principia Mathematica, which is usually referred to simply as The Principia. We shall examine Newton's three laws of motion one by one. If you are curious about these laws—and if your Latin is fairly good—you might try translating them from the original (shown in the margin). A modernized version of Newton's laws, in English, is in Study Guide 3.1.

Before looking at Newton's contribution let us first find out how some other scientists of Newton's time might have responded to some of these questions.

3.4 The Aristotelian explanation of motion.

The idea of force played a central role in the dynamics of Aristotle, twenty centuries before Newton. You will recall from Chapter 2 that there were two types of motion—"natural" motion and "violent" motion. For example, a falling stone is in natural motion. A stone being steadily lifted is in violent motion. To maintain this uniform violent motion, a force must be continuously applied. A person lifting a large stone is keenly aware that a continual force is required as he strains to hoist the stone higher.

Let us explore the idea of violent or unnatural motion a little further, for as we shall see, there were difficulties. To understand these difficulties, let us take a specific example—an arrow flying through the air. Aristotle had generalized the common-sense notion that an object cannot undergo violent motion without a mover, or something pushing on it. Thus, an arrow flying through the air must be continually propelled by a force. Further, if

Summary 3.4

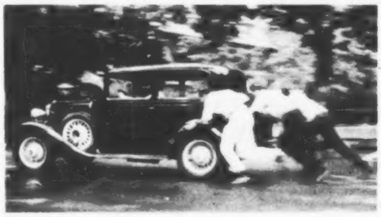
1. According to Newton's first law, neither rest nor uniform motion requires explanation; by contrast, in the Aristotelian system all motion of earthly objects requires explanation.

2. The Newtonian view eventually replaced the Aristotelian one, not because it was true in some ultimate cosmic sense but rather because it was part of a conceptual scheme that proved to be more useful.

the propelling force is removed, the arrow should immediately stop its flight and fall directly to the ground. But how can this be? Does the arrow fall to the ground as soon as it loses direct contact with the bow string? The archer —and certainly the victim—are aware that it does not.

What then is the force that propels the arrow? This force was accounted for by an ingenious suggestion: the motion of the arrow was maintained by the air itself! A commotion is set up in the air by the initial movement of the arrow; that is, as the arrow starts to move the air is compressed and pushed aside. The rush of air to fill the space being vacated by the arrow (remember that according to Aristotle a vacuum is impossible) maintains it in its flight.

To an Aristotelian, a force is necessary to sustain uniform motion. The explanation of uniform motion is reduced to identifying the origin of the force. And that is not always easy.



Keeping an object in motion at uniform (constant) velocity.

Of course, Aristotle's followers had other problems. For example, a falling acorn does not move with uniform speed—it accelerates. How is acceleration explained? Aristotelians thought the speeding up of a falling object was associated with its approaching arrival at its natural place, or home, the earth. In other words, a falling object is like the tired old horse that perks up and starts to gallop as it approaches the barn. Galileo's Aristotelian contemporaries offered a more scientific-sounding but equally false explanation for the acceleration of falling bodies. They claimed that when an object falls, the weight of the air above it increases while the column of air below it decreases, thus offering less resistance to its fall.

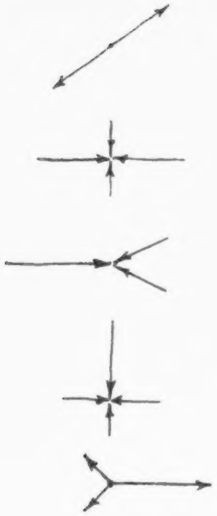
When a falling acorn finally reaches the ground, as close to the center of the earth as it can get, it stops. And there, in its natural place, it remains. Rest, the natural state of the acorn, requires no explanation. You see, the three phenomena—rest, uniform motion, and acceleration—could be explained by an Aristotelian. Now, let us examine the alternative explanation that our present understanding offers.

Study Guide 3.2

Q3 According to Aristotle, a ________ is necessary to maintain motion.

Q4 Can you come up with an Aristotelian explanation of a dry-ice puck's uniform motion across a table top?

3.5 Forces in equilibrium. Forces make things move—they also hold things still. The barrel supporting a circus elephant and the cable supporting the main span of the Golden Gate Bridge are both under the influence of mighty forces, yet they remain at rest. Apparently, more is required to initiate motion than the mere application of a force.



In which cases are the forces balanced?

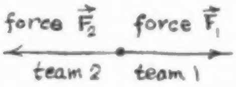
Of course, this may not be surprising to you. We have all seen children quarrelling over the same toy. If each child pulls determinedly in his own direction, the toy goes nowhere. On the other hand, if two of the children cooperate and pull together against the third, then the tide of battle shifts.

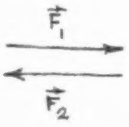
Likewise, in a tug-of-war between two teams, there are large forces exerted at each end, but the rope may not budge an inch. You might say it is all a matter of balance. If the team pulling in one direction exerts a force equal to that of the team pulling in the other direction, the forces acting on the rope are balanced and the rope does not move. The physicist would say that the rope is in equilibrium under the forces acting on it.

For reasons explained in the next section, we shall have to make a correction, and add "or in uniform rectilinear motion" wherever the word "rest" appears in this section.

The vector nature of force suggests a graphical representation of the tug-of-war or the toy-pulling episode. When we draw the lengths of the vectors representing the forces acting on the rope or toy proportional to the magnitudes of the forces, we discover a surprising result. We can predict whether or not the rope or the toy will remain at rest! In fact, if we know the forces acting on any object, we can generally predict whether an object at rest will remain at rest.

It is as simple as this: if the vectors representing the forces acting on an object at rest add up to zero, the object is in equilibrium under these forces and will remain at rest. To return to our tug-of-war, let us assume the forces are known and are accurately represented by the vectors drawn below. They are balanced; that is, the net force is zero.

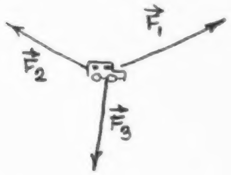


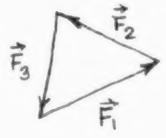


$\vec{F}_1 + \vec{F}_2 = 0$

Summary 3.5
If the sum of all forces acting on an object at rest is zero, the object will remain at rest.

This same procedure can be applied to the toy. Again, the known forces are represented by vectors and are drawn below. Is the toy in equilibrium under the forces? Yes, if the vectors add up to zero. Let's see.



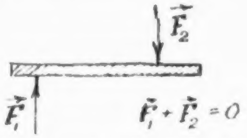


$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

Yes, indeed, the truck is in equilibrium. To obtain the answer, we merely apply the rule of vector addition. A ruler and protractor are, of course, handy tools of the trade.

We can now summarize our understanding of the state of rest as follows: if the sum of all forces acting on an object at rest is zero, the object will remain at rest. We regard rest as a condition or state in which forces are balanced.

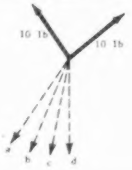
We are defining equilibrium without worrying whether the object will rotate. For example



The sum of the forces on the plank in the diagram is zero, but it is obvious that the plank will rotate.

Study Guide 3.3

Q5 Which arrow (a,b,c, or d) indicates the direction and magnitude of the force needed to balance the two 10-pound forces indicated in the diagram?



3.6 Newton's first law of motion. Were you surprised when you first pushed a dry-ice puck or some other frictionless device? Remember how it glided along after just the slightest nudge? Remember how it showed no signs of slowing down? Or speeding up? We were surprised, probably because the puck failed to live up to our everyday Aristotelian expectations.

Yet the puck was behaving quite naturally indeed. If the retarding forces of friction were absent, a gentle push would cause tables and chairs to glide across the floor just like a dry-ice puck. Newton's first law brings the eerie motion of the puck from the realm of the unnatural to that of the natural. The first law can be stated as follows:

> Every object continues in its state of rest or of uniform rectilinear motion unless acted upon by an unbalanced force.

One must think of all the forces acting on an object. If all forces, including friction, balance, the body will be moving at constant $\vec{v}$ ("rest" being a special case, namely $\vec{v} = 0$). Straight-line motion is assured if all forces on the object balance or cancel.

Summary 3.6

1. If an object is at rest or moving with uniform motion in a straight line, then one assumes that there are no unbalanced forces acting on it; any acceleration implies an unbalanced force.

2. Newtons first law is sometimes referred to as the law of inertia.

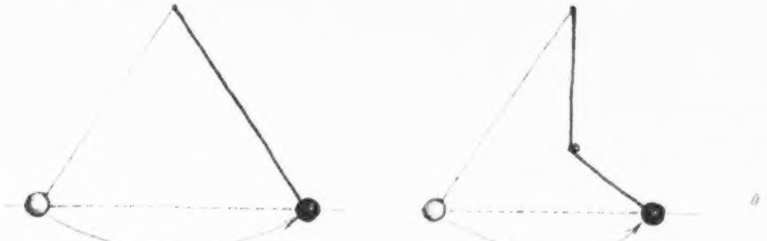
3. Implications of the first law:
a) Inertia is a basic inherent property of all objects.
b) There is no dynamical distinction between an object at rest and an object in motion.
c) To describe motion, a reference frame must be specified.
d) There is no distinction between terrestial and celestial physics. The same law applies both to objects on earth and to planets and stars.

American Journal of Physics
April, '64 "Theory of the air supported puck"

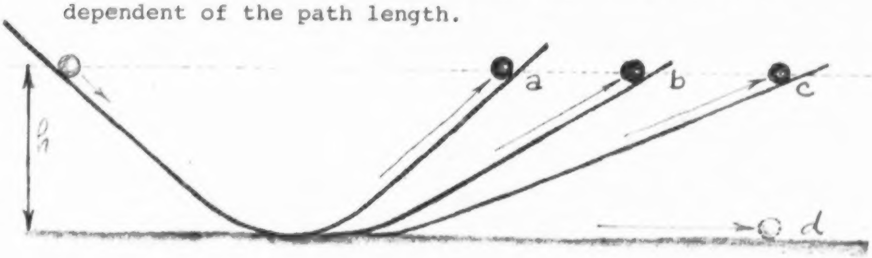
D II : Newton's first law
A : Newton's first law
L4 : A matter of relative motion

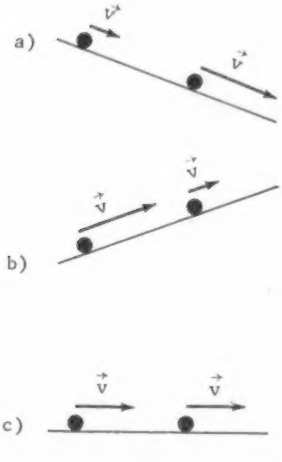
Although Newton was the first to express this law in general terms, it was clearly anticipated by Galileo. Of course, neither Galileo nor Newton had dry ice pucks, so they could not experimentally observe motion for which friction had been so significantly reduced. Instead, Galileo did a thought experiment in which he imagined the friction to be zero.

This thought experiment started with an actual observation. If a pendulum bob on the end of a string is pulled back and released from rest, it will swing through an arc rising to very nearly its starting height. Indeed, as Galileo showed, the pendulum bob will rise almost to its starting height even if a peg is used to change the path



From this observation he generates his thought experiment. A ball released from a height, h, on a frictionless incline, will roll up an adjoining incline, also frictionless, to the same height. Further, he reasons, this result is independent of the path length.





Another thought experiment. (a) A ball rolls down a smooth inclined plane; it gathers speed, i.e., $\vec{v}$ increases. (b) If it is made to roll up an incline, $\vec{v}$ decreases. (c) If the surface slants neither up nor down, i.e., is perfectly level, the ball, once started, will neither speed up nor slow down, i.e., $\vec{v}$ will remain constant.

The implications of the thought experiment illustrated in the above diagram are these: as the incline on the right is lowered from positions (a) to (b) and to (c), the ball must roll further in each case to reach its original height. In the final position (d) the ball can never reach its original height; therefore, Galileo believed the ball would roll in a straight line and at a uniform speed forever.

This tendency of objects to maintain their state of rest, or of uniform motion, is called "the principle of inertia." In fact, Newton's first law is sometimes referred to as the law of inertia. Inertia is an inherent property of all objects: the greater the inertia of an object, the greater is its resistance to a change in its state of motion. Material bodies are, so to speak, subject to a stubborn streak of nature so far as their state of motion is concerned: they continue to move with unchanging velocity (unchanging speed and direction) unless compelled by some force to do otherwise.

A consequence of Newton's first law is that if an object moves with a constant speed in a straight line, the forces acting on it are balanced. But wait. In the last section (3.5) we established that an object remains at rest as long as there are no unbalanced forces acting on it. Does this mean that the state of rest and the state of uniform motion are equivalent for dynamics? Indeed it does.

Do you understand inertia? See Study Guide 3.4.

This equivalence can be demonstrated by re-examining the quarrel over the toy. Suppose the quarrelling children were sitting on the deck of a barge that was slowly drifting, with uniform velocity, down a lazy river. Two observers—one on the barge and one on the shore—will give reports on the incident as viewed from their frames of reference. The observer on the barge will observe that the forces on the toy are balanced and will report that with respect to him it is at rest. The observer on the shore will report that the forces on the toy are balanced and that with respect to him the toy is in uniform motion. Which observer is right? They are both right. Rest or uniform motion depends on one's point of view.

You may have found Galileo's thought experiment to be convincing, but remember that neither Galileo nor Newton proved the principle of inertia. Think of how you might try to verify that principle experimentally. You could start an object moving (perhaps a dry ice disc) in a situation in which there is no unbalanced force acting on it. Then you could observe whether or not the object continued to move uniformly in a straight line, as the first law claims it should. But there are at least two drawbacks to this experiment:

1. How do you know that there is no unbalanced force acting on the object? The only answer we have is: because the object continues to move uniformly in a straight line. But that reason is merely a restatement of the first law which we wanted to prove by experiment. Surely we cannot use the first law to verify the first law!

Galileo's idea of a straight line.

Newton's idea of a straight line.

2. What is meant by a "straight line"? When Galileo wrote about uniform motion along a "straight" line, he was thinking of a line parallel to the earth's surface. Thus, an object given an initial push would move around and around the earth. Galileo's "straight line" was really a circle! Newton, on the other hand, meant an ideal straight line such as you talk about in plane geometry. Such a line is not parallel to the earth's surface; instead it continues indefinitely out into space. The line traced by a dry ice disc in any actual laboratory experiment is so short relative to the earth's circumference, that it could fit either Galileo's or Newton's meaning.

What, then, is the significance of Newton's first law of motion? For convenience let us list the important insights the first law provides.

1. It presents inertia, the tendency of an object to maintain its state of rest or uniform motion, as a basic inherent property of all objects.

2. It makes no dynamical distinction between an object at rest and an object in uniform motion. Both states are characterized by the absence of unbalanced forces.

3. It raises the whole issue of reference frames. An object stationary for one observer might be in motion for another observer; therefore, if the ideas of rest or uniform motion are to have any significance, a reference frame must be stipulated.

4. It is a general law. It emphasizes right from the start that a single scheme is being formulated to deal with motion anywhere in the universe. For the first time no distinction is made between terrestrial and celestial domains. The same law applies to objects on earth as for planets and stars.

Study Guide 3.5 and 3.6

5. The first law informs us of the behavior of objects when no force acts on them. Thus, it sets the stage for the question: what happens when a force does act on an object?

Q6 Can you give a Newtonian explanation of a dry-ice puck's uniform motion across the table top?

Q7 How does Newton's concept of inertia differ from Galileo's?

D12 Newton's laws (airtrack)
D13 Friction on acceleration
F2 PSSC film-Inertia
A Newton's second law
E9 Newton's second law

3.7 Newton's second law of motion. So far we have met two of our three objectives: the explanation of rest and of uniform motion. In terms of the first law, the states of rest and uniform motion are equivalent; they are the states that result when no unbalanced force acts on an object.

R9 "Speed"
R10 "Motion"

The last section was concluded by a list of insights provided by the first law. Perhaps you noticed that there was no quantitative relationship established between force and inertia. Newton's second law of motion enables us to reach our third objective—the explanation of acceleration—and also provides a quantitative relationship between force and inertia. We shall study these two aspects of the second law, force and inertia, individually. First we consider the situation in which different forces act on the same object, and then the situation in which the same force acts on different objects.

To emphasize the force aspect, the second law can be stated as follows:

> The net unbalanced force acting on an object is directly proportional to, and in the same direction as, the acceleration of the object.

More briefly, this can be written as:

acceleration is proportional to net force.

If we let $\vec{F}$ stand for force and, as before, let $\vec{a}$ stand for acceleration, we can rewrite this as:

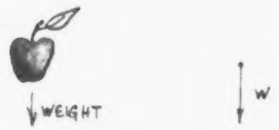
$\vec{a}$ is proportional to $\vec{F}$.

To say that one quantity is proportional to another is to make a precise mathematical statement. Here it means that if a given force causes an object to move with a certain acceleration, then twice the force will cause the same object to have twice the acceleration, three times the force will cause three times the acceleration, and so on. Using symbols, this becomes:

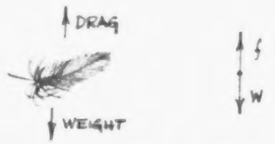
if $\vec{F}$ causes	$\vec{a}$
then $2\vec{F}$ will cause	$2\vec{a}$
$3\vec{F}$ will cause	$3\vec{a}$
$\frac{1}{2}\vec{F}$ will cause	$\frac{1}{2}\vec{a}$
$5.5\vec{F}$ will cause	$5.5\vec{a}$

and so on. So much for the effect of different forces on a single object. Now we can consider the inertia aspect of the second law, the effect of the same force acting on different objects. In discussing the first law, we defined inertia as the resistance of an object to having its velocity changed. We know from experience and observation that some objects have greater inertia than others. For instance, if you were to throw a baseball and a shot put with your full force, you know very well that the baseball would be accelerated to a greater speed than the shot put. The acceleration given to a body thus depends as much on an inherent characteristic as it does on the force applied.

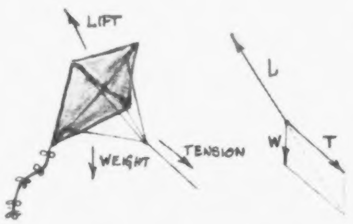
Find the net force on the body in each case:



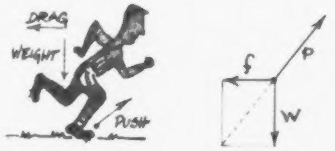
Apple falling - negligible friction



Feather falling at nearly constant speed



Kite, held suspended by wind



Man running against wind

Summary 3.7

1. Acceleration is proportional to force.

2. Acceleration is inversely proportional to mass.

3. $\vec{F}_{net} = m\vec{a}$.

4. In the mks system, the standard unit of force, 1 kg-m/sec^2, is called a newton.

(Remember: the force referred to is always the unbalanced or resultant force.)

Inertia seems to be associated with the massiveness, the amount of matter in an object. These vague ideas of "massiveness" and "amount of matter" have been replaced in physics by the quantitative concept of mass. Mass is a measure of the inertia of any object.

If you have several objects, and if you apply the same force to each, the various accelerations will not be the same. Newton's claim is that the resulting acceleration of each object is inversely proportional to its mass. Using m as a symbol for mass (a scalar quantity), and a as a symbol for the magnitude of the vector acceleration $\vec{a}$, we can write

What does it mean to say that mass is a scalar quantity?

It is a simple number with units, but without direction.

a is inversely proportional to m,

or, what is mathematically the same thing,

a is proportional to $\frac{1}{m}$.

This means that if a certain force causes a given object to have a certain acceleration, then the same force will cause: an object having twice the mass to undergo one-half the acceleration; an object having three times the mass, one-third the acceleration; an object of one-fifth the mass, five times the acceleration; and so on. Using symbols, we can express this as:

if for a given force $\vec{F}$

m will experience	a,
then 2m will experience	1/2 a,
3m will experience	1/3 a,
1/5 m will experience	5a,
2.5m will experience	0.4a,

etc.

Complete this table of the relationships between mass and acceleration for a fixed force:

mass	acceleration
m	30 m/sec^2
2m	15 m/sec^2
3m	10 "
1/5m	150 "
0.4m	75 "
45m	2/3 "
10 m	3 m/sec^2
2/5 m	75 m/sec^2

Now we can combine the roles played by force and mass in the second law into a single statement:

> The acceleration of an object is directly proportional to, and in the same direction as, the unbalanced force acting on it, and inversely proportional to the mass of the object.

Fortunately, the idea expressed in this long statement can be summarized by the equation

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

Newton's law is more accurately represented as $\vec{a} \propto \vec{F}/m$. The equality here depends upon choice of units.

which can be taken as a statement of Newton's second law. In this expression the unbalanced or net force is symbolized by $\vec{F}_{net}$. The second law may of course equally well be written in the form

$$\vec{F}_{net} = m\vec{a}.$$

Study Guide 3.7

Study Guide 3.8

This is probably the most fundamental single equation in all of Newtonian mechanics. We must not let the simplicity of the law fool us; behind that equation there is a lengthy "text."

If the law is to be useful, however, we must find a way to express force and mass numerically. But how? By measuring the acceleration which an unknown force gives a body of known mass, we could compute a numerical value for the force.• Or, by measuring the acceleration which a known force gives a body of unknown mass, we could compute a numerical value for the mass. But we seem to be going in a circle in trying to find values for force and mass—to find one we apparently need to know the other in advance.

One straightforward solution to this dilemma is to choose some convenient stable object, perhaps a certain piece of polished rock or metal, as the universal standard of mass. We arbitrarily assign it a mass of one unit. Such a standard object has, in fact, been agreed on by the scientific community. Once this unit has been selected we can proceed to develop a measure of force.

Although we are free to choose any object as a standard of mass, ideally it should be exceedingly stable, easily reproducible, and of reasonably convenient magnitude.

By international agreement, the primary standard of mass is a cylinder of platinum-iridium alloy, kept near Paris at the International Bureau of Weights and Measures. The mass of this platinum cylinder is defined as 1 kilogram.

Accurate copies of this international primary standard of mass have been deposited at the various standards laboratories throughout the world. From these, in turn, other copies are made for distribution to manufacturers and laboratories.

Now we can decide on an answer to the question of how much "push or pull" should be regarded as one unit of force. We will simply define 1 unit force as a force which when acting alone causes a mass of 1 kilogram to accelerate at the rate of 1 meter/second^2.

Imagine an experiment in which the standard 1-kilogram mass is pulled in a horizontal direction across a level, frictionless surface with a light cord, and the pull is regulated so that the 1-kilogram mass accelerates at exactly 1 m/sec^2. The force will be one unit in magnitude.

What shall we call this unit of force? According to the second law (using only magnitudes):

• Do not claim that Newton's second law is a definition of force. Remember Sagredo's comment (p. 48) that "all definitions are arbitrary."

One arbitrary definition we will make is $\vec{p} = m\vec{v}$. Newton's second law, on the other hand, is not arbitrary. It describes an experimental relationship between quantities which can be defined operationally.

See Sec. 3.11 and particularly the first two paragraphs on page 85.

$$F = ma$$

1 unit of acceleration = 1 m/sec^2.

$$\begin{aligned}\text{1 unit of force} &= \text{1 unit of mass} \times \text{1 unit of acceleration}\\ &= 1\ \text{kilogram} \times \frac{1\ \text{meter}}{\text{second}^2}\\ &= 1\ \frac{\text{kilogram} \times \text{meter}}{\text{second}^2}\\ &= 1\ \frac{\text{kg} \times \text{m}}{\text{sec}^2}.\end{aligned}$$

1 unit of force = 1 kg × m/sec^2.

Thus, 1 $\frac{\text{kg} \cdot \text{m}}{\text{sec}^2}$ of force is that quantity of force which causes a mass of 1 kg to accelerate 1 m/sec^2.

Study Guide 3.10 and 3.11

The unit kg × m/sec^2 has been given a shorter name: it is called the newton (abbreviated as N). The newton is, therefore, a derived unit which is defined in terms of a particular relationship between the meter, the kilogram and the second. These three are taken as the fundamental units of the mks system of units.

Q8 A net force of 10 N gives an object a constant acceleration of 4 m/sec^2. The object's mass is _________?

Q9 Newton's second law holds only when frictional forces are absent. (True or false.)

Q10 A 2 kg object is shoved across the floor with an initial speed of 10 m/sec. It comes to rest in 5 sec.

(a) What was the average acceleration?

(b) What was the magnitude of the force producing this acceleration?

(c) What do we call this force?

Summary 3.8

1. In free fall objects drop with uniform acceleration, therefore the second law implies that an object in free fall must be experiencing constant force.

2. The acceleration of gravity varies slightly with geographical location and with altitude.

3. Weight F_g is the magnitude of the gravitational force that an object experiences.

4. The weight of an object is directly proportional to its mass.

D14: Demonstrations with rockets
D15: Making an inertial balance
A: Accelerometer
E10: Inertial and gravitational mass

3.8 Mass, weight and gravitation. Objects may be acted upon by all kinds of forces—by a push of the hand, by a pull on a string, or by a blow from a hammer. These forces don't have to be "mechanical" or exerted by contact only, they can be due to gravitational, electric, magnetic or other actions. The laws of Newton are valid for all of them.

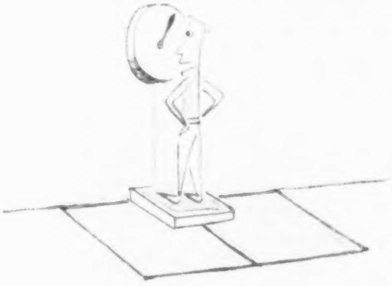
The force of gravity, which we take so much for granted, is of the kind that acts without direct contact, not only on a stone or ball that is falling near the earth, but also across empty space, for example on one of the artificial satellites around it.

We shall give the gravitational force which pulls all objects towards the earth the symbol $\vec{F}_g$. The magnitude of the gravitational pull, $\vec{F}_g$, on any particular object is, roughly speaking, the same anywhere on the surface of the earth. When we choose to be more precise, we can take into account the following facts:

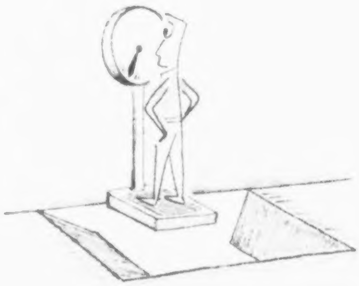
1. the earth is not exactly spherical and
2. there are irregularities in the composition of the earth's crust. These two factors cause slight variations in the gravitational force as we go from place to place. (Ge-

As Galileo said, (see Chapter 2, page 47) every term can be defined as we please, but should serve best to describe natural phenomena. There is some disagreement among physics educators about how the term "weight" is best used. We have tried to use it in a way that teachers have found to make the most sense to students, rather than following any one of the several rigorous definitions.

ologists make use of these variations in locating oil and mineral deposits.)



The term weight is used frequently in every day conversation as if it is the same as bulk or mass. In physics, we define the weight of an object as equal to the gravitational force that the body experiences. Hence weight (symbol $\vec{W}$) is a vector, as are all forces; and $\vec{W} = \vec{F}_{grav}$, by definition. When you stand on a bathroom scale to "weigh" yourself, your weight is the downward force the planet exerts on you. The bathroom scale is only registering the force with which it is pushing up on your feet and to keep you in balance, and this will be equal in magnitude to your weight if the scale is fixed and is not accelerating. If, on the other hand, the gravitational force on you is the only force you experience, and there is no other force on you that balances it, then you must be in free fall motion!



This is what would happen if the bathroom floor suddenly gave way as you stand on the scale (forgive the absurd "thought experiment"!). You and the scale would both fall down at equal rates, as all bodies do, pulled down by their weight. Your feet would now barely touch the scale, if at all; and if you looked down you would see that the scale registers zero since it is no longer pushing up on you. This does not mean you have lost your weight—that could only happen if the earth suddenly disappeared, or if you are taken to interstellar space. No, $\vec{F}_{grav}$ acts as before, and keeps you in free fall; it just means a bathroom scale does not measure your weight if it is accelerating.

We are now in a position to deepen our insight into Galileo's experiment on falling objects. Galileo's experiments indicated that every object (at a given locality) falls with uniform acceleration. And what causes a uniform acceleration? A constant net force—in this case, in free fall, just $\vec{F}_g$ or $\vec{W}$. Newton's second law gives us the relationship between this force and the resulting acceleration and we can write

Study Guide 3.15

$$\vec{F}_g = m\vec{a}_g$$

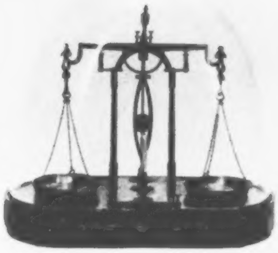
where m is the mass of the falling object and $\vec{a}_g$ is the acceleration resulting from the gravitational force $\vec{F}_g$. Thus we would conclude from Newton's second law that as long as the gravitational force is constant, the resulting acceleration is constant.

Galileo, however, did more than just show that all objects fall with uniform acceleration: he showed that all objects fall with the same uniform acceleration. Regardless of an object's weight, $\vec{F}_g$, it falls with the same acceleration $\vec{a}_g$. Is this consistent with the above relation, $\vec{F}_g = m\vec{a}_g$? It is only consistent provided there is a direct proportionality between weight and mass: if m is doubled, $\vec{F}_g$ must double, if m triples, $\vec{F}_g$ must triple. This is a profound result indeed: weight and mass are entirely different concepts—

weight is the gravitational force on an object (hence weight is a vector)

mass is a measure of the resistance of an object to changes in motion, a measure of inertia (mass is a scalar)

—yet the magnitudes of these two quite different quantities are proportional in a given locality.



What does it measure--mass or weight?

It measures mass by comparing the weight of an unknown object to the weight of a set of standard objects with known mass. (Strictly speaking, then, it is the gravitational mass and not the inertial mass which is measured.)

Study Guide 3.17

As a specific example, let us compare the magnitude of the weight and the mass of a 1-kg object and a 2-kg object. The respective weights $\vec{W}_1$ and $\vec{W}_2$, can be computed as follows (at the surface of the earth):

$$\vec{W}_1 = \vec{F}_g = m_1\vec{a}_g = (1\text{ kg})(9.8\text{ m/sec}^2) = 9.8\text{ N}$$

$$\vec{W}_2 = \vec{F}_g = m_2\vec{a}_g = (2\text{ kg})(9.8\text{ m/sec}^2) = 19.6\text{ N}$$

We see again that the ratio of the magnitudes of the weights (19.6:9.8 = 2:1) is the same as the mass ratio. These ratios will only be the same, however, if both objects are at the same location. For example, if the 1-kg object is placed at a higher altitude, its weight will be diminished but its mass will not; and so the weight ratio between it and the 2-kg object will change while the mass ratio remains 2:1. In other words, weight depends on position, but mass does not.

Q11 An astronaut is orbiting the earth in a space capsule. The acceleration of gravity is half its value on the surface of the earth. Which of the following statements is true?

(a) His weight is zero.
(b) His mass is zero.
(c) His weight is half its original value.
(d) His mass is half its original value.
(e) His weight is the same.
(f) His mass is the same.

Q12 A boy jumps from a table top. Halfway between the table top and the floor, which of these statements (for Q11) is true?

3.9 Newton's third law of motion. Newton's first law describes motion of objects when they are in a state of equilibrium, that is, when the resultant force acting on them is zero. The second law tells how their motion changes when the resultant force is not zero. Neither of these laws indicates what the origin of the force is.

For example, in the 100-meter dash, an Olympic track star will go from rest to nearly his top speed in a very short time. With high speed photography his initial acceleration

D16: Action – reaction (rope) I
D17: Action – reaction (rope) II
D18: Reaction force of a wall

A: Action-reaction model
T8: Tractor-log paradox

Summary 3.9
1. Whenever two bodies interact, the forces they exert on each other are equal in magnitude and opposite in direction. 2. The forces always exist in pairs. 3. The forces always arise as a result of interactions between objects.

4. Action and reaction begin and end at precisely the same time and act on different objects.*

5. The third law is universally applicable.

*(One of the halves of an action-reaction pair can be a field rather than an "object".)

could be measured. Also, we could measure his mass. With mass and acceleration known, we could use $\vec{F} = m\vec{a}$ to find the force acting on him. But where does the force come from? It must have something to do with the runner himself, but can he exert a force on himself as a whole? Can you lift yourself by your own bootstraps?

Newton's third law of motion helps us to explain just such puzzling situations. First, let us examine the third law to see what it claims. In Newton's words,

> To every action there is always opposed an equal reaction: or, mutual actions of two bodies upon each other are always equal and directed to contrary parts.

This is a rather literal translation. It is generally agreed, however, that the word force may be substituted for both the word action and the word reaction in Newton's statement.

The most startling idea to come out of this statement is that forces always exist in pairs. Indeed, any thought of a single unaccompanied force is without any meaning whatsoever. On this point Newton wrote:

> Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone.

This suggests that forces always arise as a result of interactions between objects: object A pushes or pulls on B while at the same time object B pushes or pulls with precisely equal amount on A. These paired pulls and pushes are always equal in magnitude and opposite in direction.

The terms action and reaction are arbitrary, as is the order of their naming. The action does not cause the reaction. The two coexist. And most important, they are not acting on the same body. In a way, they are like debt and credit: one is impossible without the other; they are equally large but of opposite sign, and they happen to two different objects.

We can describe the situation where A exerts a force on B and at the same time B exerts on A an equal and opposite force. In the efficient shorthand of algebra we may write

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

This is the equivalent of Newton's statement that,

> Whenever two bodies interact, the forces they exert on each other are equal in magnitude and opposite in direction.

Note, now, what the third law does not say—for this, too, is of importance. It does not speak of how the push or pull is applied, whether it is through contact (if we could

The Principia was written in Latin, although in Newton's day scholars were beginning to use their native language more and more in their writings. The English language itself has always been changing, and so what constitutes the most accurate translation of seventeenth-century Latin into twentieth-century English is not beyond dispute.

In the collision between the ball and the club, the force the ball exerts on the club is equal and opposite to the force the club exerts on the ball.

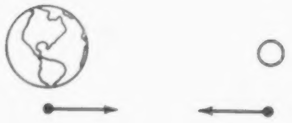
define that word) or by magnetic action or electrical action. Nor does the law require that the force be either an attraction or repulsion. The third law really does not depend on any particular kind of force. Indeed, what makes the third law extremely valuable, is its universal nature.

Study Guide 3.19

Q13 A piece of fishing line will break if the force exerted on it is greater than 500 N. Will the line break if two people at opposite ends of the line pull on it, each with a force of 300 N?

D19: Newton's third law
D20: Action-reaction (car)
D21: Action-reaction (nail)
D22: Action-reaction (jumping)

3.10 Using Newton's laws of motion. We have discussed Newton's three laws of motion in some detail. In doing so, we saw that each law is important in its own right. The first law emphasizes the modern point of view for the study of motion: what requires explanation is not uniform motion, but change of motion. The first law stresses that what one must account for is why an object speeds up or slows down or changes direction. The second law asserts that the rate of change of velocity of an object is related to both the mass of an object and the net force applied to it. In fact, the very meanings of force and mass are bound up in the second law. The third law is a statement of the force relationship between interacting objects.



The force on the moon due to the earth is equal and opposite to the force on the earth due to the moon.

Summary 3.10
1. The first law is perfectly general and is intended to apply to planets and stars no less than to objects on the earth.

Despite their individual importance, Newton's three laws are most powerful when used together to explain complex phenomena. Let us examine a few specific examples that illustrate the use of these laws.

Example 1. On Monday, September 12, 1966, a dramatic experiment based on Newton's second law was carried out high over the earth. In this experiment, the mass of an orbiting Agena rocket was determined by accelerating it with a push from a Gemini spacecraft. After the Gemini spacecraft made contact with the Agena rocket, the aft thrusters on the Gemini, calibrated to give a thrusting force of 890 N, were fired for 7 sec. The change in velocity of the spacecraft and rocket was 0.93 m/sec. The mass of the Gemini spacecraft was 3360 kg. What was the mass of the Agena?

2. Newton's second law allows us to give quantitative answers to questions about motion. Such answers lead to new questions and new insights. 3. The third law applies universally to all interacting objects whether or not there is any obvious physical connection between them.

A force of magnitude 890 N acts on a total mass M where

$$M = M_{Gemini} + M_{Agena} \text{ or } M = 3360 \text{ kg} + M_{Agena} .$$

The magnitude of the acceleration is given as follows:

$$a = \frac{\Delta v}{\Delta t}$$

$$= \frac{.93 \text{ m/sec}}{7 \text{ sec}}$$

$$= 0.13 \text{ m/sec}^2.$$

Newton's second law gives us the relation

$$\bullet T = Ma$$

or

$$T = (M_{Agena} + 3360 \text{ kg})a$$

where T is the thrust. Solving for M_{Agena} gives

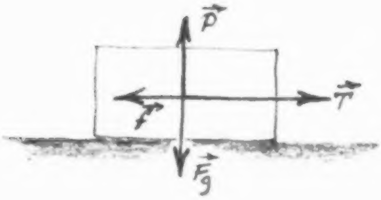
$$M_{Agena} = \frac{T}{a} - 3360 \text{ kg} = \frac{890 \text{ N}}{.13 \text{ m/sec}} - 3360 \text{ kg}$$

$$M_{Agena} = 6850 \text{ kg} - 3360 \text{ kg} = 3490 \text{ kg}.$$

The mass of the Agena was known to be approximately 3660 kg which means there was a 5% error in the measurement. (This experiment was performed to determine the feasibility of this technique as a means for finding the mass of a foreign satellite while it is in orbit.)

In this equation, T is used for force since it is a thrusting force.

•Students dislike using different symbols to represent the same quantity, or using the same symbol to represent different quantities. However, physicists are like Humpty Dumpty, who once said, "When I use a word it means exactly what I choose it to mean, neither more nor less... [it's a] question of which is to be master, that's all." (Through the looking glass, Lewis Carroll.



Example 2. A case of books whose mass is 8.0 kg rests on a table. What constant horizontal force $\vec{T}$ is required to give it a velocity of 6 m/sec in 2 sec, starting from rest, if the friction force $\vec{f}$ between the moving case and the table is constant and is equal to 6 newtons? (Assume all forces act at the center of the case.)

In solving problems such as this, it is always helpful to make a sketch showing the forces acting. The forces acting on the case are the frictional force $\vec{f}$, the force $\vec{T}$, the force of gravity $\vec{F}_g$ (the case's weight), and the force of the table on the case $\vec{P}$. The case pushes down on the table with a force $\vec{F}_g$ and the table pushes back on the case with a force $\vec{P}$ (Newton's third law). Therefore

What is your interpretation of this equation?

$$\vec{F}_g = -\vec{P}$$

or

$$\vec{F}_g + \vec{P} = 0.$$

For the forces parallel to the surface we can write

$$\text{unbalanced force} = \vec{T} - \vec{f} = \vec{T} - 6\text{N}$$

From the second law we have

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{T} - 6\text{N} = m\vec{a}$$

$$\vec{T} = m\vec{a} + 6\text{N}$$

Summary 3.11
1. All motion and hence Newton's laws of motion must be specified with regard to some frame of reference.

2. All the forces of nature are manifestations of four basic interactions.

The magnitude of the acceleration is

$$a = \frac{\Delta v}{\Delta t} = \frac{6\ \text{m/sec}}{2\ \text{sec}} = 3\ \text{m/sec}^2.$$

Thus we can now determine the magnitude of the horizontal force.

Study Guide 3.13, 3.14, 3.16, 3.18

$$T = ma + 6.0\ \text{N} = (8.0\ \text{kg})\ (3\ \text{m/sec}^2) + 6.0\ \text{N}$$

$$T = 24\ \text{N} + 6\ \text{N} = 30\ \text{N}.$$

3.11 Nature's basic forces. Our study of Newton's laws of motion has increased our understanding of objects at rest, in uniform motion, and accelerating. However, we have accomplished much more in the process. Newton's first law alerted us to the importance of reference frames. What you observe depends upon your point of view—your frame of reference. A critical analysis of the relationship between descriptions from different frames of reference was a forerunner of the theory of relativity.

Newton's second law alerted us to the importance of forces. In fact, it presents us with a mandate: when we observe acceleration, find the force! For example, when we recognize that an orbiting satellite is accelerating, we look for a force. We might begin this search by giving the force a name, for example, gravitational force.

Here we gain insight into the second law, which equates intrinsic properties (mass) and behavior (acceleration) of an object with properties of the objects environment (force).

But a name alone adds nothing to our basic understanding. We really want to know what determines the force acting on a satellite. Does the force depend on the earth? Obviously it does. Does the force depend on the satellite's position? On its velocity? On the time? Answers to questions such as these can be summarized in a force law which describes the force in terms of those factors it depends on. A force law provides a basis for understanding the way in which the earth and a satellite interact with each other. Knowing the force law, the physicist claims to "understand" the nature of the interaction.

Gravitational attraction is just one of the basic ways in which objects interact. It is exciting to realize that there appear to be very few of these basic interactions. In fact, physicists now believe there are just four. Does it surprise you to think there are so few? Imagine—all we observe in nature is the consequence of just four basic interactions. In terms of our present understanding, all the forces of nature—subnuclear and nuclear, atomic and molecular, terrestrial and solar, galactic and extragalactic—are the manifestations of these four basic interactions.

There is, of course, nothing sacred about the number four. The number might be reduced or enlarged due to new discoveries. In fact, physicists hope that as they gain further insight into these basic interactions, two (or more) of them might be seen as the consequence of something even more basic.*

*Einstein spent many years trying to find a link between gravity and electromagnetism.

The first interaction, the gravitational, becomes important only on a very large scale, when there are tremendous amounts of matter involved. It literally holds the universe together. The second interaction concerns electric and magnetic processes. These processes are most important on a small scale—the atomic and molecular scale. We know the force laws governing gravitational and electromagnetic interactions; therefore they are fairly well "understood." The situation changes completely when we consider the remaining basic interactions. They are still the subject of vigorous research today. The third interaction (the so-called "strong" interaction) somehow holds the nucleus together.

"The Starry Night", 1889, by Vincent van Gogh.
Collection, The Museum of Modern Art, New York.

The fourth interaction (the so-called "weak" interaction) governs certain reactions among subnuclear particles.

We do, of course, have other names for forces. One of the most common, yet one of the least understood, is the frictional force. This subtle force fooled people for centuries into thinking that an object required a "pusher" or "puller" if it was to remain in motion. Yet, the frictional force is undoubtedly an electrical type of force; that is, the atoms on the surfaces of the objects sliding or rubbing against each other interact electrically. In this case, too, we seem to be able to understand all our observations of nature in terms of just a few basic interactions.

We shall be encountering some of these ideas again. We shall meet the gravitational force in Unit 2, the electrical and magnetic forces in Unit 4 and the nuclear force in Unit 6. In all these cases remember that a force plays the same role regardless of its origin; that is, an object sensitive to the force will be accelerated.

The knowledge that there are so few basic interactions is both surprising and encouraging. It is surprising because at first glance the world seems so complicated; it is encouraging because our elusive goal—an understanding of nature—seems nearer.

The intuition that all of nature's phenomena are interlinked on a grand scale is shared by scientists as well as artists.

3.1 Newton's First Law: Every object continues in its state of rest or of uniform rectilinear motion unless acted upon by an unbalanced force.

Newton's Second Law: The acceleration of an object is directly proportional to, and in the same direction as, the unbalanced force acting on it, and inversely proportional to the mass of the object.

Newton's Third Law: To every action there is always opposed an equal reaction: or, mutual actions of two bodies upon each other are always equal and directed to contrary parts.

3.2 The Aristotelian explanation of motion should not be dismissed lightly. Great intellects of the Renaissance period such as Leonardo da Vinci, who, among other things, designed artillery for launching projectiles, apparently did not challenge the Aristotelian explanation. One reason for the longevity of Aristotle's ideas is that they are so closely aligned with our common-sense ideas. In what ways do your common-sense notions of motion agree with Aristotle's? Discussion

3.3 Three children, Karen, Keith and Sarah are each pulling on the same toy.

Karen pulls toward the east with a force of 8 units.

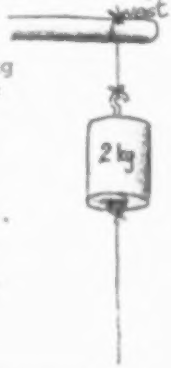
Sarah pulls toward the north with a force of 6 units.

Keith pulls in a direction 30° south of west with a force of 12 units.

a) Is there a net (i.e., unbalanced) force on the toy? Yes

b) If there is a net force, what is its magnitude and direction? 4.4 units west

3.4 A 2 kg mass is suspended by a string. A second string is tied to the bottom of the mass.



a) If the bottom string is pulled with a sudden jerk, the bottom string breaks. Discussion

b) If the bottom string is pulled with a steady pull, the top string breaks. Discussion

Explain.

(Try this experiment for yourself. You might tie thread to a brick, or a hammer, or a pipe wrench.)

3.5 In terms of Newton's first law, explain:

a) why people in a moving car lurch forward when the car suddenly decelerates;

b) what happens to the passengers of a car that makes a sharp and quick turn;

c) why, when a coin is put on a phonograph turntable and the motor is started, the coin flies off when the turntable reaches a certain speed. Why doesn't it fly off before? Discussion

3.6 a) You exert a force on a box, but it does not move. How would you explain this? How might an Aristotelian explain it?

b) Suppose now that you exert a greater force, and the box moves. Explain this from your point of view and from an Aristotelian point of view. Discussion

3.7 Assume that the floor of a laboratory could be made perfectly horizontal and perfectly smooth. A block of wood is placed on the floor and given a small push. Predict the way in which the block will move. How would this motion differ if the whole laboratory were moving with constant velocity during the experiment. How would it differ if the whole laboratory were accelerating along a straight line? If the block were seen to move in a curved path along the floor, how would you explain this? Discussion

3.8 A body is being accelerated by an unbalanced force. If the magnitude of the net force is doubled and the mass of the body is reduced to one-third of the original value, what will be the ratio of the second acceleration to the first? 6/1

3.9 Hooke's Law says that the force exerted by a stretched or compressed spring is directly proportional to the amount of the compression or extension. As Robert Hooke put it in announcing his discovery:

> ...the power of any spring is in the same proportion with the tension thereof: that is, if one power stretch or bend it one space, two will bend it two, three will bend it three, and so forward. Now as the theory is very short, so the way of trying it is very easie.

If Hooke says it's "easie," then it might well be so. You can probably think immediately of how to test this law using springs and weights. Try designing such an experiment; then after checking with your teacher, carry it out.

Discussion

Hooke's experiment is described in his own words in W. F. Magie, A Source Book in Physics, McGraw-Hill, 1935.

3.10 If you have dynamics carts available, here is one way of doing an experiment to test the inverse proportionality between acceleration and mass:

a) Add load blocks to one or the other of two carts until the carts balance when placed on opposite platforms of a laboratory balance. Balance a third cart with one of the first pair. Each cart now has mass m. (State two main assumptions involved here.)

b) Accelerate one cart on a level surface using the rubber-band technique; that is, pull the car with the rubber band keeping it stretched a constant amount. Any other method can also be used that will assure you that, within reason, the same force is being applied each time. Record the position of the cart at equal time intervals by means of stroboscopic photography.

c) Repeat the last step in all details, but use two carts hooked together. Repeat again using all three carts hooked together. In all three cases it is crucial that the applied force be essentially the same.

d) Determine the value of acceleration for masses of m (1 cart), 2m (2 carts), and 3m (3 carts).

e) Prepare a graph of a vs. m, of a vs. $\frac{1}{m}$, and of $\frac{1}{a}$ vs. m. Comment on your results. Discussion

3.11 Complete this table:

a)	1.0	N	1.0	kg	1.0	m/sec^2
b)	24.0	N	2.0	kg	12.0	m/sec^2
c)		N	3.0	kg	8.0	m/sec^2
d)		N	74.0	kg	0.2	m/sec^2
e)		N	0.0066	kg	130.0	m/sec^2
f)	72.0	N		kg	8.0	m/sec^2
g)	3.6	N		kg	12.0	m/sec^2
h)	1.3	N		kg	6.4	m/sec^2
i)	30.0	N	10.0	kg		m/sec^2
j)	0.5	N	0.20	kg		m/sec^2
k)	120.0	N	48.0	kg		m/sec^2

Table

3.12 Recount in detail what steps you must take (in idealized experimentation) to determine the unknown mass m (in kilograms) of a certain object if you are given nothing but a frictionless horizontal plane, a 1-kg standard, an uncalibrated spring balance, a meter stick, and a stopwatch. Discussion

3.13 A certain block is dragged with constant velocity along a rough horizontal table top, by means of a spring balance horizontally attached to it which reads 0.40 N, no matter what the velocity happens to be. This means that the retarding frictional force between block and table is 0.40 N and not dependent on the speed. When the block is given a constant acceleration of 0.85 m/sec^2, the balance is found to read 2.1 N. Compute the mass of the block. $m = 2.0$ kg

3.14 A sled has a mass of 4440 kg and is propelled by a solid propellant rocket motor of 890,000 N thrust which burns for 3.9 seconds.

$a = 2.0 \times 10^2$ m/sec^2 .... a) What is the sled's average acceleration and maximum speed?

$v_{max} = 7.8 \times 10^2$ m/sec^2

b) The data source states that this sled has a maximum acceleration of 30g ($=30 \times a_g$). How can that be, considering the data given? a varied

c) If the sled travels a distance of 1530 m while attaining a top speed of 860 m/sec (how did it attain that high a speed?!), what is its average acceleration? $\bar{a} = 2.4 \times 10^2$ m/sec^2

3.15 Discuss the statement that while the mass of an object is the same everywhere, its weight may vary from place to place.

3.16 A 75 kg man stands in an elevator. What force does the floor exert on him when the elevator

F=860N ... a) starts moving upward with an acceleration of 1.5 m/sec^2?

F=750N ... b) When the elevator moves upward with a constant speed of 2.0 m/sec?

F=640N ... c) When the elevator starts accelerating downward at 1.5 m/sec^2?

The same values as (a), (b), (c). d) If the man were standing on a bathroom (spring) scale during his ride, what readings would the scale have in parts a, b, and c?

Discussion e) It is sometimes said that the "apparent weight" changes when the elevator accelerates. What could this mean? Does the weight really change?

3.17 A replica of the standard kilogram is constructed in Paris and then sent to the National Bureau of Standards in Washington. Assuming that this secondary standard is not damaged in transit, what is (a) the same mass

(b) $F(\text{Paris}) = 9.809$ N

(c) $F(\text{Washington}) = 9.801$ N

a) its mass in Washington,
b) its weight in Paris and in Washington. (In Paris, a_g=9.81 m/sec^2; in Washington, a_g=9.80 m/sec^2.)

3.18 Consider the system consisting of a 1.0 kg ball and the earth. The ball is dropped from a short distance above the ground and falls freely. We can take the mass of the earth to be approximately 6.0×10^{24} kg.

a) Make a vector diagram illustrating the important forces acting on the members of the system.
b) Calculate the acceleration of the earth in this interaction.
c) Find the ratio of the magnitude of the ball's acceleration to that of the earth's acceleration (a_b/a_e).

3.19 In terms of Newton's third law assess the following statements:

a) You are standing perfectly still on the ground; therefore you and the earth do not exert equal and opposite forces on each other.
b) The reason that a jet airplane cannot fly above the atmosphere is that there is no air to push against, as required by the third law.
c) The mass of object A is 100 times greater than that of object B, but even so the force it (A) exerts on B is no greater than the force of B on it.
d) C, D, and E are three objects having equal masses; if C and D both push against E at the same time, then E exerts only one-half as much force on C as C does on E. Discussion

3.20 Consider a tractor pulling a heavy log in a straight line. On the basis of Newton's third law, one might argue that the log pulls back on the tractor just as strongly as the tractor pulls the log. But why, then, does the tractor move?

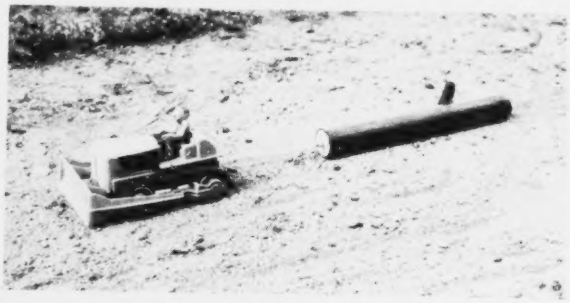
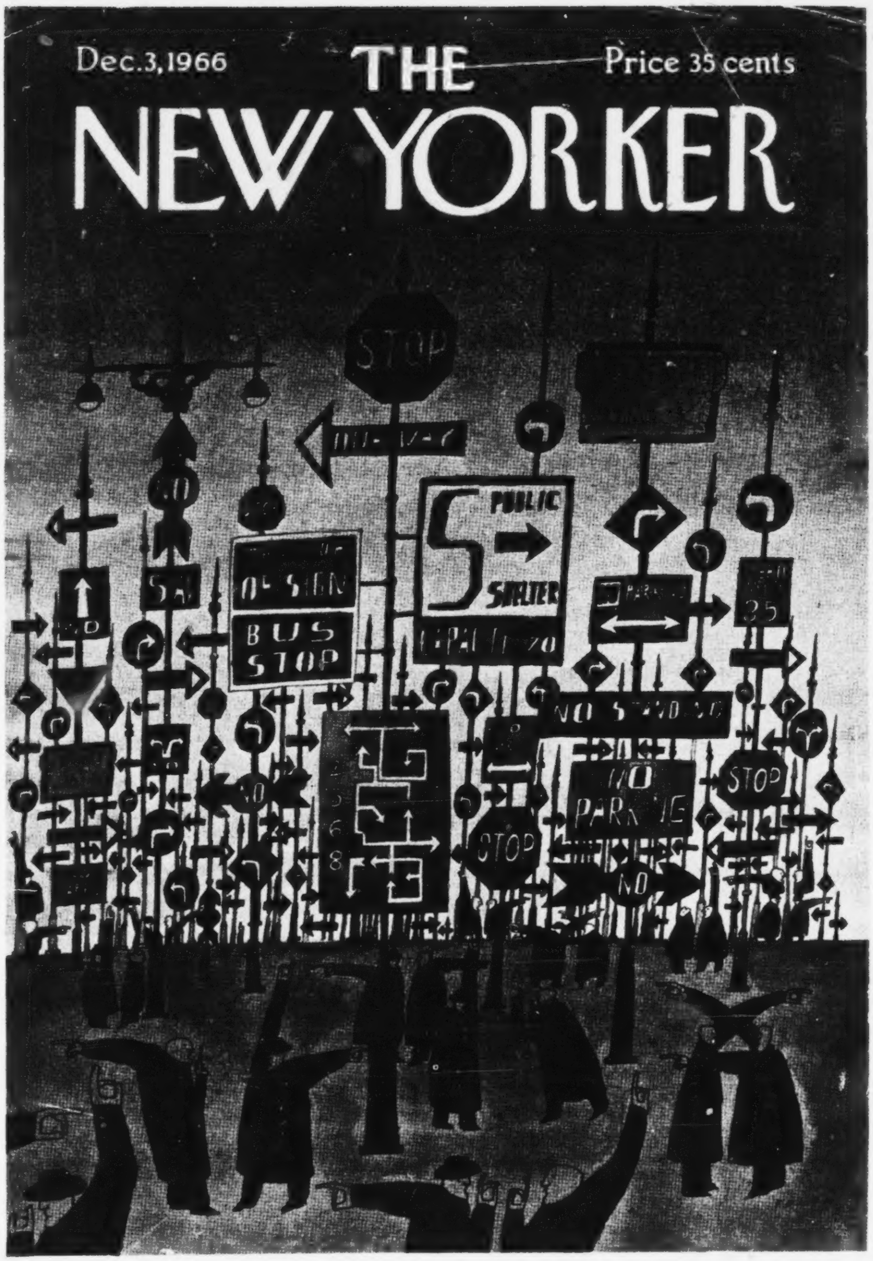


diagram ...

1.7×10^{-24} m/sec^2......

$a_b/a_c = 6 \times 10^{24}$......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Chapter 4 Understanding Motion

Summary 4.1

1. A motion that appears complex often can be broken down into components which are each relatively simple to analyze and describe.

2. Concepts about rectilinear motion can be usefully applied to the study of complex motions.

3. Projectile motion and uniform circular motion are simple but representative types of curvilinear motion.

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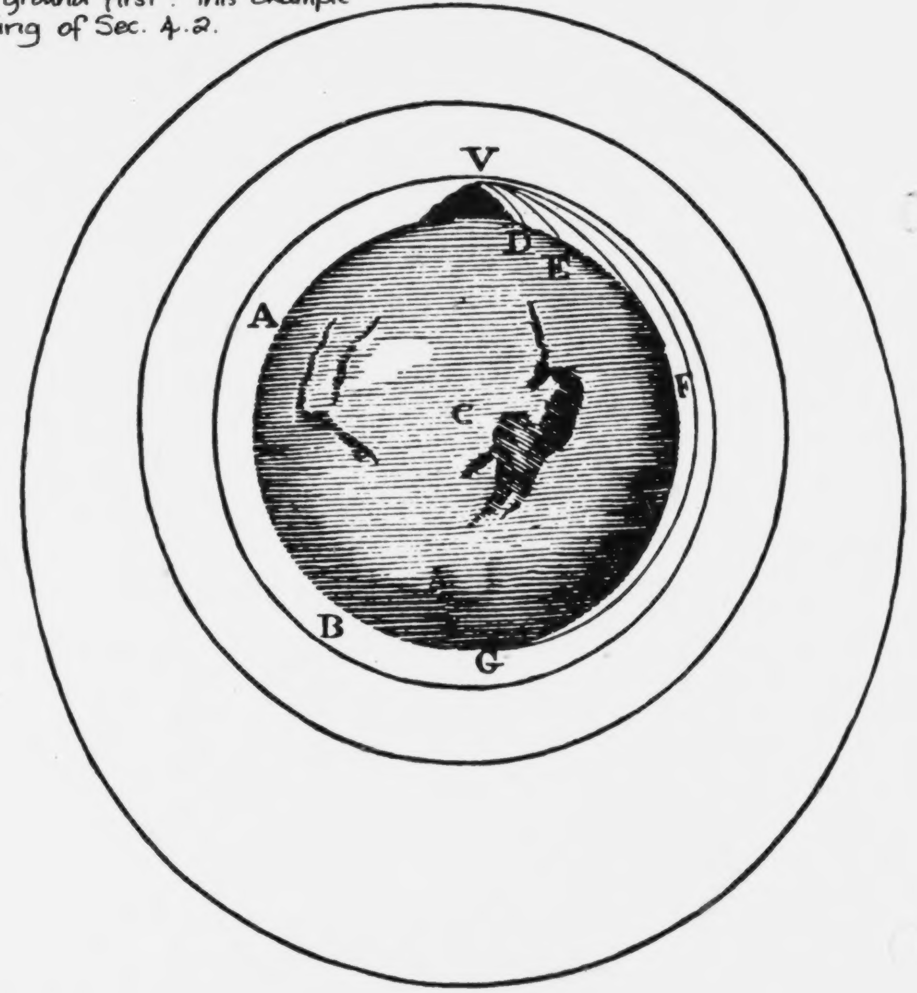
To stimulate discussion: If a bullet is shot horizontally and another bullet dropped at the same instant, which bullet will strike the ground first? This example is discussed at the beginning of Sec. 4.2.

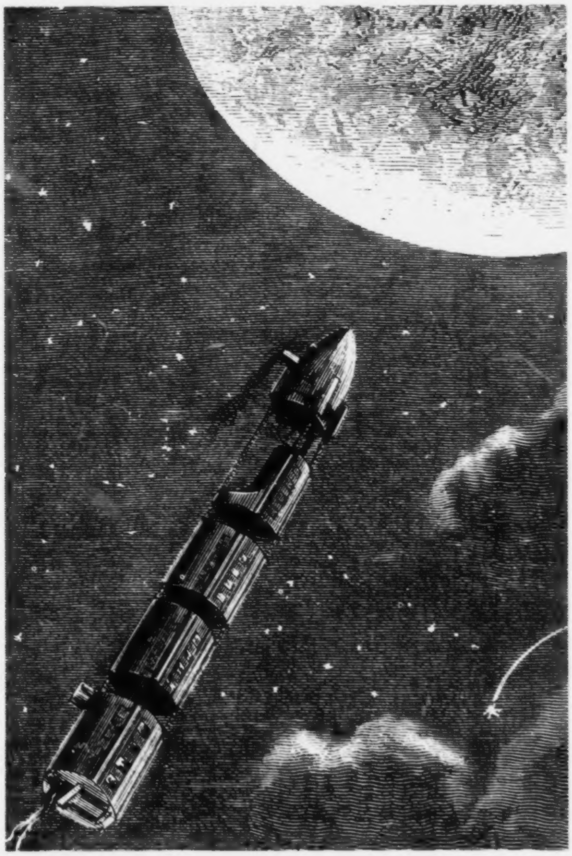
"...a stone that is projected is by the pressure of its own weight forced out of the rectilinear path, which by the initial projection alone it should have pursued, and made to describe a curved line in the air; and through that crooked way is at last brought down to the ground; and the greater the velocity is with which it is projected, the farther it goes before it falls to the earth. We may therefore suppose the velocity to be so increased, that it would describe an arc of 1, 2, 5, 10, 100, 1000 miles before it arrived at the earth, till at last, exceeding the limits of the earth, it should pass into space without touching it." [Newton's Principia]



4.1 A trip to the moon. Imagine a Saturn missile taking off in the early morning hours from its launching pad at Cape Kennedy. It climbs in a curved path above the earth, passing through the atmosphere and beyond. Successive stages of the missile burn out leaving finally an instrument capsule hurtling through the near vacuum of space toward its destination 240,000 miles away. Approximately 65 hours after take-off, the capsule circles the moon and plummets to its target—the center of the lunar crater Copernicus.

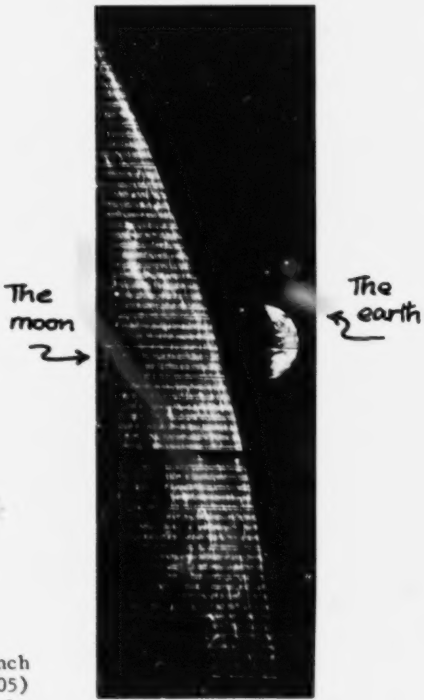
As you first think about it, you are likely to be struck by the complexity of such a voyage. The atmospheric drag at the beginning of the flight depends upon the missile's speed and altitude; the rocket's thrust changes with time. You must consider the changing gravitational pulls of the sun, the earth, and the moon as the capsule changes its position relative to them. Besides the forces, you must consider the facts that the rocket's mass is changing and that it is launched from a spinning earth, which in turn is circling the sun. Furthermore, the target—the moon—is moving around the earth at a speed of about 2,300 miles per hour.

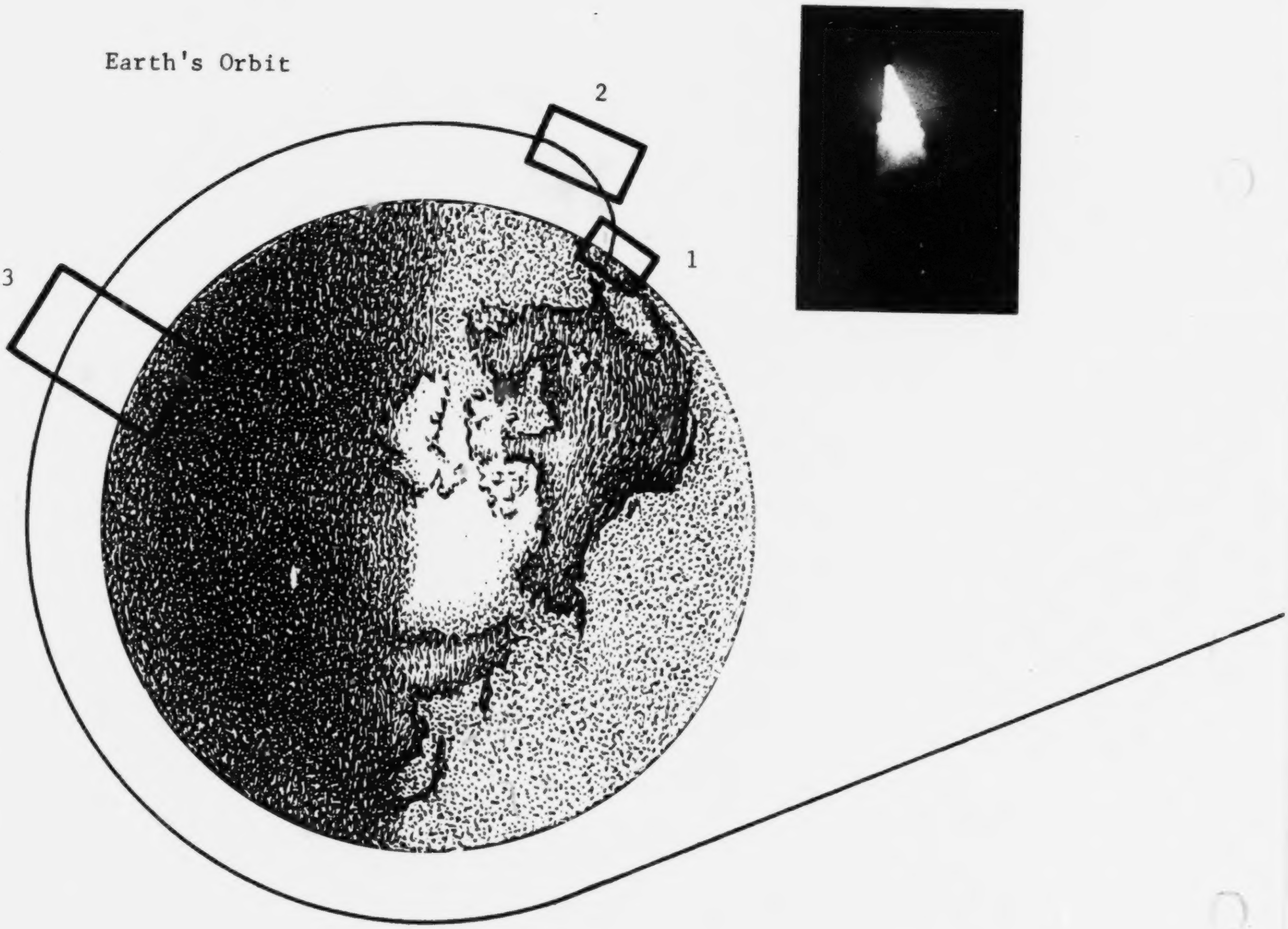
The complexities of a rocket flight from earth to moon are indeed great and the amount of computation enormous—which is why NASA, the National Aeronautics and Space Administration, uses high speed electronic computers to help analyze and control the flight path. Though complicated in its totality, this flight can be broken down into small portions which are each relatively simple to analyze and describe. What we have learned in earlier chapters will be useful in this task.



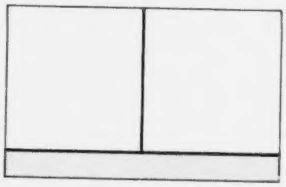
Over 100 years ago, the French author Jules Verne (1828-1905) portrayed how technology might be employed to place a man on the moon. In two prophetic novels, Verne launched three intrepid spacemen to the moon by means of a gigantic charge fixed in a steel pipe deep in the earth.

The world's first view of the earth taken by a spacecraft from the vicinity of the moon.



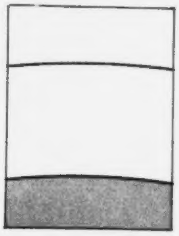


The earth-moon trip shown in the figure above can be divided into these six parts.

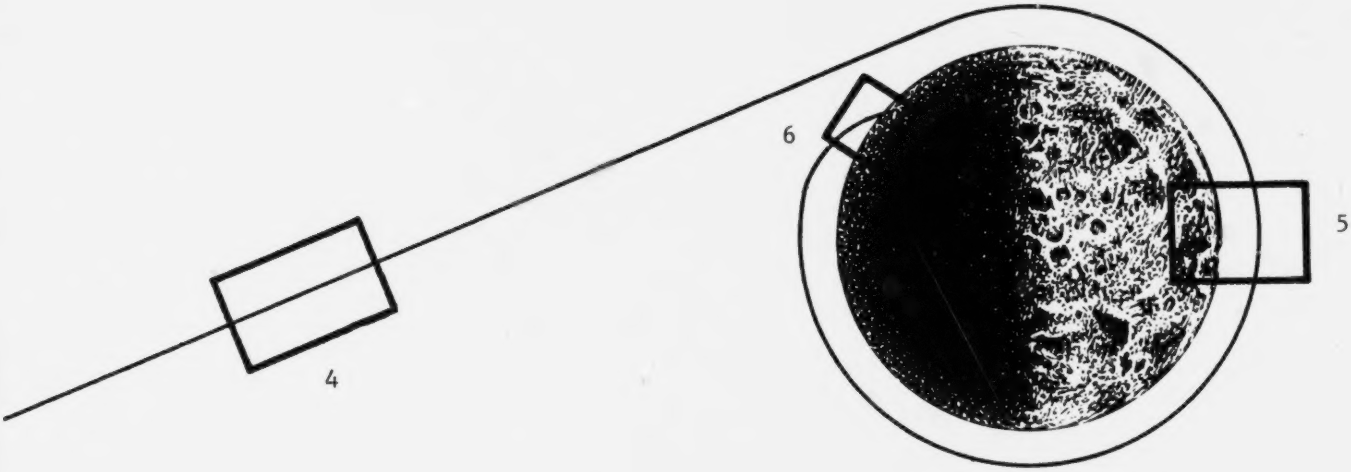


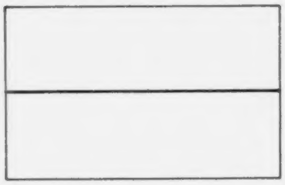
Part 1. The rocket accelerates vertically upward from the surface of the earth. The force acting on the rocket is essentially constant. The mass of the rocket, however, is decreasing. The value of the acceleration at any instant can be computed using Newton's second law.

Part 2. The rocket, still accelerating, follows a curved path as it enters into an orbit about the earth.



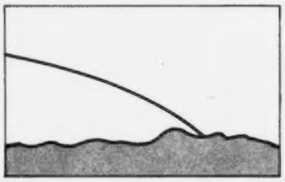
Part 3. In an orbit 115 miles above the earth's surface, the rocket circles at a constant speed of 17,380 miles/hr. The minimum escape velocity is 24,670 miles/hr; therefore, by accelerating in the direction of its path when it has reached the bottom of the semi-circular arc, the rocket can now thrust into distant space.





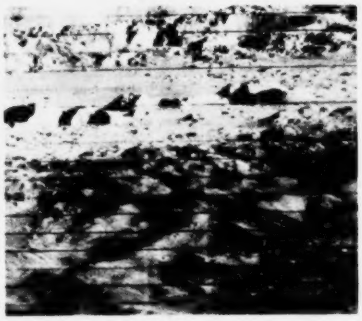
Part 4. In flight between earth and moon, only occasional bursts from the capsule's thrusters are required to keep it on course. Between these correction thrusts, the capsule moves under the influence of the gravitational forces of earth, moon, and sun. We know from Newton's first law that the capsule would move with constant velocity if it were not for these forces.

Part 5. The capsule is moving with constant speed of 1 mile/sec in a circular path 50 miles above the moon's surface.



Part 6. The capsule is accelerating toward the surface of the moon. It follows an arching path before landing in the crater Copernicus.

Let us analyze in greater detail the last two parts of this trip—the capsule circling the moon and then falling to the moon's surface—since they are examples of circular motion and projectile motion, two important classes of motion. How shall we go about this? Must we travel to the moon, set up our cameras on the edge of the crater Copernicus, and make a stroboscopic record of the path of the capsule as it streaks through the lunar vacuum and crashes into the moon's surface? Not at all! We now realize, thanks to Galileo • and Newton, that we can learn about the behavior of moving objects beyond our reach by studying the motion of objects near at hand.



• Remember the last paragraph of Sec. 2.1. Galileo did not clock falling objects - he studied a ball rolling down an incline. To learn about projectiles we might observe a ball rolling off the edge of a table.

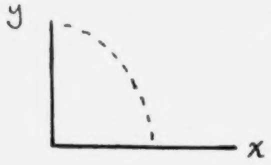
Summary 4.2

1. A projectile is an object that coasts freely, moving independently or nearly independently of the material, if any, that it encounters.

2. A projectile that is launched horizontally falls toward the earth with the same acceleration as an object that is dropped; the vertical motions of both objects are described (over reasonably small distances) by the equation $y = a_g t^2/2$.

4.2 Projectile motion. Imagine a rifle mounted on a tower with its barrel parallel to the ground. Imagine also that the ground over which the bullet will travel is level for a very great distance. Suppose further that at the instant a bullet leaves the rifle, a second, identical bullet is dropped from the same height as the muzzle of the rifle. The second bullet has no horizontal motion relative to the ground. Which bullet will reach the ground first? Do we need to know something about the muzzle velocity of the bullet and the height of the tower before we can answer this question?

To avoid confusion in notation, we let the displacement in the horizontal direction be x and the displacement in the vertical direction be y. This leads to the set of axes.



Consider first the motion of the bullet that is dropped. As a freely falling object, it accelerates toward the ground with uniform acceleration. Hence, in a time t it will fall a distance, y, given by

$$y = \tfrac{1}{2}a_g t^2$$

where a_g is the acceleration due to gravity.

The bullet that is fired horizontally from the rifle is an example of a projectile. Any object that is given an initial velocity and whose subsequent path is determined solely by the gravitational force and by the resistance of the air is a projectile. The path followed by a projectile is its trajectory. As the gunpowder explodes, the bullet is driven by the force of expanding gases and accelerated very rapidly until it reaches the muzzle of the rifle. On reaching the muzzle these gases escape and no longer push the bullet. At this moment, however, the bullet has a very large horizontal speed, v_x. The air will slow the bullet slightly, but we shall ignore that fact in our development

3. The downward acceleration of a projectile does not affect its horizontal motion, which is uniform and is described by the equation $x = v_x t$.

4. The superposition principle holds that the motion of a projectile can be considered as a combined motion of its horizontal and vertical motions, which are independent of each other.

and imagine an ideal case with no air friction. As long as air friction is ignored, there is no net force acting in the horizontal direction. Hence, the horizontal speed will remain constant. From the instant the bullet leaves the muzzle, we would expect its horizontal motion to be described by the equation

$$x = v_x t.$$

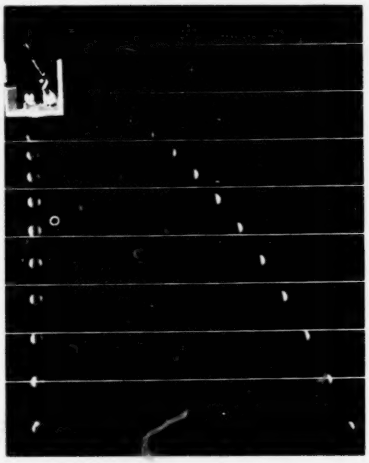
As soon as the bullet leaves the gun, however, it becomes an unsupported body and falls toward the earth as it moves forward. Can we use the same equation to describe its fall that we used to describe the fall of the dropped bullet? That is, can we use $y = \frac{1}{2} a_g t^2$ to describe the fall of the high speed bullet? We believe, of course, that the bullet will fall to the ground, for any other answer would be contrary to our experience. But, whether it will fall at the same rate as the bullet with no horizontal motion is not clear. Nor, for that matter, can we be sure how falling will affect the bullet's horizontal motion. These doubts raise a more fundamental question that goes beyond just the behavior of bullets; namely, is the vertical motion of an object affected by its horizontal motion? Or vice versa?

To answer these questions, we can carry out a real experiment similar to our thought experiment. We can use a special laboratory device designed to fire a ball in a horizontal direction at the moment that a second ball is released to fall freely from the same height. We set up our apparatus so that both balls are the same height above a level floor. The experiment is started. Although the motions of the balls may be too rapid for us to follow with the eye, we hear only a single sound as they strike the floor. This result suggests that the vertical motion of the projected ball is unaffected by its horizontal velocity.

Let us examine a stroboscopic photograph of this experiment. Equally spaced lines in the background aid our examination. Look first at the ball which was released without any horizontal motion. You see that it is accelerated because as it moves it travels a greater distance between successive flashes. Careful measurement of the photograph shows that the acceleration is uniform to within the precision of our measurements.

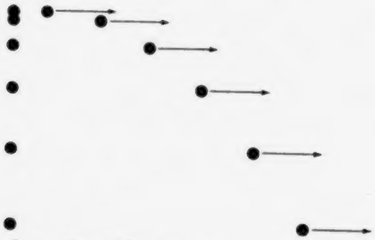
Now compare the vertical positions of the ball fired to the right with the vertical positions of the ball which is falling freely. The horizontal lines show that the dis-

The Physics Teacher
October, '66
"Water Parabola"



The two balls in this stroboscopic photograph were released simultaneously. The one on the left was simply dropped from rest position; the one on the right was given an initial velocity in the horizontal direction.

T9: Projectile motion
A : Speed of water stream, superposition principal pictures (projectile)

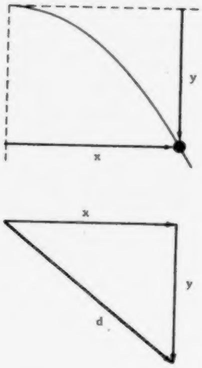


A projectile has a constant horizontal velocity v_x.

tances of fall are the same for corresponding times. The two balls obey the same law for motion in a vertical direction. That is, at every instant they both have the same constant acceleration, a_g, the same downward velocity, and the same vertical displacement.

We can use the strobe photo to see if the downward acceleration of the projectile affects its horizontal velocity by measuring the horizontal distance between successive images. We see that the horizontal distances are essentially equal. Since the time intervals between images are equal, we can conclude that the horizontal velocity v_x is constant.

We now have definite answers to our questions. The horizontal motion of the ball does not interfere with the vertical motion, and vice versa. The two motions are completely independent of each other. This experiment can be repeated from different heights, and with different muzzle velocities, but the results will always show that the horizontal motion is independent of the vertical motion.



The displacement $\vec{d}$ of an object is a vector giving the straight-line distance from beginning to end of an actual path; $\vec{d}$ can be thought of as made up of a horizontal (x) and vertical (y) component of displacement, that is, $\vec{d} = \vec{x} + \vec{y}$ (added vectorially).

The independence of motions at right angles has interesting consequences. For example, it is easy to predict the displacement and the velocity of a projectile at any time during its flight. We need merely to consider the horizontal and vertical aspects of the motion separately and then add the results—vectorially. We can predict the positions x and y and the speeds v_x and v_y at any instant by application of the appropriate equations. For the horizontal component of motion

$$v_x = \text{constant}$$

$$x = v_x t$$

and for the vertical component of motion,

$$v_y = a_g t$$

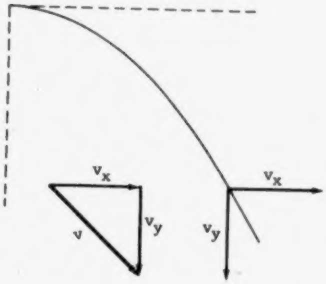
$$y = \tfrac{1}{2} a_g t^2 .$$

Because x, y, and d and v_x, v_y and v are the sides of right triangles, the magnitude d of the total vector displacement $\vec{d}$ can be written as

$$d = \sqrt{x^2 + y^2}$$

and the magnitude v of the velocity $\vec{v}$ can be written as

$$v = \sqrt{v_x^2 + v_y^2} .$$



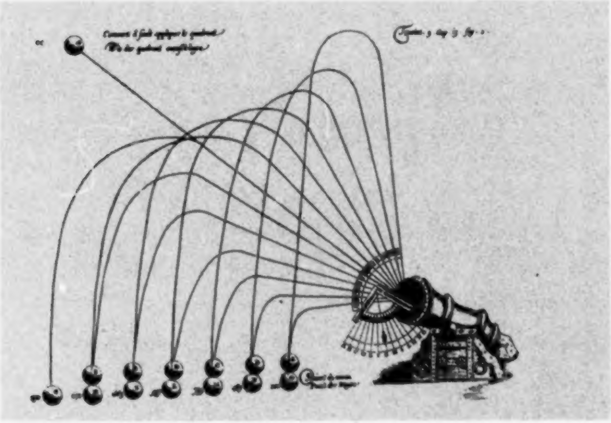
See Study Guide 4.3.

Q1 A projectile is launched horizontally with a muzzle velocity of 1,000 m/sec from a point 20 m above the ground. How long will it be in flight? How far, horizontally, will the projectile travel?

T9: Projectile motion
T10: Path of a projectile
F3: PSSC film – Free fall and projectile motion
A: Generating a cycloid, waterdrop parabola

4.3 What is the path of a projectile? It is easy to see that a thrown object, such as a rock, follows a curved path. But there are many kinds of curves, and it is not so easy to see which kind of curve a projectile traces. For example, arcs of circles, ellipses, parabolas, hyperbolas and cycloids (to name only a few geometric figures) all provide likely looking curved paths.

Ufano, a contemporary of Galileo, held a common belief about projectile trajectories. He thought that a projectile rises along a rather flat path, and then drops suddenly. Ufano was wrong, but more important is the fact that by direct observation of the moving object itself one could not determine the details of the trajectory.



The path taken by a cannon ball according to a drawing by Ufano (1621). He shows that the same horizontal distance can be obtained by two different firing angles. Gunners had previously found this by experience. What angles give the maximum range?

New knowledge about the path of a projectile was gained when the power of mathematics was applied to the problem. This was done by setting out to derive an equation that would express the shape of the path. Only a few steps are involved. First let us list equations we already know for a projectile launched horizontally.

$$x = v_x t$$

and

$$y = \frac{1}{2} a_g t^2 .$$

We would know the shape of the trajectory if we knew the height of the projectile above the ground for any horizontal distance from the launch point; that is, if we knew y for any value of x. We can find the height, y, for any horizontal distance, x, by combining our two equations in a way that eliminates the time variable. Solving the horizontal distance equation for t, we get

$$t = \frac{x}{v_x} .$$

Summary 4.3

1. By an argument based on known equations and the superposition principle, it can be deduced that (as long as a_g can be considered constant) the distance a horizontally projected object falls is proportional to the square of the distance that it moves horizontally.

2. The mathematical curve represented by this relationship,
$$y \propto x^2,$$
is a parabola; hence the conclusion, later brought out by experiments, that the trajectory of a projectile is a parabola.

3. Because we can express physical relationships in terms of mathematical equations, we are able to manipulate the equations and gain unexpected physical insight.

Because t means the same in both equations we can substitute for t in the vertical distance equation to obtain

Specialized equations such as these are not to be memorized.

$$y = \tfrac{1}{2}a_g\left(\frac{x}{v_x}\right)^2.$$

In this last equation there are two variables of interest, x and y, and three constant quantities: the number ½, the uniform acceleration of free fall a_g, and the horizontal speed v_x which is constant for any one flight. Bringing these constants together, we can write the equation as

$$y = \left(\frac{a_g}{2{v_x}^2}\right) x^2$$

or

$$y = kx^2$$

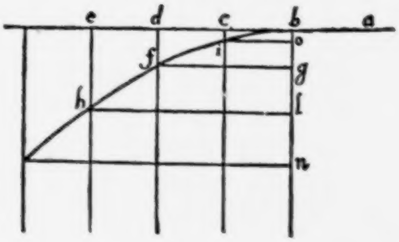
where

$$k = \left(\frac{a_g}{2{v_x}^2}\right).$$

•Point out to students that it is not unusual to lump a number of constants and assign K, k or c as the constant.

This equation relates x and y for the trajectory. We can translate it as: the distance a projectile falls (vertically) is proportional to the square of how far it moves sideways (horizontally).

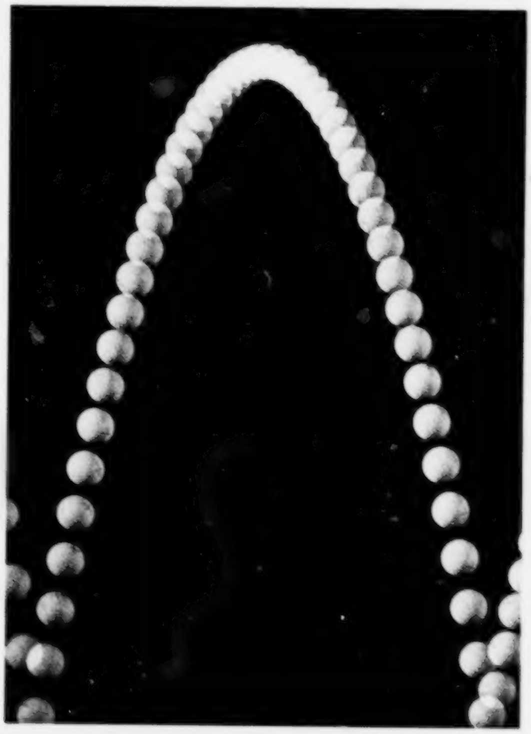
The mathematical curve represented by this relationship between x and y is called a parabola. Galileo deduced the parabolic nature of the trajectory (by an argument similar



The parabolic path of a projectile fired horizontally to the left as deduced by Galileo on theoretical grounds in his Dialogues Concerning Two New Sciences. What is the relation between distances bo, og, and gl; and why?

They are approximately in the ratio

1 : 3 : 5 : 7....



to the one we used). With this discovery, the study of projectile motion became much simpler, because the geometric properties of the parabola had been established centuries earlier by Greek mathematicians.

Here we find a clue to one of the important strategies in modern science. When we express the features of a phenomenon quantitatively and cast the relations between them into equation form, we can use the rules of mathematics to manipulate the equations, and open the way to unexpected insights.

Galileo insisted that the proper language of nature is mathematics, and that an understanding of natural phenomena is aided by translating our qualitative experiences into quantitative terms. If, for example, we find that trajectories have a parabolic shape, we can apply all we know about the mathematics of parabolas to describe—and predict—trajectories. There is always a need for well-developed systems of pure mathematics which the physicist may use to express in precise form his conceptions of natural phenomena.

"Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it." (Discoveries and Opinions of Galileo, translated by Stillman Drake, Anchor Books, pp. 237-238.)

Moreover, the physical scientist often tries to use methods from another branch of science, or from mathematics, to find a solution for his particular problem. For example, just as Galileo applied the already known mathematics of parabolas to estimate actual projectile motions, so the modern sound engineer solves problems in acoustics using mathematical schemes developed independently by electrical engineers. Whatever the methods of science may be, many ideas and concepts can often be extended from one specialty to another with fruitful results.

We can now apply our theory of projectile motion to the descent of a space capsule onto the moon's surface. The retro rockets of the orbiting capsule are fired to decrease its speed. After the retro rockets are turned off, the capsule's horizontal velocity (the velocity component parallel to the moon's surface) remains constant and the capsule falls freely under the influence of the moon's gravity. The path followed by the capsule with respect to the moon's surface is a parabola. Space-flight engineers are able to apply these ideas to land a space capsule on a desired moon target.

See Study Guide 4.4.

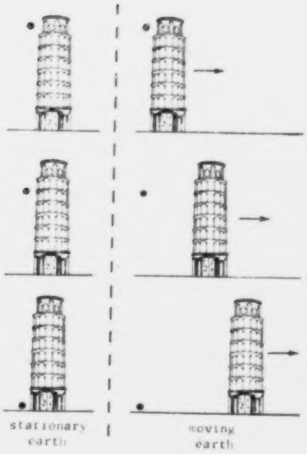
Q2 In the derivation of the path followed by a projectile, what assumptions have been made?

Write an equation for the trajectory of a projectile launched horizontally on the moon.

Q4 What is the constant in your equation in Q3?

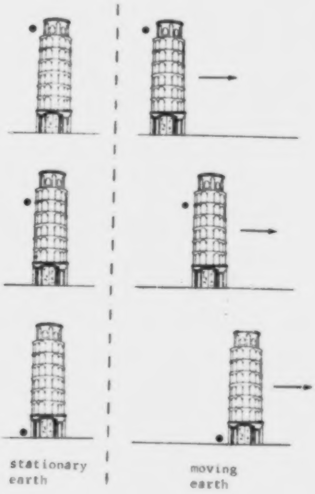
D23: Frames of reference. L6: Galilean relativity II, object dropped from aircraft.
D24: Inertial vs non-inertial reference frames. L7: Galilean relativity III, projectile fired vertically.
L4: A matter of relative motion. F4: PSSC film - Frames of reference.
L5: Galilean relativity I, ball dropped from mast of ship.

4.4 *Galilean relativity*. Galileo's work on projectiles illustrates the importance of reference frames. As you will see in Unit 2, Galileo ardently supported the idea that the reference frame for discussing motions in our planetary system be the one fixed to the sun, not the earth, and that therefore the earth both rotates on its own axis and moves in a path around the sun. For many scientists of Galileo's time, this was not an easy idea to accept. If the earth moved, they said, a stone dropped from a tower would not land directly at its base. As earth moves through space, they argued, the stone would be left behind while falling through the air, and consequently would land far behind the base of the tower. But this is *not* what happens, so many of Galileo's critics believed that the tower and the earth cannot be considered to be in motion.



The critics of Galileo claimed that if the earth moved, a dropped ball would land beyond the foot of the tower.

To answer these critics, Galileo first assumed that during the time of fall, the tower and the ground supporting it were moving forward equally with some uniform horizontal velocity v_x. He then claimed that the stone being held at the top of the tower also had the same horizontal velocity v_x, and that this velocity was not affected by the fact that the stone moves vertically upon being released. In other words, the falling stone behaves like any other projectile: the horizontal and vertical components of its motion are independent of each other. Since the stone and tower have the same v_x, the stone will not be left behind as it falls. Therefore, *whether the speed of the earth is zero or not*, the stone should land at the foot of the tower. So, the fact that falling stones are not "left behind" does not mean the earth is standing still.



Galileo argued that as the ball also shared the motion of the earth, an observer on earth could not tell whether or not the earth moved.

Similarly, Galileo said, an object released from a crow's nest at the top of a ship's straight mast will land at the foot of the mast whether the boat is standing still in the harbor or moving with constant velocity through quiet water. To a sailor standing on the ship, the trajectory will appear to be a straight vertical line in either case. To a person standing on shore, however, the trajectory appears to be a straight vertical line when the ship is stationary, and a curved line when the ship is moving. Obviously, the frame of reference of the observer must be taken into account when analyzing the motion of objects. Galileo's explanation of the differing descriptions of the falling object was that the sailor on the deck of the moving ship is sharing the horizontal velocity of both the ship and the falling object. Sailor, ship, mast and object are all moving horizontally, so he cannot notice this component of the motion. By con-

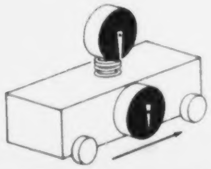
Summary 4.4
1. The motion of the observer must be taken into account when analyzing the motion of objects.

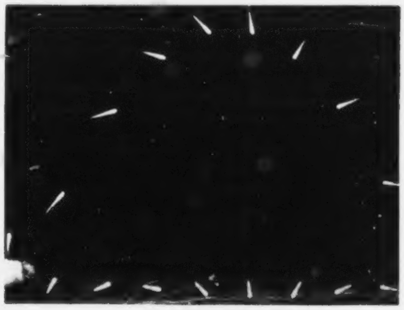
2. So far as experiments in mechanics are concerned, a laboratory that moves with constant velocity is the same as a stationary laboratory; physical relationships in one will hold true in the other.

trast, the observer on shore does not have the horizontal velocity v_x of the ship and object, and so he can see both the horizontal and vertical velocities of the falling object. These velocities, as we know, add vectorially to give a parabola.

The same ideas apply not only to falling bodies but also to projectiles in general. For example, if an object is projected vertically upward from a cart, it will fall back into the cart whether the cart is continuously moving at constant velocity or is standing still. From this, and equivalent observations, has come a most valuable generalization, usually called the Galilean relativity principle: any mechanical experiment will give the same result for any observer moving with constant velocity no matter what the magnitude and direction of the velocity. In other words, it is impossible to tell by any kind of mechanical experiment whether or not one's laboratory (reference frame) is really at rest or is moving with some constant velocity.

From the Galilean relativity principle, it follows that the laws which describe mechanical experiments are the same in a reference frame at rest or in a reference frame moving with a constant velocity. Therefore, the laws for the description of the motion of projectiles would be found to be the same whether these laws are obtained by experiments inside a ship moving with constant velocity or at the dock; whether on a stationary earth or on an earth which, during any mechanical experiment on projectiles, is moving with virtually a constant velocity. In all these cases, we are in inertial frames of reference and we would arrive at a set of equations identical to the ones we have encountered in this and the earlier chapters.





Two special clocks are attached to a cart. While the cart is moving at a constant speed, one of the clocks is sprung straight upwards from it and the subsequent motion of the two clocks is photographed under a stroboscopic light source. How do the horizontal positions of the two clocks compare in successive images?

The questions in Study Guide 4.5 and 4.6 deal with Galilean relativity.

Before turning to circular motion, consider the famous "monkey in the tree" problem. It is described in Study Guide 4.7.

Q5 Compare the results of Galileo's inclined plane experiment performed in an elevator under the following circumstances:

a) elevator at rest.
b) elevator moving uniformly upward.
c) elevator moving uniformly downward.
d) elevator accelerating uniformly upward.
e) elevator accelerating uniformly downward.

Q6 For which experiment in Q5 would a_g appear to be the largest?

4.5 Circular motion. A projectile launched horizontally from a tall tower strikes the earth at a point determined by the speed of the projectile, the height of the tower and the acceleration due to the force of gravity. As the projectile's launch speed is increased, it strikes the earth at points farther and farther from the tower's base. (The assumptions we made in the analysis of projectile motion such as a "flat"

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April, '63
"Teaching the dynamics of uniform circular motion"

D25: Uniform circular motion
A: Distortion due to rotating reference frame
Determining the speed of a pellet gun pellet

E11: Circular motion I or
E12: Circular motion II

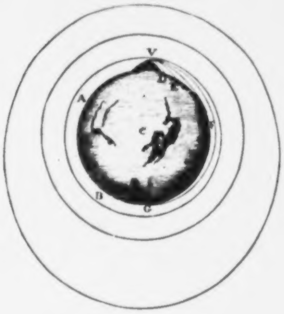
Summary 4.5
1. Whether or not an object moving along a circular path appears to an observer to have circular motion depends upon the relative position and motion of the observer.
2. Revolution is the act of travelling along a circular path; rotation is the act of spinning.

earth which in turn implies a fixed direction of the gravitational force are no longer valid.) If we suppose the launch speed to be increased even more, the projectile would strike the earth at points ever farther from the tower, till at last it would rush around the earth in a near circular orbit. At this orbiting speed, the "projectile" is traveling so fast that its vertical fall just keeps pace with the receding surface of the curved earth.

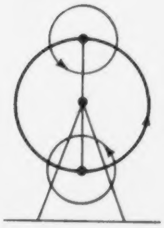
What horizontal launch speed is required to put an object into a circular orbit about the earth? We shall be able to answer this question quite easily when we learn more about circular motion. We will also then be able to consider our problem of the capsule circling the moon.

The simplest kind of circular motion is uniform circular motion, that is, motion in a circle at constant speed. If you drive a car around a perfectly circular track so that at every instant the speedometer reading is forty miles per hour, you are executing uniform circular motion. But you will not be doing so if the track is any shape other than circular, or if your speed changes at any point.

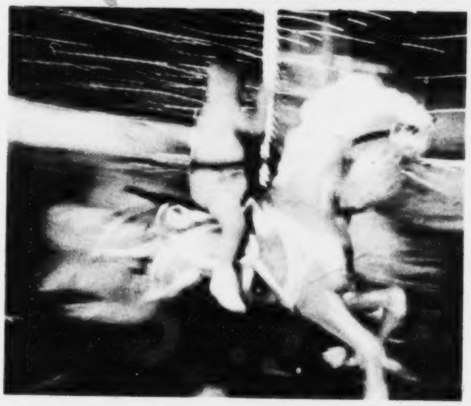
How does one find whether an object in circular motion is moving at constant speed? The answer, surely, is to apply the same test we use in deciding whether or not an object traveling in a straight line does so with constant speed. We measure the instantaneous speed at many different moments and see whether the values are the same. If the speed is constant, we can describe the circular motion of an object by means of two numbers: the radius R of the circle and the speed v along the path. Instead of the speed, however, we shall



In discussing circular motion it is useful to keep clearly in mind a distinction between revolution and rotation. We define these terms differently: revolution is the act of traveling along a circular path; rotation is the act of spinning without traveling at all. A point on the rim of a phonograph turntable travels a long way; it is revolving about the axis of the turntable. But the turntable as a unit does not move from place to place: it merely rotates. In some situations both processes occur at once; for example, the earth rotates about its own axis while it also revolves (in a nearly circular path) around the sun.



The circular motion of a double ferris wheel.



3. The simplest kind of circular motion is uniform circular motion; it can be described by two numbers: for example, radius of curvature and velocity. 4. Frequency f is defined as $f = 1/T$. 5. Therefore $v = 2\pi R/T = 2\pi Rf$.

use a quantity easier to measure: either (1) the time required by an object to make one complete revolution, or (2) the number of revolutions the object completes in a stated interval of time. These latter two concepts have have been given names. The time required for an object to complete one revolution in a circular path is called the period of the motion. The period is denoted by the letter T. The number of revolutions completed by the same object in a specified time is called the frequency of the motion. Frequency will be denoted by the letter f.

6. The two numbers which describe uniform circular motion can be any two of these three: radius, velocity, frequency (or period).

Many commercial record turntables are designed to rotate at frequencies of 16 2/3 rpm (called transcription speed), 33 1/3 rpm (for LP's), 45 rpm (pop records), and 78 rpm (old-fashioned). What is the period corresponding to each of these frequencies?

In these terms we can describe a car moving with uniform speed on a circular track. Let us suppose the car takes 20 seconds to make one lap around the track. Thus, T = 20 seconds. Alternatively, we might say that the car makes 3 laps per minute, that is, 3/60 = 1/20 laps per second. Therefore, f = 1/20 revolutions/sec or more briefly f = 1/20 sec^{-1}. In this last expression the symbol sec^{-1} stands for 1/sec, or "per second." When the same time unit is used, the relationship between frequency and period is

$$f = \frac{1}{T}.$$

Any convenient units may be used. Radius may be expressed in terms of centimeters, kilometers, miles, or any other distance unit. Period may be expressed in seconds, minutes, or years. Correspondingly, the frequency may be expressed as "per second," "per minute," or "per year." The most widely used units of radius, period and frequency in scientific work are meter, second and per second.

Table 4.1

A comparison is shown below of the frequency and period for various kinds of circular motion. Note the changes in units. Can you put all the values in the table in seconds and per sec?

Phenomena	Period	Frequency
Electron in atom	10^{-16} sec	10^{16} per sec
Ultra-centrifuge	0.00033 sec	3000 per sec
Hoover Dam turbine	0.33 sec	3 per sec
Rotation of earth	24 hours	0.0007 per min
Moon around the earth	30 days	0.001375 per hour
Earth about the sun	365 days	0.0027 per day

• An exercise in conversion factors. Lots of problems involve these units (sec and sec^{-1}).

If an object is in uniform circular motion, a person who knows the frequency of revolution f and the radius R of the path can compute the speed v of the object without difficulty.

The distance traveled in one revolution is simply the perimeter of the circular path, that is, $2\pi R$. The time for one revolution is just the period T. Thus since

$$\text{speed} = \frac{\text{distance traveled}}{\text{time elapsed}}$$

by substitution we can get

$$v = \frac{2\pi R}{T} .$$

To reformulate this circular motion equation in terms of frequency f we rewrite it as

$$v = (2\pi R)\,\frac{1}{T} .$$

Now since by definition

$$f = \frac{1}{T} ,$$

we find that

$$v = (2\pi R)(f) = 2\pi Rf.$$

_If_ the body is in uniform circular motion, the speed computed with the aid of this equation is both the instantaneous speed and the average speed. If the motion is nonuniform, the formula gives only the _average_ speed. The _instantaneous_ speed can be determined only if we are able somehow to find $\Delta d/\Delta t$ from measurements of very small segments of the path.

Let us now see how this equation can be used. We can calculate the speed of the tip of a helicopter rotor blade as the helicopter sits on the ground. On one model, the main rotor has a diameter of 7.6 m and a frequency of 450 revolutions/minute under standard conditions. Thus f = 450 per minute and R = 3.8 m, so

$$v = 2\pi Rf$$

$$v = 2\pi(3.8)(450) \text{ meters/minute}$$

$$v = 10{,}700 \text{ meters/minute,}$$

See Study Guide 4.9 and 4.11.

or about 400 mph.

Q7 A phonograph turntable makes 90 revolutions in 120 seconds.

a) What is its period (in seconds)?
b) What is its period (in minutes)?
c) What is its frequency in cycles per second?

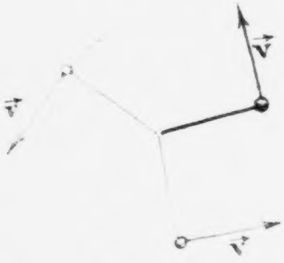
Q8 What is the period of the minute hand of an ordinary clock? If the hand is 6.0 cm long, what is the linear speed of the tip of the minute hand?

Summary 4.6

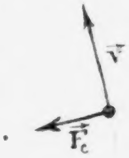
1. In uniform circular motion, velocity is constant in magnitude and tangent to the path at any instant.

2. In uniform circular motion, the magnitude of the acceleration is constant and the direction is always toward the center. 3. For an object in uniform circular motion, the vectors $\vec{v}$ and $\vec{a}$ are always at right angles to each other.

4.6 Centripetal acceleration. Let us assume that a stone, whirling on a string, is moving with uniform circular motion. The speed of the stone is constant. The velocity, however, is continuously changing because the direction of motion is continuously changing. At any instant, the direction of the velocity is tangent to the circular path. Since the velocity is changing, the stone is accelerating.

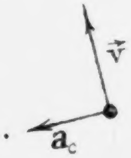


To keep the stone moving in a circular path, that is, to produce an acceleration, a force is needed. In the case of the whirling stone, a force is exerted on the stone by the string. If the string were suddenly cut, the stone would go flying off with the velocity it had at the instant the string was cut. As long as the string holds together, the stone is forced into a circular path.



The direction of the force acting on the stone is along the string. Thus the force is always pointing toward the center of rotation. This kind of force—always directed toward the center of rotation—is called a centripetal force. (The adjective centripetal literally means "tending toward the center.")● We shall give centripetal force the symbol $\vec{F}_c$. In uniform circular motion, the centripetal force always makes a right angle with the instantaneous velocity. As long as the force and the instantaneous velocity are at right angles, the magnitude of the velocity (that is the speed) does not change.

●or 'center seeking'



From Newton's second law we know that force and acceleration are in the same direction. Thus, the acceleration of the stone moving with constant speed along a circular path must, like the force, be directed toward the center of rotation. Furthermore, like the force, the acceleration always makes a right angle with the instantaneous velocity. We shall call this acceleration centripetal acceleration and give it the symbol $\vec{a}_c$. Any object moving along a circular path has a centripetal acceleration.

We know the direction of centripetal acceleration. What is its magnitude? We can determine the magnitude of the centripetal acceleration by an analysis of the figures on the next page. Assume the stone is moving in a circle of radius R. At any instant, the stone has a velocity, it has an acceleration, and it has a force exerted on it by the string. In order to keep the stone moving with constant speed in a circular path, a definite relationship between the magnitudes of the velocity, v, and the centripetal acceleration a_c, must exist. We can find what this relationship is by treating a small part of the circular path as the combination of

4. In rectilinear motion, $\vec{v}$ and $\vec{a}$ are always parallel or antiparallel.

5. In projectile motion, the angle between $\vec{v}$ and $\vec{a}$ is 90° only at the top of the trajectory.

6. In uniform circular motion, $a = v^2/R$.

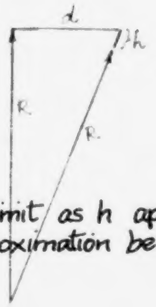
T11 : Centripetal acceleration - graphical treatment
T12 : Centripetal acceleration derivation
F5 : PSSC film - Vector kinematics
A : Centripetal force cork-in-bottle accelerometer

a tangential motion and an acceleration toward the center. To follow the circular path, the stone must accelerate toward the center through a distance h in the same time that it would move a tangential distance d. The stone, with speed v, would travel a horizontal distance d given by d = vt. In the same time t, the stone, with acceleration a_c, would travel toward the center a distance h given by $h = \frac{1}{2}a_ct^2$. (We can use this last equation because at t = 0, the stone's velocity toward the center is zero.)

We can now apply the Pythagorean Theorem to the triangle in the figure above.

$$R^2 + d^2 = (R + h)^2 = R^2 + 2Rh + h^2.$$

When we cancel the like terms on each side of the equation, we are left with



$$d^2 = 2Rh + h^2.$$

We can simplify this expression by making an approximation: since h is very small compared to R, h^2 will be very small compared to Rh. So we shall neglect h^2 and write

$$d^2 = 2Rh.•$$

• In the limit as h approaches 0, the approximation becomes exact.

However, we know d = vt and $h = \frac{1}{2}a_ct^2$ so we can substitute for d^2 and for h. Thus

$$(vt)^2 = 2R\left(\tfrac{1}{2}a_ct^2\right)$$

$$v^2t^2 = Ra_ct^2$$

$$v^2 = Ra_c$$

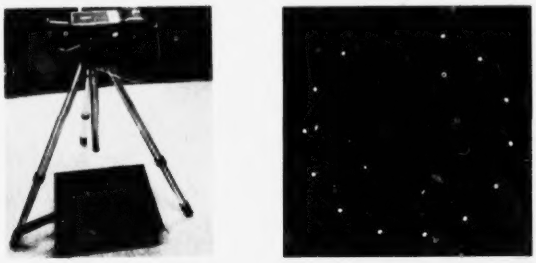
or

$$a_c = \frac{v^2}{R}.$$

This is the magnitude of the centripetal acceleration for an object moving with a speed v on a circular path of radius R.

Let us verify this relationship. A photograph has been made of a blinky• which was placed on a rotating phonograph turntable. The photograph and the actual setup are shown below. The blinky travels in a circular path with constant speed. The centripetal force in this case is the frictional force acting between the blinky and the surface of the phonograph turntable.

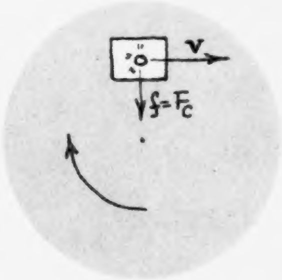
• A blinky is simply a black box with a neon bulb on top that flashes periodically.



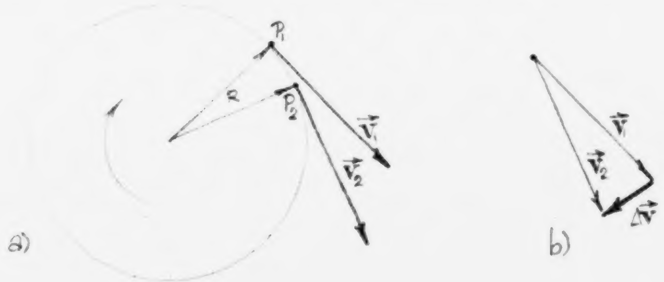
(a) The laboratory equipment for the rotating blinky experiment. (b) A photographic record of one revolution by the blinky. The blinky had a frequency of 9.4 sec and its path has a radius of 10.6 cm.

Another way to express $f=9.4$ per sec is $f=9.4\ \text{sec}^{-1}$.

We shall determine the acceleration of the blinky by two methods. The first method makes use of the basic definition of acceleration, $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$. The second makes use of the equation $a_c = \frac{v^2}{R}$.



As the blinky travels around its circular path, it may be at position P_1 at some instant and at position P_2 a short time later. At each such position its velocity can be represented by a vector. Since the circular motion is uniform, the arrows representing $\vec{v}_1$ and $\vec{v}_2$ must be equal in length. However, the vectors $\vec{v}_1$ and $\vec{v}_2$ differ in direction. What is the difference between the vectors? The figure below shows the two vectors (arranged with the tails at the same point, to make the comparison easier) and makes clear that they differ by the vector labeled $\Delta\vec{v}$. That is, $\Delta\vec{v}$ has a direction and a magnitude such that



$$\vec{v}_1 + \Delta\vec{v} = \vec{v}_2.$$

In words, this equation means that in the short time interval Δt in which the blinky travels from P_1 to P_2, it must acquire a new component of velocity—a component having the direction and magnitude of $\Delta\vec{v}$. The direction of $\Delta\vec{v}$ is toward the center of the circle.

For vanishingly small angles, the triangle with sides v_1, v_2, Δv is similar to the triangle with sides R, R, Δs; therefore

$$\frac{\Delta v}{v} = \frac{\Delta s}{R}.$$

But, for such a small angle, $\Delta s = v\Delta t$. So

$$\frac{\Delta v}{v} = \frac{v\Delta t}{R}$$

and $$\frac{\Delta v}{\Delta t} = \frac{v^2}{R}$$

By rearrangement, this becomes

$$\Delta v = \vec{v}_2 - \vec{v}_1,$$

which is the definition of "change in velocity."

From measurements on the revolving blinky, we have determined the speed v. Using an appropriate scale factor (in this case 1.0 cm stands for 45 cm/sec) we plotted the velocity at points P_1 and P_2 and determined Δv by a direct length measurement. The magnitude of the change in velocity was found to be 20 cm/sec. The rest is straight calculation. The time interval Δt between flashes was .11 sec, and therefore:

Can you determine the speed of the blinky from the data given?

$$\text{the magnitude of the acceleration} = \frac{\text{the magnitude of the change in velocity}}{\text{the change in time}}$$

$$= \frac{20\ \text{cm/sec}}{.11\ \text{sec}}$$

$$= 190\ \text{cm/sec}^2.$$

How is Δt = .11 sec obtained from the information that f = 9.4 flashes/sec? $\Delta t = \frac{1}{f}$

Thus, by a combination of graphical and algebraic steps, we found the magnitude of the acceleration the blinky underwent as it revolved on the turntable.

There were 9.4 blinks/sec and a total of 14 blinks; therefore the period T must be

$$\frac{14\ \text{blinks}}{9.4\ \text{blinks/sec}},$$

or

$$T = 1.5\ \text{sec}.$$

Let us now use the equation, $a_c = \frac{v^2}{R}$, to find the centripetal acceleration of the blinky and compare it to the results obtained by the graphical method. The information we have is R = 10.6 cm and T = 1.5 sec. The speed of the blinky is given by

$$v = \frac{2\pi R}{T}.$$

Substituting in numerical values we get

$$v = \frac{2(3.14)(10.6)\ \text{cm}}{1.5\ \text{sec}}$$

$$= 44\ \text{cm/sec}.$$

We can substitute this value for speed into the expression we derived for the acceleration.

For this and most other problems on uniform circular motion, it is only necessary to remember and understand

$$v = \frac{2\pi R}{T},$$

$$f = \frac{1}{T}, \text{ and}$$

$$a_c = \frac{v^2}{R}.$$

$$a_c = \frac{v^2}{R}$$

$$= \frac{(44.4\ \text{cm/sec})^2}{10.6\ \text{cm}}$$

$$= \frac{1971\ \text{cm}^2/\text{sec}^2}{10.6\ \text{cm}}$$

$$= 190\ \text{cm/sec}^2.$$

The answers obtained by the two methods agree.

If v^2/R is the magnitude of the centripetal acceleration, then from Newton's second law we can conclude that mv^2/R is the magnitude of the centripetal force. The hammer thrower in the photograph is exerting a tremendous centripetal force to keep the hammer moving in a circle as he speeds it up. From the distance the hammer travels, we can estimate its speed at release. To keep the 16-pound hammer in a circle at the release speed requires over 500 pounds of force!

Let us return to our space flight. The space capsule in Part 5 of our earth-moon flight is orbiting the moon in a circle at a constant speed. From the radius of the orbit and the capsule's speed, we can compute the centripetal acceleration and, if we know the capsule's mass, the centripetal

Summary 4.7
1. Rotational motion has been one of the central intellectual and practical concerns of man.

force. What is the origin of the centripetal force? If you do not already know, you will find out in Unit 2. By knowing what the centripetal force is, space engineers can work the problem backwards to determine the speed the capsule must have for a particular lunar orbit.

See Study Guide 4.12, 4.14, and 4.15 for further thoughts on centripetal acceleration.

Q9 In the last section we calculated that the tip of a helicopter rotor blade ($f = 450\ \text{min}^{-1}$ and $R = 3.8$ m) was moving about 10,700 m/sec. Find centripetal acceleration of the tip.

4.7 The motion of earth satellites. Nature and technology provide many examples of the type of motion where an object is in uniform circular motion. The wheel has been a main characteristic of our civilization, first as it appeared on crude carts and then later as an essential part of complex machines. The historical importance of rotary motion in the development of modern technology has been described by the historian V. Gordon Childe:

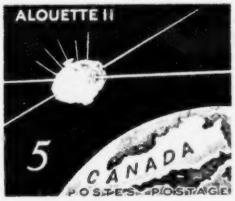
> Rotating machines for performing repetitive operations, driven by water, by thermal power, or by electrical energy, were the most decisive factors of the industrial revolution, and, from the first steamship till the invention of the jet plane, it is the application of rotary motion to transport that has revolutionized communications. The use of rotary machines, as of any other human tools, has been cumulative and progressive. The inventors of the eighteenth and nineteenth centuries were merely extending the applications of rotary motion that had been devised in previous generations, reaching back thousands of years into the prehistoric past....
> [V. Gordon Childe "Rotary Motion" in The History of Technology, ed. Charles Singer, E. J. Holmyard, and A. R. Hall, Vol. I (New York: Oxford University Press, 1953) p. 187.]

We shall see in Unit 2 that another rotational motion, that of the orbiting planets around the sun, has also been one of the central concerns of man throughout recorded history.

Chariot. Alberto Giacometti, 1950. Courtesy Museum of Modern Art.

2. An earth satellite in a circular orbit does not fly off into outer space because it is constantly being accelerated toward the earth.

Since the kinematics and dynamics of all uniform circular motion are the same, we can apply what we have learned to the motion of artificial earth satellites in circular (or nearly circular) paths. The satellite selected for study here is Alouette, Canada's first satellite, which was launched into a nearly circular orbit by a Thor-Agena B rocket on September 29, 1962.



Alouette I still provides useful data upon command. Alouette II was placed in orbit in late 1965.

Alouette is orbiting at an average distance of 4,593 miles from the center of the earth. Its closest approach to the earth is 620 miles, and its farthest distance from the earth

3. On the other hand, it does not fall back to the earth because its tangential velocity is not being reduced by the perpendicular acceleration; it is only changing direction.

Table 4.2 Some information on selected artificial earth satellites.

Name	Launch date	Weight (lb)	Period (min)	Height (miles) Perigee-Apogee	Remarks
Sputnik 1 1957α (USSR)	Oct. 4, 1957	184	96.2	142-588	First earth satellite. Internal temp., pressure.
Explorer 7 1958α (USA)	Jan. 31, 1958	30.8	114.8	224-1573	Cosmic rays, micrometeorites, internal and shell temps., discovery of first Van Allen belts.
Lunik 3 1959θ (USSR)	Oct. 4, 1959	959	22,300	30,000-291,000	Transmitted photographs of far side of moon.
Vostok 1 1961μ (USSR)	Apr. 12, 1961	10,416	89.34	109-188	First manned orbital flight (Major Yuri Gagarin; one orbit).
Midas 3 1961σ (USA)	July 12, 1961	3,500	161.5	2,129-2,153	Almost circular orbit.
Telstar 1 1962αε (USA)	July 10, 1962	170	157.8	593-3,503	Successful transmission across the Atlantic: telephony, phototelegraphy, and color and black and white television.
Alouette 1 1962βα (USA-CANADA)	Sept. 29, 1962	319	105.4	620-640	Joint project between NASA and Canadian Defense Research Board; measurements in ionosphere.
Luna 4 1963-08 (USSR)	Apr. 2, 1963	3,135	42,000	56,000-435,000	Passed 5,300 miles from moon; very large orbit.
Vostok 6 1963-23 (USSR)	June 16, 1963	"about 5 tons"	88.34	106-134	First orbital flight by a woman; (Valentina Terishkova; 48 orbits).
Syncom 2 1963-31 (USA)	July 26, 1963	86	1,460.4	22,187-22,192	Successfully placed in near-synchronous orbit.

is 640 miles. Since this path is so nearly circular, we will treat it as a circle in our analysis of the satellite's motion. The speed of Alouette can be taken to be constant for our purposes, since it varies less than one mile per minute above or below the average speed of 275 miles per minute.

Now let us compute the orbital speed and centripetal acceleration of Alouette. The relationship $v = 2\pi R/T$ allows us to find the speed of any object moving uniformly in a circle if R and T are known. To determine a satellite's speed, we need to know its distance R from the center of the earth and its period T.

Tracking stations located in many places around the world maintain a record of any satellite's position in space. From the position data, the satellite's distance above the earth at any time and its period of revolution are found. By means of such tracking, we know that Alouette moves at an average height of 630 miles above sea level, and takes 105.4 min to complete one circular orbit. Adding 630 miles to the earth's radius, 3,963 miles, we obtain R = 4,593 miles, and

$$v = \frac{2\pi R}{T} = \frac{2\pi(4{,}590)\text{ mi}}{105\text{ min}} = \frac{28{,}800\text{ mi}}{105\text{ min}} = 275\text{ mi/min}.$$

This is equivalent to 16,500 mi/hr, or 7,150 m/sec.

The last equation can be used to find the speed of any satellite, for example, that of our moon. The average distance from the center of the earth to the center of the moon is approximately 2.39×10^5 mi, and the moon takes an average of 27 days, 7 hrs, 43 min to complete one revolution around the earth with respect to the fixed stars. Thus

$$v = \frac{2\pi(2.39 \times 10^5)\text{ mi}}{3.93 \times 10^4\text{ min}} = 38.1\text{ mi/min},$$

or roughly 2,280 mi/hr.

If we wish to calculate the centripetal acceleration of Alouette, we can use the value of v found above along with the relationship $a = \frac{v^2}{R}$. Thus

$$a = \frac{(275\text{ mi/min})^2}{4{,}590\text{ mi}} = 16.5\text{ mi/min}^2$$

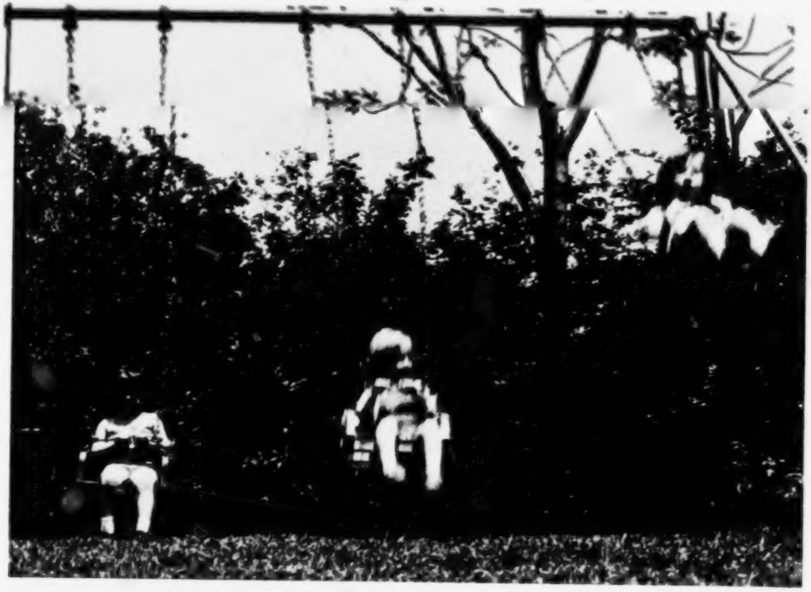
This is the equivalent of 7.42 m/sec^2. What force gives rise to this acceleration? (Hint: the acceleration of a falling stone at the surface of the earth is 9.80 m/sec^2.)

In Chapter 2 we found that the acceleration due to gravity at the earth's surface was about 9.8 m/sec^2. Here we have just calculated the acceleration of Alouette toward the center of the earth to be about 7.4 m/sec^2. Calculate the moon's centripetal acceleration.

Earlier we asked the question, "What horizontal launch speed is required to put an object into a circular orbit about the earth?" Can you answer this question now? If not, turn to Study Guide 4.23 for help.

Summary 1. Simple harmonic motion (SHM) is an example of a more complex motion. 2. It can be described in terms of circular motion. 3. Examples of SHM are vibrating tuning forks, pendulums, objects suspended from springs, etc. 4. Forces that obey Hooke's law give rise to SHM.

A vibrating string



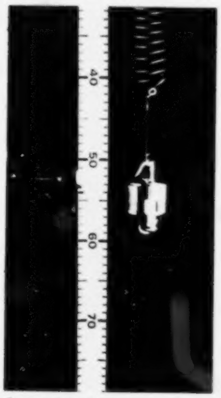
Swinging children

D 26 : Simple harmonic motion
D 27 : Simple harmonic motion (airtrack)

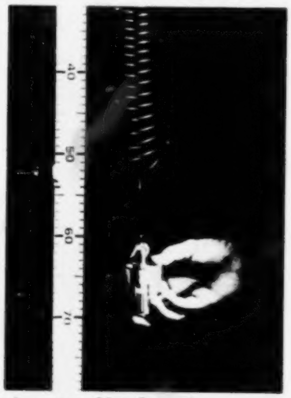
Simple Harmonic Motion

Back-and-forth motions similar to the swinging child and the vibrating guitar string are common. There are rocking boats, swaying trees, and vibrating tuning forks. There are clock pendula and quivering diving boards. What are the details of such oscillatory motions? What kind of force is acting on an oscillating object? No new concepts are needed to answer these questions, so let us proceed.

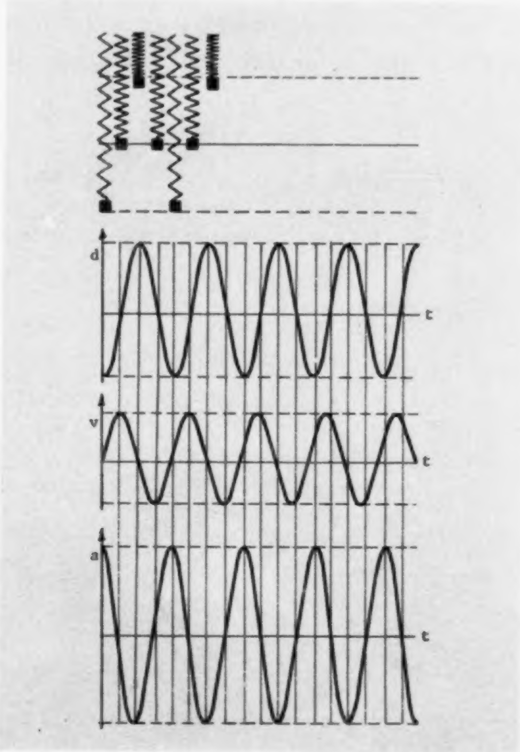
The mass on the spring pictured below is in equilibrium. If we displace the mass vertically from its equilibrium position, a force is exerted on the mass by the spring. This force, F_s, tends to restore the mass to its equilibrium position. Let us displace the mass, release it and observe its motion. We observe that the mass oscillates back and forth through its equilibrium position. If we start a timer at the instant of release, we could represent the displacement of the mass at any time on a graph. The displacement of the mass ranges from a maximum in one direction to a maximum in the other direction; that is, from $+ d$ to $-d$.



A mass in equilibrium



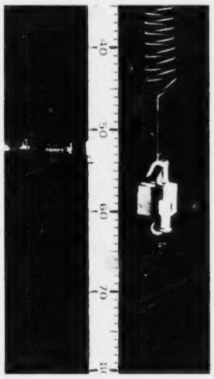
A mass displaced from equilibrium



As the mass approaches its maximum displacement, it slows down, stops and then speeds up in the opposite direction. The speed of the mass is the greatest as it passes through the equilibrium position. This information can also be represented graphically. The displacement-time, velocity-time and acceleration-time graphs are shown below.

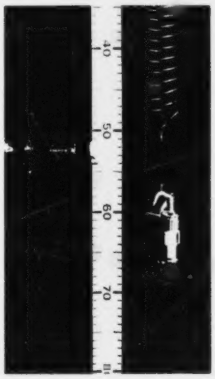
These graphs give us the kinematic details of the motion. From the graphs we see that the velocity is a maximum when the displacement is a minimum. Further, we see that when the displacement is a maximum in one direction, the acceleration is a maximum in the other direction.

What about the force exerted on the mass by the spring? By combining the information in the acceleration-time graph with Newton's second law, we know that the force is varying in both magnitude and direction. We can determine how the force varies by an experiment shown in the photographs below. In this experiment forces of known magnitudes—0.5 N and 1.0 N—were applied to the mass. From the photographs we can measure the displacements of the mass resulting from the known forces. This measurement tells us the force that the spring is exerting on the mass at these two displacements.



F_s=0.5 N

d=3.7 cm



F_s=1.0 N

d=7.5 cm

That is, when

$d = 3.7$ cm then $F_s = 0.5$ N

and when

$d = 7.5$ cm then $F_s = 1.0$ N.

A close look at these results seems to indicate that $F_s \propto d$ or $F_s = kd$ where k is a constant of proportionality. (Verify for yourself that F_s is proportional to d. Remember Study Guide 3.9.)

One additional piece of information is needed before we fully understand the spring force F_s. What is the relation between the directions of $\vec{F}_s$ and $\vec{d}$? When the displacement is in the downward direction, the spring force is in the upward direction and vice versa. In other words, the force $\vec{F}_s$ is in the opposite direction of $\vec{d}$. We can now write the force law which expresses the nature of the force exerted by the spring on the mass. This force law is

$$\vec{F}_s = -k\vec{d}.$$

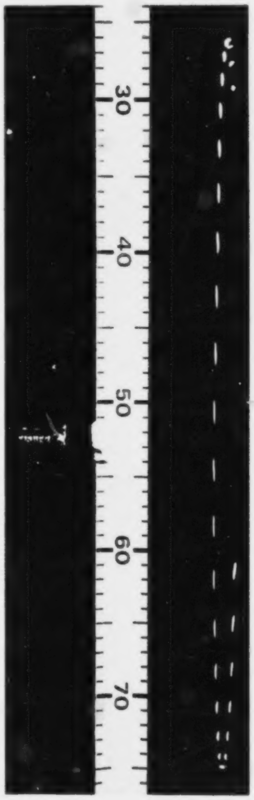
The minus sign indicates the opposite directions of $\vec{F}_s$ and $\vec{d}$.

The back-and-forth motion resulting from the force $\vec{F}_s = -k\vec{d}$ is called simple harmonic motion. Are the swinging child and the vibrating guitar string examples of simple harmonic motion? Are they examples of motion for which the motivating force is proportional to the displacement? The answer to both of these questions is "only approximately." There are other forces which tend to slow down and bring to rest the swinging child and the guitar string. In other words, the back-and-forth motion is damped. If the damping forces are not large, these motions and many others besides closely approximate simple harmonic motion.

SUGGESTED ACTIVITY

The stroboscopic photograph at the right shows the position of a light attached to the mass at time intervals of 1/30 second. The mass is 0.52 kg.

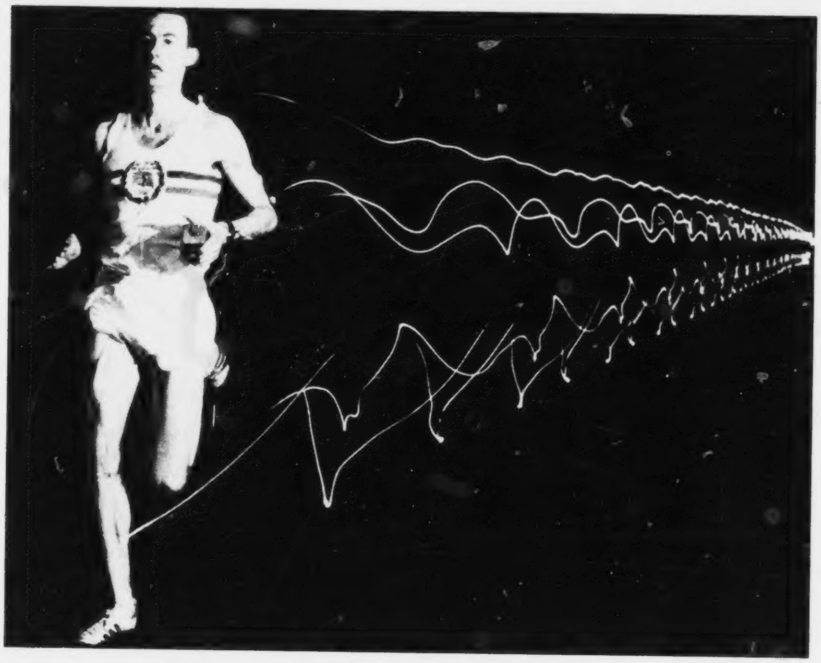
1. What is the equilibrium position?
2. Construct a displacement-time graph.
3. Measure the slope of the displacement-time graph at several different times and construct a velocity-time graph.
4. Determine the acceleration of the mass when it is positioned half-way between the maximum displacement and the equilibrium position.
5. What is the force exerted on the mass by the spring at the same point chosen in (4) above?
6. Does Newton's second law hold?
7. For additional suggested activities see Study Guide 4.22.



4.8 What about other motions? So far we have described rectilinear motion, both uniform and accelerated, projectile motion, uniform circular motion, and simple harmonic motion. Taken together, these descriptions are useful in clarifying much of interest in the world of motion. Even so, it is clear that we still have avoided many complicated kinds of motion that may interest us. For instance, consider these:

Summary 4.8
All motions can be analyzed using the concepts of position, time, velocity and acceleration, even though these concepts were developed from a study of relatively simple types of motion.

a) the motion in a pattern of water ripples;
b) the motion of the Empire State Building;
c) the motion of a small dust particle as it zig-zags through still air;
d) a person running.



Even if we have not treated these motions directly, what we have done so far is of real value. The methods for dealing with motion which we have developed in this and the preceding chapters are important because they give us means for dealing with any kind of motion whatsoever. All motion can be analyzed in terms of position, velocity, and acceleration.

When we considered the forces needed to produce motion, Newton's laws supplied us with concise yet very general answers. Later, when we discuss the elliptical motion of planets, and the hyperbolic motion of an alpha particle passing near a nucleus, we shall be able to infer the magnitude and direction of the forces acting in each case.

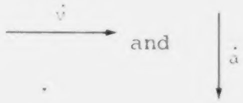
On the other hand, when we know the magnitude and direction of the force acting on an object, we can determine what its change in motion will be. If in addition to this, we know the position and velocity of an object, we can reconstruct how it moved in the past and we can predict how it will move in the future. Thus, Newton's laws provide a comprehensive view of forces and motion. It is not surprising that Newton's work was greeted with astonished wonder. Such wonder is aptly expressed in Alexander Pope's oft-quoted couplet:

> Nature and Nature's laws lay hid in night,
> God said, "Let Newton be!" and all was light.

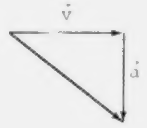
4.1 Using symbols other than words, give an example of each of these:

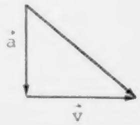
a) a scalar.
b) a vector.
c) the addition of two scalars.
d) the addition of two vectors.
e) the addition of three vectors.
f) the subtraction of one scalar from another.
g) the subtraction of one vector from another. Discussion

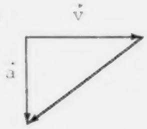
4.2 For a given moving object the velocity and acceleration can be represented by these vectors:

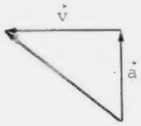


The sum of these two vectors is:

(a)


(b)


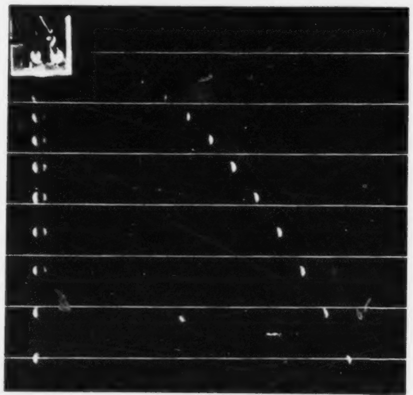
(c)


(d)


e) They cannot be added.

Answer (e)

4.3 A sphere is launched horizontally, as shown below. Suppose the initial speed v_x is 3.0 m/sec. Where is the projectile (displacement), and what is its speed and direction (velocity) 0.5 sec after launching?



a) x = 1.5m, y = 1.25 m, d= 1.9 m at angle 40° below horizontal.

b) v = 5.7 m/sec at angle 59° below horizontal.

4.4 If a raindrop accelerated at a constant rate of 9.8 m/sec^2 from a cloud 1 mile up what would be its speed just before striking the ground. Does a raindrop accelerate at a constant rate over a 1 mile fall? v = 177 m/sec. No.

4.5 An airplane has a gun that fires bullets at the speed of 600 mph when tested on the ground with the plane stationary. The plane takes off and flies due east at 600 mph. Which of the following claims are correct, if any? In defending your answers, refer to Galilean relativity.

True … a) When fired directly ahead the bullets move eastward at a speed of 1200 mph.
True … b) When fired in the opposite direction, the bullets drop vertically downward.
True … c) If fired vertically downward, the bullets move eastward at 600 mph.
Discussion

4.6 Two persons watch the same object move. One says it accelerates straight downward, but the other claims it falls along a curved path. Invent a situation in which they both could be right. Discussion

4.7 A hunter points his gun barrel directly at a monkey in a distant palm tree. Where will the bullet go? If the animal, startled by the flash, drops out of the branches at the very instant of firing, will it then be hit by the bullet? Explain. Discussion

4.8 If a broad jumper takes off with a speed of 10 m/sec at an angle of 45° with respect to the earth's surface, how far would he leap? If he took off from the moon's surface with that same speed and angle, what would be the length of his leap. The gravitational acceleration of a body at the moon's surface is $\frac{1}{6}$th of that at the earth's surface.
(a) 10.2 meters (b) 61.2 meters

4.9 Contrast rectilinear motion, projectile motion, and uniform circular motion by:

a) defining each.
b) giving examples.
c) comparing the velocity-acceleration relationships. Discussion

4.10 You are inside a uniformly accelerating moving van. If when the van is traveling at 10 mph (and still accelerating) you dropped a ball from the roof of the van onto the floor, what would be the ball's path relative to the van? What would be its path relative to a person driving past the van in the opposite direction of the van at a uniform speed? What would be its path relative to a person standing on road? Discussion

4.11 An object in uniform circular motion makes 20 revolutions in 4.0 sec.

a) What is its period T? 2 sec
b) What is its frequency f? 5 cps
c) If the radius of rotation is 2 meters, what is its speed? 62.8 m/sec

4.12 Two blinkies were placed on a rotating turntable and photographed from directly overhead. The result is shown in the figure below. The outer blinky has a frequency of 9.4 flashes/sec and is located 15.0 cm from the center. For the inner blinky, the values are 9.1 flashes/sec and 10.6 cm.

a) What is the period of the turntable?
b) What is the frequency of rotation of the turntable? Is this a standard phonograph speed? $f = 32\ \text{min}^{-1}$
c) What is the linear speed of the turntable at the position of the outer blinky? $v = 50$ cm/sec
d) What is the linear speed of the turntable at the position of the inner blinky? $v = 35$ cm/sec
e) What is the linear speed of the turntable at the center? $v = 0$ cm/sec
f) What is the angular speed of each blinky in degrees/sec? Are they equal? $\omega = 190° \text{ sec}^{-1}$ They are equal
g) What is the centripetal acceleration experienced by the inner blinky? $a_c = 120$ cm/sec^2
h) What is the centripetal acceleration experienced by the outer blinky? $a_c = 160$ cm/sec^2

4.13 These questions are asked with reference to Table 4.2 on page 112.

a) Are the distances to apogee and perigee given as height above the surface of the earth or distance from the center of the earth? Above the surface
b) Which satellite has the most nearly circular orbit? Syncom 2
c) Which are the most eccentric? How did you arrive at your answer? Lunik 3 discussion
d) Which satellite in the table has the longest period? Luna 4
e) What is the period of Syncom 2 in hours. 24.34 hr
f) How does the position of Syncom relative to a point on the earth change over one day. remain overhead
g) Which satellite has the greater centripetal acceleration, Midas 3 or Syncom 2? Midas 3
h) What is the magnitude of the centripetal acceleration of Vostok 6. Express answer in m/sec^2. 9.36 m/sec^2

4.14 The following table shows the period and the mean distance from the sun for the three planets that most nearly go in a circular orbit.

Planet	Mean distance (r) from sun (in A.U.)	Period (T) in years
Venus	0.72	0.62
Earth	1.00	1.00
Neptune	30.06	164.8

(A.U. = astronomical unit = the mean distance of the earth from the sun; 1 A.U. = 92.9 × 10^6 miles.)

a) What is the average orbital speed for each planet (in A.U./year)?
b) Calculate the centripetal acceleration for each planet in A.U./yr^2.
c) Can you see any relationship between the mean distance and the centripetal acceleration a_c? [Hint: Does it appear to be (1) $a_c \propto r$, or (2) $a_c \propto 1/r$; or (3) $a_c \propto r^2$; or (4) $a_c \propto 1/r^2$? How can a graph help you to decide?]

a) v(Venus) = 7.3 A.U./yr
v(Earth) = 6.28 A.U./yr
v(Neptune) = 1.14 A.U./yr

b) a_c (Venus) = 74 A.U./yr^2
a_c (Earth) = 39.5 A.U./yr^2
a_c (Neptune) = 4.37×10^{-2} A.U./yr^2

c) a_c is proportional to $\frac{1}{r^2}$

4.15 Explain why it is impossible to have an earth satellite orbit the earth in 80 minutes. Does this mean that it is impossible for an object to circle the earth in less than 80 minutes? Discussion. No.

4.16 The intention of the first four chapters has been to describe "simple" motions and to progress to the description of "complex" motions. Organize the following examples into a list from the simplest to the most complex, making whatever idealizing assumptions you wish. Be prepared to say why you placed any one example ahead of those below it, and to state any assumptions you made.

A "human cannon ball" in flight
A car going from 40 mph to a complete stop
A redwood tree
A child riding a ferris wheel
A rock dropped 3 m
A woman standing on an escalator
A climber ascending Mt. Everest Discussion

4.17 Could you rank the above examples if you were not permitted to idealize? If yes, how would you then rank them? If no, why not? Discussion

4.20 Compare the centripetal acceleration of the tire tread of a motor scooter wheel (diameter 1 ft) with that of a motorcycle wheel (diameter 2 ft) if both vehicles are moving at the same speed.

$a_1/a_2 = 2/1$

4.21 Our sun is located at a point in our galaxy about 30,000 light years (1 light year = 9.46×10^{12} km) from the galactic center. It is thought to be revolving around the center at a linear speed of approximately 250 km/sec. a) What is the sun's centripetal acceleration with respect to the center of our galaxy? b) If the sun's mass is taken to be 1.98×10^{30} kg, what centripetal force is required to keep it moving in a circular orbit about the galactic center? c) Compare the centripetal force in b) with that necessary to keep the earth in orbit about the sun. (The earth's mass is 5.98×10^{24} kg and its average distance from the sun is 1.495×10^{8} km. What is its linear speed in orbit?)

a) $a_c = 2 \times 10^{-10}$ m/sec^2

b) $F_s = 4 \times 10^{20}$ N

c) $F_e = 3.55 \times 10^{22}$ N

4.18 Using a full sheet of paper, make and complete a table like the one below. Table

Concept	Symbol	Definition	Example
		Length of a path between any two points as measured along the path	
			Straight line distance and direction from Detroit to Chicago
speed			
	$\bar{v}$		
			An airplane flying west at 400 mph at constant altitude
		Time rate of change of velocity	
	a_g		
Centripetal acceleration			
			The drive shaft of some automobiles turns 600 rpm in low gear
		The time it takes to make one complete revolution	

4.19 The diameter of the main wheel tires on a Boeing 727 fan jet is 1.26 m. The nose wheel tire has a diameter of 0.81 m. The speed of the plane just before it clears the runway is 86.1 m/sec. At this instant, find the centripetal acceleration of the tire tread, for each tire.

a_c (main) = 1.18×10^{4} m/sec^2

a_c (nose) = 1.8×10^{4} m/sec^2

4.22 Here are a list of some possible investigations into simple harmonic motion.

1. How does the period of a pendulum depend upon
   - a) the mass of the pendulum bob? Independent
   - b) the length of the pendulum? T increases with l
   - c) the amplitude of the swing (for a fixed length and fixed mass)? Independent
2. How does the period of an object on the end of a spring depend upon
   - a) the mass of the object? T increases with mass
   - b) the spring constant, k, where the spring constant k is defined as the slope of the graph of force versus spring extension? Its units are newtons/meter. T decreases for stiffer spring

4.23 The centripetal acceleration experienced by a satellite orbiting at the earth's surface (air resistance conveniently neglected) is the acceleration due to gravity of an object at the earth's surface (9.8 m/sec^2). Therefore, the speed required to maintain the satellite in a circular orbit must be such that the centripetal acceleration of the satellite is 9.8 m/sec^2. This condition can be expressed as follows. 85 min; 17650 mi/hr

$$a_c = \frac{V^2}{R} = a_g = 9.8 \text{ m/sec}^2$$

R, the radius of the earth, is 6.38×10^6 meters

$$a_g = 9.8 \text{ m/sec}^2$$

$$V^2 = 9.8 \text{ m/sec}^2 \times 6.38 \times 10^6 \text{ m}$$

$$= 62.5 \times 10^6 \text{ m}^2/\text{sec}^2$$

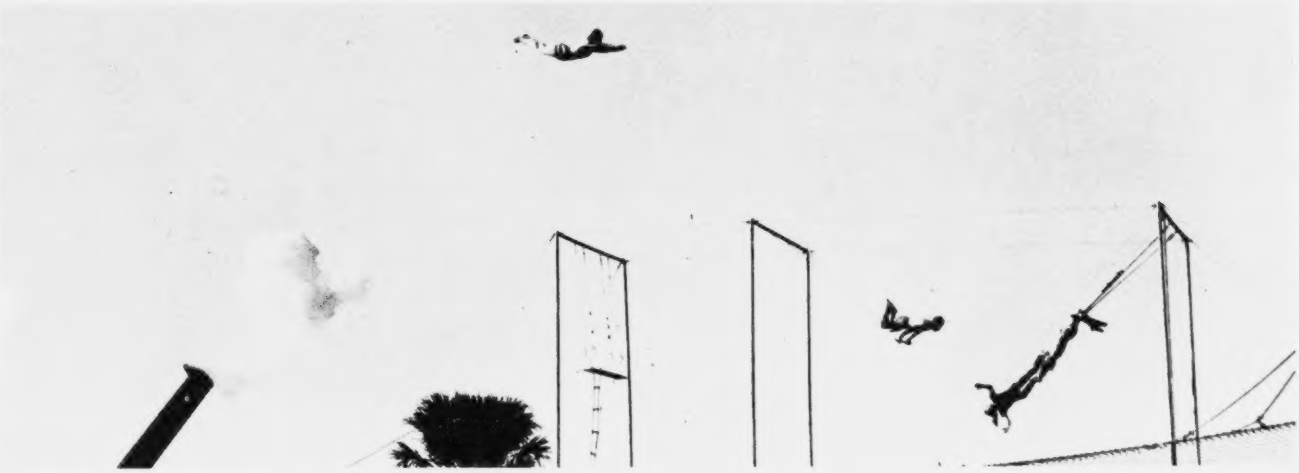
$$V = 7.85 \times 10^3 \text{ m/sec}$$

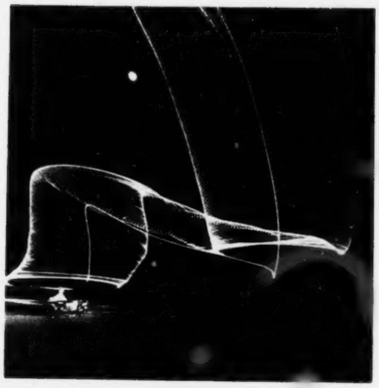
What is the period T of this orbit?

What is the satellite's speed expressed in miles per hour? (Hint: 1,000 meters = .61 miles.)

4.24 The thrust of a Saturn Apollo launch vehicle is 7,370,000 newtons (approximately 1,650,000 lbs) and its mass is 540,000 kg. What would be the acceleration of the vehicle relative to the earth's surface at lift off? How long would it take for the vehicle to rise 50 meters? The acceleration of the vehicle increases greatly with time (it is 47 m/sec^2 at first stage burnout), even though the thrust force does not increase appreciably. Explain why the acceleration increases. 13.6 m/sec^2; 2.71 sec; discussion

4.25 Write a short essay on one of the following pictures. Discussion





Epilogue The purpose of this Unit was to deal with the fundamental concepts of motion. We decided to start by analyzing particularly simple kinds of motion in the expectation that we might discover the "ABC's" of physics. With these basic ideas it was hoped we could turn our attention back to some of the more complex (and more interesting) features of the world. To what extent were these expectations fulfilled?

We did find that a relatively few basic concepts allowed us to gain a considerable understanding of motion. First of all, we found that useful descriptions of the motion of objects can be given using the concepts of distance, displacement, time, speed, velocity and acceleration. If to these we add force and mass and the relationships expressed in Newton's three laws of motion, it becomes possible to account for observed motion in an effective way. The surprising thing is that these concepts of motion, which were developed in extraordinarily restricted circumstances, can in fact be so widely applied. For example, our work in the laboratory centered around the use of sliding dry ice pucks and steel balls rolling down inclined planes. These are not objects to be found in the everyday "natural" world. Even so, we found that the ideas obtained from those experiments could be used to deepen our understanding of objects falling near the earth's surface, of projectiles, and of objects moving in circular paths. We started by analyzing the motion of a piece of dry ice moving across a smooth surface and ended up analyzing the motion of a space capsule as it circles the moon and crashes to its surface.

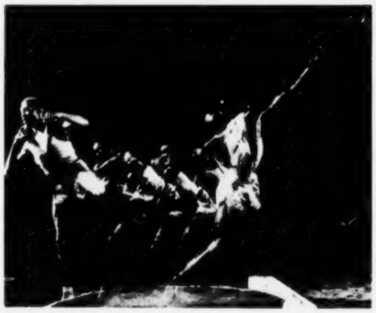
In other words, we really have made substantial progress. On the other hand, we cannot be satisfied that we have all of the intellectual tools necessary to understand all of the phenomena that interest us. We will find this to be especially true as we turn our attention away from interactions involving a relatively few objects of easily discernable size, and to interactions involving countless numbers of submicroscopic objects, i.e., molecules and atoms. Thus in Unit 3 we shall add to our stock of fundamental concepts a few additional ones, particularly those of momentum, work and energy.

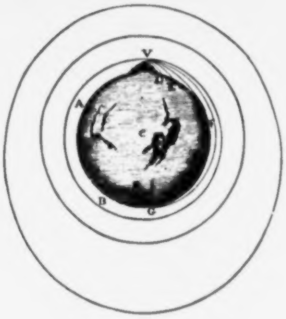
In this Unit we have dealt primarily with concepts that owe their greatest debts to Galileo, Newton and their followers. If space had permitted, we should also have included the contributions of René Descartes and the Dutch scientist, Christian Huyghens. The mathematician and philosopher,

A. N. Whitehead has summarized the role of these four men and the significance of the concepts we have been dealing with in this Unit in the following words:

> This subject of the formation of the three laws of motion and of the law of gravitation [which we shall take up in Unit 2] deserves critical attention. The whole development of thought occupied exactly two generations. It commenced with Galileo and ended with Newton's *Principia*; and Newton was born in the year that Galileo died. Also the lives of Descartes and Huyghens fall within the period occupied by these great terminal figures. The issue of the combined labours of these four men has some right to be considered as the greatest single intellectual success which mankind has achieved. [*Science and the Modern World*]

The revolution Whitehead speaks of, and the subject of this Unit, was important for many reasons, but most of all because it led to a deeper understanding of celestial motion. For at least 25 centuries man has been trying to reduce the complex motions of the stars, sun, moon, and planets to an orderly system. The genius of Galileo and Newton was in studying the nature of motion as it occurs on earth and then assuming that the same laws would apply to objects in the heavens beyond man's reach. Unit 2 is an account of the immense success of this idea. We shall trace the line of thought, starting with the formulation of the planetary problems by the ancient Greeks, through the work over a 100-year span of Copernicus, Tycho Brahe, Kepler, and Galileo, that provided a planetary model and several general laws for planetary motion, to Newton's magnificent synthesis of terrestrial and celestial physics in his Law of Universal Gravitation.





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Answers to End of Section Questions

Chapter 1

Q1 .5 cm/sec from 0 to 2 seconds; .33 cm/sec from 2 to 5 seconds; 2 cm/sec from 5 to 6 seconds.

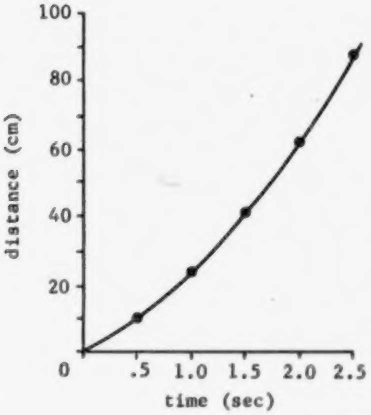
Q2 .66 cm/sec.

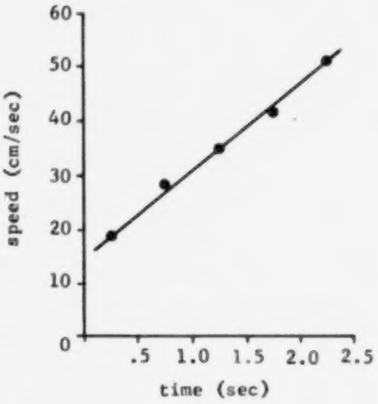
Q3 Interpolation means estimating value between data points; extrapolation means estimating values beyond data points.

Q4 The average speed over an interval of time Δt is $\Delta d/\Delta t$; instantaneous speed means in principle the speed at a point, and in practice is defined as the average speed for an interval so small that the average speed wouldn't change if the interval were made smaller.

Q5 The following table summarizes the data in the photo on page 28. The two accompanying graphs are based on the same data.

position	s	t	v
0	0 cm	0 sec	
			19 cm/sec
1	9.5	0.5	
			28
2	23.5	1.0	
			35
3	41	1.5	
			42
4	62	2.0	
			51
5	87.5	2.5	





Why doesn't the speed-time graph pass through the origin?

Q6 40,000 miles/hour^2 or 12 mph/sec.

Q7 -8 miles/hour^2.

Chapter 2

Q1 He could not measure v.

Q2 d = vt can only be used if v is constant. In acceleration motion v is not constant and the two equations cannot be combined.

Q3 c

Q4 c (a case can also be made for (a) or (b))

Q5 a

Chapter 3

Q1 Speed is a scalar quantity having only a magnitude while velocity is a vector quantity having both a magnitude and a direction.

Q2 a) $\Delta\vec{v}$ = 14.1 m/sec southeast
b) $\vec{a}$ = 2.8 m/sec^2 southeast

Q3 Force

Q4 According to Aristotelian physics a force is needed to maintain a motion. One possible (but slightly unbelievable) explanation would be "air currents circulate around the puck and push it along."

Q5 d

Q6 The net force acting on the puck is zero. Therefore the velocity does not change.

Q7 Galileo's "straight" lines were actually great circles about the earth. Newton's straight lines were straight.

Q8 2.5 kg

Q9 False

Q10 a) 2 m/sec^2
b) 4 N
c) Friction

Q11 c and f

Q12 e and f

Q13 No. The force "pulling the string apart" is still only 300 N.

Chapter 4

Q1 a) 2 seconds
b) 2,000 meters

Q2 a) No air resistance
b) The motion in the horizontal direction has no effect on the motion in the vertical direction.

Q3 $y = \frac{a_g}{2(v_x)^2} \times x^2$ where a_g is the acceleration due to gravity at the moon's surface.

Q4 $\frac{a_g}{2(v_x)^2}$

Q5 In cases a, b and c the results would be identical. In cases d and e the acceleration of the ball would be constant but the acceleration would be greater in case d than in case e.

Q6 Case d

Q7 a) 1.33 seconds/cycle
b) .022 minutes
c) .75 cycles/sec

Q8 3.1 m/sec^2

Q9 .77 m/sec^2

Brief Answers to Study Guide

Chapter 1

1.2 a) 6 cm/sec
b) 15 miles
c) 15 sec
d) 3 m/sec
e) 40 miles/hr
f) 40 miles/hr
g) 5.5 sec
h) 8.8 m

1.3 1.99 miles/hr

1.5 2.7×10^8 seconds

1.6 a) 1.65 m/sec
b) 3 m/sec

1.12 3.15×10^5 cm/sec

1.14 a) Approximately 25 meters
b) No

1.17 40 mph

Chapter 2

2.8 a) True
b) True
c) False
d) True (if air resistance is present)

2.15 c) 8 hours

2.16 a) 4.9 m
b) 9.8 m/sec
c) 14.7 m

2.17 a) 10.2 m/sec
b) 15.1 m
c) 2.04 sec
d) 20.4 m
e) 20 m/sec

2.18 a) 20.4 m/sec
b) 18.8 m/sec
c) 4.08 sec
d) 81.6 m
e) 0 (It is on the ground.)
f) 40 m/sec

2.19 a) 2 m/sec^2
b) 2 m/sec
c) 2 m/sec
d) 4 m
e) 2 m/sec
f) 4 sec

2.20 a) 56.8 m/sec^2
b) 710 m (approximately)
c) 189 m/sec^2 (about 19.5 g's)

2.23 a) 4.30 welfs/surgs^2
b) a_g = 980 cm/sec^2 or 9.8 m/sec^2. The planet Arret could be similar to the planet earth.

Chapter 3

3.3 a) Yes
b) 4.2 units 20° south of west

3.8 6:1

3.13 2 kg

3.14 a) $a = 201$ m/sec^2 $v = 790$ m/sec
b) The mass of the rocket decreases as propellent leaves the rocket.
c) 220 m/sec^2 (The acceleration is not uniform.)

3.16 a) 850 N
b) 735 N
c) 622 N
d) 850 N, 735 N, 622 N
e) The bathroom scale indicates a weight change.

3.17 a) 1 kg
b) 9.81 N, 9.80 N

3.18 b) 1.6×10^{-24} m/sec
c) 6.0×10^{24}:1

Chapter 4

4.2 e

4.3 a) $x = 1.5$ m, $y = 1.25$ m, $\vec{d} = 1.9$ m at angle 40° below horizontal
b) $\vec{v} = 5.7$ m/sec at angle 59° below horizontal

4.8 a) 10.2 meters
b) 61.2 meters

4.11 a) 0.2 seconds
b) 5 cps
c) 62.8 m/sec

4.13 a) Height above surface
b) Syncom 2
c) Lunik 3 and Luna 4
d) Luna 4
e) 24.3 hours
f) Remains almost directly above that spot
g) Midas 3
h) 9.4 m/sec^2

4.19 For the nose wheels, $a_c = 1.8 \times 10^4$ m/sec^2.

4.20 The centripetal acceleration of the scooter wheel would be twice that of the motor cycle wheel.

4.21 a) $a_c = 2.2 \times 10^{-10}$ m/sec^2
b) $F = 4 \times 10^{20}$ N
c) $F = 3.55 \times 10^{22}$ N
d) $v = 2.98 \times 10^4$ m/sec

Acknowledgments

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Project Physics **Teacher's Guide**

An Introduction to Physics 1 **Concepts of Motion**

Authorized Interim Version 1968-69

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This teacher guide is the authorized interim version of one of the many instructional materials being developed by Harvard Project Physics, including text units, laboratory experiments, and readers. Its development has profited from the help of many of the colleagues listed at the front of the text units.

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Introduction to Project Physics

Project Physics has as one of its major goals the development of course materials that will help to bring a larger proportion of high-school students to the study of physics, and to make this study more meaningful and enjoyable to students and instructors.

With the aid of experienced physicists and teachers, materials for a physics course have been developed that will be instructive and appealing to a wide variety of students, including those already intent on scientific careers, those who may not go to college and those who in college will concentrate in the humanities or social studies. For a detailed explanation of the background, aims, contents and teaching procedures, refer to the May 1967 issue of The Physics Teacher.

For the last group in particular, it is necessary to show that physics is neither an isolated body of facts and theories with merely vocational usefulness nor a glorious entertainment restricted to an elite group of specialists. Rather, it should be seen as an always unfinished creation at the forefront of human ingenuity. Moreover, what has been achieved in physics has always influenced, sooner or later, man's whole cultural life. To be ignorant of physics may therefore leave such students unprepared for their time.

The course being developed by Harvard Project Physics treats physics as a lively and fundamental science in its own right. But it also shows physics as an activity which is closely related to the achievements both in other sciences and outside science itself. This is not a minor or negligible aspect of the course; on the contrary, it reflects what physics truly is. By presenting physics in this way it may be possible to catch the interest of that large untapped group of students who now do not enroll in a physics course at all because their inclinations and talents lead them away from what they perceive as narrowly pre-professional physics courses.

This approach has the endorsement of outstanding and thoughtful physicists today. For example, the American Nobel Prize physicist, I. I. Rabi—who is also a member of the Advisory Committee of Harvard Project Physics—has stated:

> ...I believe, basically, we have not been cautious enough of the meaning of science in our generation, to teach it in a way which would be understood and appreciated and felt by the students. We have very little of the positive values of science outside of the applications which are obvious to anybody living in this age. In other words, my claim is, and this is something we should discuss, that we have not been teaching our science in a humanistic way. We have been teaching science at every level, in a certain sense, as a certain bag of tricks which the bright boy or girl could learn and show off with, or at least get a great deal of pleasure out of—the same kind of pleasure, but not quite as sharp, as he would get out of plane geometry.
>
> Now, science is a very different thing...it is an adventure of the whole human race to learn to live in and perhaps to love the universe in which they are. To be a part of it is to understand, to understand oneself, to begin to feel that there is a capacity within man far beyond what he felt he had, of an infinite extension of human possibilities—not just on the material side (whatever that may mean, because the more we study the material side, the more and more it recedes)....
>
> So what I propose as a suggestion for you is that science be taught at whatever level, from the lowest to the highest, in the humanistic way. By which I mean, it should be taught with a certain historical understanding, with a certain philosophical understanding, with a social understanding and a human understanding in the sense of the biography, the nature of the people who made this construction, the triumphs, the trials, the tribulations.*

The new Project Physics course has six general divisions: concepts of motion, motion in the heavens, energy, electromagnetism, models of the atom, and the nucleus. Each of the six separately bound units of the basic text has its own conceptual structure which holds its four chapters together as an integral unit. Each unit makes some connection to its neighbor by means of its prologue or epilogue—one outlining the important considerations that will be encountered in the chapters ahead, the other leading into the next unit while consolidating the work just studied. It also follows from the goals of the course that there are specific features which give it a different form from most existing physics courses. As the occasion arises, the text or the supplementary book of readings stresses the humanistic background of the sciences: how modern physical ideas have developed and who the men and women were who made key contributions; the effect which physics has had on other sciences, especially chemistry and astronomy; and the fact that the progress of physics contributes to contemporary technology and in turn is stimulated by it. The course will also touch on scientific methodology as illustrated by specific developments in physics.

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*From the address at AAAS meeting of Educational Policies Commission, 27 December 1966, Washington, D.C., The Physics Teacher, May 1967, p. 197.

Introduction to Project Physics Learning Materials

Like all high-school science courses that intend to attract substantial numbers of students, Harvard Project Physics has developed a wide range of learning materials. These include texts, student handbooks of laboratory and other activities, laboratory equipment, programmed instruction, tests, film loops (and some longer films), overhead transparencies and books of selected readings. These varied learning materials make possible a more effective use of your capacities as a teacher to make allowances for individual differences and environmental contingencies. In a word, they permit you to shape the course to fit the diverse needs and interests of your students.

In the next several sections you will find comments regarding each of the various Project Physics learning materials. Each of the materials has been designed to perform special and sometimes multiple functions in the course. You should develop a clear notion of what roles these materials can play in your classes and for your students, as such an understanding is essential for their effective use.

Text

One of the important learning materials in the Project Physics course is the "textbook." It is, however, only one and should not be thought of as anything more than one component of the course. It shares with the laboratory, the films, the handbook, and the rest the task of helping the student learn some significant amount of physics.

If the text has any unique function to serve, it is as a systematic guide for the student. It presents the main concepts to be dealt with in the course and establishes the logical and developmental relationships among those concepts. For this reason the text is frequently referred to by the Project Physics staff as "the student guide." A road map may be enormously helpful in establishing direction and perspective, but reading a map is not an adequate substitute for an actual drive through the mountains, or even for a good motion picture of such a drive; so it is that the physics text alone is not able to supply the rich variety of insights and experiences needed to fill out the course and make it intellectually and emotionally enriching. Thus it would be a mistake to assume that students can master topics just because they are introduced or used in the text. From time to time, indeed, the text treatment of a topic (e.g., vectors), is deliberately shallow or incomplete just because there exist in the course other instructional materials which can deal with that topic more effectively. The overprinted student text pages in this guide, as well as the chapter organizational charts and the interleaved pages, attempt to clarify for you the relationships between the text and the other course materials.

The Physics Reader

There are six sets of collections of articles from periodicals and text books, bound in paperback Readers, one for each of the six units of the text. Descriptions of reader articles will be found in the Aid Summaries section of each Teacher Guide. Also, there is a very brief summary of the content of each reader article right under the title in the reader itself. Biographical information on the authors is contained in the back of the readers.

We suggest that the readers be made available to students at all times. If some signout system is necessary, it should be the responsibility of the students and require a minimum amount of paperwork on your part. This does not mean, however, that the readers can be put on a shelf and just left there for the students to read when they "get interested." An important element in making the reader attractive to students is the manner in which you make reference to specific articles. For example, you might try, now and then, reading short quotations from articles during class time—without, however, letting this take much time from other class activities. But most of all, you should become as familiar as possible with the reader, so that you can refer students to specific articles when their interest in a certain topic is revealed during class discussion.

The reader is not a textbook. It is not even a supplementary textbook. You need not require that every student read a certain article. You should assign reader articles only occasionally, but guide students to these articles in such a way that they discover by themselves, how interesting many of them are.

The very nature of the reader—a collection of many different types of articles by authors living at different times and working in different fields—makes some part of the reader potentially appealing to every student. This potential can be distorted if the student gets the feeling that the reader is just one more piece of the package which must be studied and "learned" by exam time.

Your attitude toward the reader, and the manner in which you present it to your class, will determine how the reader is accepted by the student.

Laboratory and the Student Handbook

The students' experiences with nature in the Project Physics course are provided primarily through student experiments, teacher-group experiments and teacher demonstrations—and only secondarily by means of film loops, sound films, transparencies and programmed instruction materials. In addition, many opportunities are suggested in the Student Handbook for interested students to carry out investigations or activities on their own in school or at home. From this collection of activities every student should find something appealing.

Teacher demonstrations present phenomena with the student cast in the role of spectator—although hopefully not as an entirely passive one!

In teacher-group experiments students are encouraged to make observations and collect data, participate in the analysis and interpretation of the experiment and enter into a dialogue with the teacher concerning the meaning and interpretation of the data. This technique can furnish precise data or opportunities for you to make persuasive observations in response to student arguments or questions.

Student experiments are exercises performed by the student himself in the laboratory. Here the individual student, or a small group of students, constructs and manipulates the apparatus, carries through the required steps, answers questions as they arise, and each student draws his own conclusions. Individual results vary, of course, and thus at times only the pooling of results from all of the students (or student groups) will lead to useful conclusions from which generalizations can be made. Throughout the course, the purpose of the experiment and plan of investigation should comprise the majority of your pre-lab discussions, while the analysis should always be a part of post-lab discussions.

Description and comments on each of these exercises can be found in this guide.

Whatever form the experimental work at a given point in the course takes—teacher demonstrations, teacher-group experiments or student experiments—you should decide ahead of time what the main purposes of the activity are. In the Project Physics course, any given experiment may serve one or more of the following student purposes:

1. to become familiar with some of the phenomena with which the course deals. Thus, even before they "understand" them, students should encounter cases of uniform acceleration, wave refraction, spectra and the like.

2. to provide one of several approaches to the understanding of an important physical concept. Some students seem to learn best through laboratory activity, whereas others can learn by reading, or viewing films, or discussion, or even listening to lectures. In this sense, laboratory is simply one of the learning media.

3. to learn something about the nature of experimental inquiry and the role of the laboratory in the advancement of scientific knowledge.

4. to learn some things about the physical world other than by written or oral assertion, i.e., in a sense to "discover" something.

5. to have a pleasant experience. One of the advantages of science (and art and drama) courses over many others is that there are active, non-bookish ways for the student to become involved. With help and reassurance it is likely that even the most apparatus-shy youngster can learn to operate effectively in the laboratory and to enjoy doing so. It is probably important for students' long-run achievement in the course that the laboratory activity be a pleasant experience even for those who do not become terribly good at it.

The Student Handbook is a somewhat unique kind of document. It is much more than a laboratory guide. You should look through it carefully to become familiar with the details of its content and format and particularly with the instructions given to the student for its use. The handbook should serve to guide your students into interesting investigations beyond the usual classroom laboratory activities. It lists a variety of materials to assist him in learning more effectively the conceptual material treated in the text itself. Also, the handbook should help you to tailor the Project Physics course to the interests and strengths of individual students.

Overhead Projector Transparencies

Experimental versions of transparencies for overhead projectors are available. Some are standard diagrams; others present items too complex or time-consuming to put on the blackboard.

The colored overlays are attached to a standard mount with a 9" × 7½" opening. These can be used on overhead projectors with a 10" × 10" stage or larger. In many cases you will want to write on the plastic sheet to add information, fill in data, or to complete graphs. For this purpose we suggest Pentel fiber pens or some other type easily erased with a moist cloth.

Most transparencies refer specifically to test material. They are numbered consecutively, but may, however, be used with several sections or chapters. (For suggestions see the organization page preceding each chapter.) You are encouraged to use these transparencies in other ways than those suggested. Descriptions of individual transparencies may be found on the interleaved sheets of this guide in the sections where their use is suggested. Also, the descriptions are collected together in the resource section for each unit.

Programmed Instruction

Programmed instruction presents a carefully graded sequence of tasks to the student, a sequence which assures a high degree of success and then provides him with information about the correctness of his response or answer. This enables the student to learn by himself and increases the probability that he will stay on the right track.

Sample questions for each program booklet appear at the beginning of the booklet. Students are instructed to try the sample questions and, if they have difficulty, to complete the appropriate booklet. Programs can be used either in class or as homework. After taking a program, and sometimes during it also, the student may still have questions; however, his need for help from you in solving problems at the end of the chapter or in learning certain difficult concepts from the text will decrease. No less than with other learning materials, you are urged to go through the programs yourself if you intend to use them; this will alert you to the student questions which are likely to arise.

It is important that students refrain from spending too much time on programmed instruction at first, since the degree of concentration required results in a negative reaction if the student has not become accustomed to it. Sessions of fifteen minutes or less are best at first. This is one of the reasons why the project programs are brief. Another reason is that each program deals only with a restricted set of concepts or skills. Still another reason for short programs is that any given program is not meant to stand alone, being rather but one of an array of learning materials dealing with a topic.

Sound Films (16mm)

Project Physics has not become deeply engaged in the production of 16 mm sound films. One reason is that several good films already exist that are appropriate for the course. The ones that we know about have been keyed into the course and noted in several places in this guide. You should, of course, use (and add to the guide list) other films that you find relevant and effective.

The project has now completed one film, "People and Particles." This film is suitable for general viewing very early in the course, and again very late in the course. (Some notes for students will be sent to you before you receive the film. A copy of these notes accompanies this guide; see page 72. More extensive notes for teachers will be included in the Unit 6 Guide.) In production are two additional films. One is, in effect, a tour of the Cambridge Electron Accelerator; the other is a biographical film on Enrico Fermi. If they are ready for tryout before the end of this school year, you will be notified.

The Multi-Media System Schedules

The materials and media described above provide the teacher with many options—so many that their use can become a problem. To assist the teacher and to explore the effects of media on teaching and learning physics, an integrated multimedia system has been developed for each of the various units. A systems approach for Unit 1 was tested in nine trial classes last fall (1967) with some success; trial versions of the multi-media systems approach for all six units will be tried out this year on a somewhat enlarged but still limited basis. These schedules represent one of many ways of incorporating available media and materials into the course, and also suggest different approaches to teaching and learning which these media make possible.

One objective of the multi-media approach is to encourage and make possible individualized instruction and a variety of experience in the classroom; it also emphasizes the need to introduce physical phenomena before generalizing and abstracting physical laws from them. The systems make use of such techniques as "laboratory stations" for qualitative experience with phenomena, "small group discussions" to encourage a high level of individual participation, and various classroom activities designed to reinforce attention to historical, philosophical and sociological aspects of physics.

Film Loops

Approximately fifty 8 mm single-concept film loops have been made so far by Project Physics. These loops are closely integrated with our other course materials, and are intended both to complement and to supplement them. The loops, packaged in plastic cartridges, are of 3 to 4 minutes running time.

Some of the loops are purely qualitative demonstrations in physics, but the majority are quantitative. For the best results, the student should not just passively view these films, but rather he should take data from the projected images and then analyze them himself.

Instructions for the specific uses of the loops are found in the annotated Student Handbook section at the end of this guide. In addition, the contents of the loops are summarized in the Aid Summaries section of the Guide.

Evaluation and Testing

The flexibility of Project Physics requires that each student have the opportunity to demonstrate achievement in the areas of his interests and in ways appropriate to those interests. A variety of techniques can and should be used to supplement information obtained from written tests.

Project Physics has developed different types of tests for use with your students. There are six test booklets, one for each unit, each of which contains four different achievement tests. Two of the tests in each booklet are part multiple choice, part essay-and-problem. One of the tests is entirely essay-and-problem, and the other is entirely multiple choice. The four tests for each unit are independent of each other and easy to locate in the test booklet.

Suggestions for evaluation and testing are developed further in a separate booklet, Evaluation Guide. This booklet contains information to help you use and score the Project Physics tests, and suggestions for supporting the flexibility of the course through an equally flexible evaluation scheme. Part I, Concerning Evaluation, discusses this scheme. Part II has suggested answers to the Project Physics tests with some information regarding the nature of the test items. The tests are grouped by units, and additional questions are given as suggestions for teachers who wish to construct their own tests.

How to use the Teacher Guide

The Teacher Guide is intended to be a versatile and flexible document. It is not meant to restrict you in any way. Use the guide material selectively, adding to or subtracting from it as you go along, thereby essentially designing your own unique guide.

The guide is in loose-leaf format to increase its flexibility. You should arrange the various pages for your own most efficient use. You may find it helpful to attach tabs to the edges of some pages for quick reference. The yellow organizational pages suggest the pacing of topics and list the materials available for each section of the text; they may be interleaved with the corresponding chapters of the overprinted version of the text.

The organizational pages suggest the pacing of topics and lists the materials available for the treatment of each section of the text. The annotated student materials contain many cross-references to experiments, activities, film loops, reader articles, etc. The commentary sheets give more detailed suggestions for the development of each section and summaries of the associated materials.

Choose the particular learning materials appropriate to your supply of equipment, your time allotment and your evaluation of their worth to individual members of your class; there is much more material in the course than any teacher can use in one year. Experiment with this guide, use it and enjoy it—and do not be alarmed if a year or two go by before you feel really "at home" with it.

SUGGESTED YEAR TIME SCHEDULE

A unit time perspective (an average only)

UNIT 1	UNIT 2	UNIT 3	UNIT 4	UNIT 5	UNIT 6
30 days	24 days	30 days	30 days	25 days	20 days

Darken in those non-school days as dictated by YOUR school calendar. Then add the above average unit times to obtain an approximate finishing date.

Academic School Calendar

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
AUGUST																															
SEPTEMBER																															
OCTOBER																															
NOVEMBER																															
DECEMBER																															
JANUARY																															
FEBRUARY																															
MARCH																															
APRIL																															
MAY																															
JUNE																															

Overview of Unit 1

How do things move? *Why* do things move? The principal task of Unit 1 is to provide answers to these questions. A secondary task is to provide insight into the way scientists go about their work.

The first question, *how* do things move, is the basic question of *kinematics*. This question is answered gradually, starting with a very simple motion and proceeding to more complex motions.

Most of Chapter 1 is spent developing the tools to describe straight-line motion. The key concepts are average speed and instantaneous speed. The chapter concludes by making an analogy between the change in position with time (speed) and the change in speed with time (an example of acceleration).

Chapter 2 extends our description of motion to accelerating objects—an object in free fall. We follow Galileo through his own analysis as he seeks to confirm that the speed of a freely falling object is proportional to the elapsed time of its fall. By using Galileo as an example, Chapter 2 also serves to provide the student with some understanding of the scientist as a person carrying on his work within a social milieu.

The second question, *why* do things move, is the fundamental question of *dynamics*. Newton provided the answer to this question with his three laws of motion. These three laws are developed in Chapter 3. Vector concepts are introduced and are used throughout the remainder of the unit.

The final chapter in Unit 1 brings together the concepts learned in the first three chapters and applies them to projectile motion and uniform circular motion. Simple harmonic motion is introduced as a special topic.

Chapter 1 begins by citing an old maxim: "to be ignorant of motion is to be ignorant of nature." Indeed, kinematics and dynamics are to physics what grammar is to language or what scales are to music. The techniques learned in Unit 1 will be used throughout the course.

MULTI-MEDIA DAILY PLAN UNIT I

INTRODUCTION TO MULTI-MEDIA	PSSC FILM # 0307 Frames of Reference Small-group discussion on film	LAB STATIONS Uniform Motion	TEACHER PRESENTATION Time, distance and speed
1a Text: Unit I Prologue	2a Reader: On Being the Right Size, Haldane	3a Text: 1.1 through 1.4	4a Text: 1.5 through 1.7
LAB STATIONS Accelerated Motion	TEACHER PRESENTATION Velocity and Acceleration	PSSC FILM # 0107 Straight-Line Kinematics Small-group discussion on film	Organization of Naked-Eye Astronomy lab (E1)
5a Text: 1.8 Selected Problems Ch. I	6a Text: 2.1 through 2.4	7a Text 2.5 through 2.7 Program: Vectors I	8a Reader: The Scientific Revolution, Butterfield
E5 Seventeenth-Century Experiment	SMALL-GROUP Problem solving session	SMALL-GROUP DISCUSSION of Reader articles	TEACHER PRESENTATION: Galileo Quiz
9a Text: 2.8 through 2.10	10a Reader articles	11a Program: Vectors II	12a Reader: Galileo's Discussion Holton and Roller
LAB STATIONS Force, Mass and Acceleration	TEACHER-LED DISCUSSION Newton's laws	Small-group problem-solving session	PSSC FILM # 0302 Inertia (26 min) Discussion of film
13a Text 3.1 through 3.5 Program: Vectors III	14a Text: 3.6 through 3.9	15a Text: 3.10 and 3.11	16a Text: 4.1 and 4.2
Demonstration: Measuring acceleration of gravity	PSSC FILM #0307 (28 min.)	LAB STATIONS Complex Motion	
17a Text: 4.3 and 4.4	18a Reader: How the Scientific Revolution .... Willey	19a	20a Text: Epilogue Unit I
Student presentations of lab outcomes	Class review Unit I	Unit I exam	Discuss Unit I exam
21a Unit I Review	22a Unit I Review	23a	24a

Details of the Multi-Media Schedule

Day 1

This day is to be used to explain the multi-media system, and to charge students with the responsibility of self-directed instruction.

Day 2

Show the first 13 minutes of the film "Frames of Reference." Divide class at random into small groups, pass out 3 or 4 open-ended questions related to film, and use the rest of the period for discussion.

Day 3

Lab Stations: Uniform Motion

Students are to make qualitative observations of objects undergoing uniform motion. Students spend 8 to 10 minutes at each station. Brief instruction of what to look for at each station will be helpful.

1. balloon pucks on glass tray
2. pucks on dylite beads
3. dynamics carts with with accelerometer (D2)
4. polaroid photograph of tractor, blinky
5. film loops, e.g. L8 and L9.

Day 4

Teacher presentation on time, distance and speed.

Various demonstrations, transparencies and examples may be used to clarify the concepts and their measurements.

Day 5

Lab Stations: Accelerated Motion

1. dynamics cart with accelerometer (D4)
2. strobe photo of free-fall
3. transparency T5
4. analysis of strobe photo (D3)
5. analysis of hurdle race (L9)

Day 6

Teacher presentation: Velocity and Acceleration

A synthesizing discussion to tie together all the loose ends generated by the previous few days' activities.

Day 7

PSSC Film: Straight-Line Kinematics

Show first 17 minutes of film. (Stop film when Haefner returns to his lab.) Provide students with some open-ended questions and break into small groups for discussion.

Day 8

Organization for Experiment 1

The naked-eye astronomy lab (E1) requires taking of data systematically over a period of several weeks. Assign students specific objects on which to gather data, e.g., sun, moon, specific stars, planets, etc.

Day 9

Seventeen-Century Experiment (E5)

Day 10

Problem-solving session in small groups

Select problems from end of Chapter 1, and assign some to each of the small groups. As groups work, circulate and observe to see which students are in need of additional help.

Day 11

Small-group discussion of reader selections according to different topics; students choose which group they wish to join, each on a different article. Discussion may need priming, teacher should be familiar with all reader articles.

Day 12

Teacher presentation: Galileo

This lecture should touch upon the life and times of Galileo, but also on the need to verify theories by performing controlled experiments.

Day 13

Lab Stations: Force, Mass and Acceleration

1. inertia (D11)
2. the dependence of acceleration upon force and mass PSSC #III-2
3. changes in velocity with a constant force PSSC #III-1
4. Newton's laws (airtrack) (D12)
5. tractor-log paradox (T8)

Day 14

Teacher-led discussion: Newton's Laws

Clarify points still unclear about Newton's three laws of motion.

Day 15

Small-group problem solving

Selected problems from Chapters 2 and 3 are given. Have better students help poorer students.

Day 16

Film: PSSC #0302, Inertia

Follow with small-group discussion of film. Challenging guide questions for discussion are essential.

Day 17

Demonstration: Measuring Acceleration of Gravity

This demonstration (E7) can be done by student or teacher, with common data shared by whole class. Alternate: Use photos from Day 5, station #2.

Day 18

PSSC Film #0307, Frames of Reference (28 min)

Show the whole film, in post-film discussion bring out concept of inertial frames.

Day 19

Lab Stations: Complex Motion

1. trajectory apparatus (E8-1)
2. strobe photo of body on spring (D3)
3. trajectories (E8-2)
4. circular motion (E11-1)
5. circular motion (E11-2)

Students are to try apparatus at each station, making qualitative observations as usual.

Day 20

Lab stations, continued

Same stations as Day 19, but students are to pick one experiment and do it quantitatively.

Day 21

Students report to rest of class on results of experiments on Day 20. Urge that presentations be very short to allow plenty of discussion time.

Day 22

Class review for Unit 1 Exam

Day 23

Unit 1 Exam

Day 24

Discuss Unit 1 Exam

Chapter 1 Schedule Blocks

Each block represents one day of classroom activity and implies a 50-minute period. The words in each block indicate only the basic material under consideration.

Chapter 1: The Language of Motion

Read 1.1, 1.2, 1.3, 1.4

What measurements can be made to allow us to talk about speed?

Read 1.5, 1.6

Using graphs to interpret motion

Read Experiment

Lab motion

Analyze Lab

Post-lab and/or problem seminar

Read 1.7

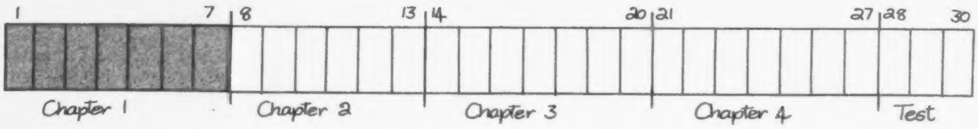
Instantaneous speed

Read 1.8

Acceleration

Review

Test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Chapter 1
Chapter 2
Chapter 3
Chapter 4
Test

Chapter 1 Resource Charts

		Easy		Hard	
1.1: The motion of things					E1: Naked eye astronomy E2: Regularity and time E3: Variations in Data D1: Recognizing simple motion
1.2: A motion experiment that does not quite work		2			
1.3: A better motion experiment		4 5		8	D2: Uniform motion using Accelerometer and dynamics cart
1.4: Leslie's "50" and the meaning of average speed			3 6 7	17	
1.5: Graphing motion		11			E4*: Uniform motion
1.6: Time out for a warning		13	14		
1.7: Instantaneous speed			9 15	16	D3: Instantaneous speed, using strobe photographs of mass on a spring
1.8: Acceleration by comparison			19	18	D4: Uniform acceleration using liquid accelerometer

Chapter 1 Resource Charts

R7: Motion in Words
R8: Representation of Motion

T1: Stroboscopic measurements

A1: Electronic stroboscope
A2: Making frictionless pucks

L8: Analysis of a hurdle race — Part I

L9: Analysis of a hurdle race — Part II
R13: Dynamics of a Golf Club

T2: Graphs of various motions

T3: Instantaneous speed
T4: Instantaneous rate of change
R9: Speed — Sawyer

T1: Stroboscopic measurements
T2: Graphs of various motions
F1: Straight-line kinematics

R1: The Value of Science

R14: Bad Physics in Athletic Measurements

Chapter 1 Experiment Summaries

Before or during Chapter 1; continue through Unit 2.

E1 Naked-Eye Astronomy

Suggestions are made for simple observations of the (apparent) motions of the sun, moon, stars and planets through the sky.

In poor viewing areas a planetarium visit may be substituted. A suggested program is given elsewhere in the Teacher Guide.

Equipment

Optional:
Shadow theodolite*
SC-1 Constellation chart*
Star & Satellite path finder*
Celestial calendar*

Early in course. Before or during Chapter 1.

E2 Regularity and Time

Students are asked to compare various repetitive events with one that they chose as a standard. E.g., blinky, pendulum, dripping tap, heartbeat, etc., are all compared with metronome. No clocks allowed. Raises the questions "What is regular?" and "How is time defined?"

Equipment

Dragstrip (chart recorder)*
blinky
pendulum
metronome etc.

Early in course, before or during Chapter 1.

E3 Variations in Data

Introduces students to the fact that variations in experimental data exist and suggests some of the courses.

Equipment

A wide variety of objects to count, measure, weigh, etc. See Teacher Guide notes on this experiment for detailed suggestions

E4 Uniform Motion

The students photograph and measure the motion of a puck sliding on plastic beads at constant speed. Alternatively they can use an air track glider, an air puck, a tractor-pushed blinky, a film loop or even the photograph in the textbook.

Sections 1.2 and 1.3 are necessary background. If the optional section at the end on graphing is done, read also Secs. 1.4 and 1.5.

The decision as to whether the data proves uniform motion or not raises an important question on the handling of experimental uncertainties.

Major equipment for version described:

Flat smooth surface (ripple tank)
Plastic beads*
Puck or other smooth-bottomed disc*
Polaroid camera*
Rotating disc strobe*
Light source*
Millimeter ruler for measuring picture*
Blinky*
Baby bulldozer*

*Equipment supplied to experimental schools by Harvard Project Physics during 1967-1968.

Chapter 2 Schedule Blocks

Each block represents one day of classroom activity and implies a 50-minute period. The words in each block indicate only the basic material under consideration.

Chapter 2: Free-Fall — Galileo Describes Motion

Read 2.1

Aristotelian theory of motion

Read 2.2, 2.3, 2.4, 2.5, 2.6

Galileo's theories of motion

Read 2.7 and Experiment

Accelerated motion

Read 2.8 and Analyze data

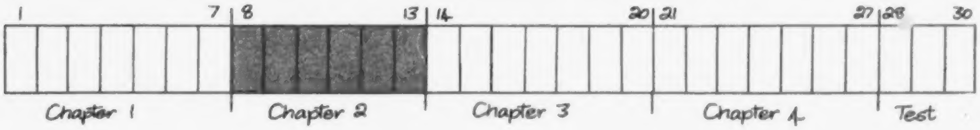
Post-Lab and/or problem seminar

Read 2.9, 2.10

Galileo's extensions

Review

Test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Chapter 1
Chapter 2
Chapter 3
Chapter 4
Test

Chapter 2 Resource Charts

	Easy		Hard	
2.1 The Aristotelian theory of motion	13			D5: Comparative fall rates of light and heavy objects.
2.2: Galileo and his times				
2.3: Galileo's "Two New Sciences"	14	5		D6: Coin and feather
2.4: Why study the motion of freely falling bodies?				
2.5: Galileo chooses a definition for uniform acceleration				
2.6: Galileo could not test his hypothesis directly				
2.7: Looking for logical consequences of Galileo's hypothesis		6 16	7 17 18 19 20	
2.8: Galileo turns to an indirect test	1 2 12	3 4		E5: A Seventeenth Century Experiment * or A Twentieth Century Version E6: of Galileo's Experiment.
2.9: How valid was Galileo's procedure?	8 9 10	11 15		E7: Measuring Acceleration of Gravity.
2.10: The consequences of Galileo's work on motion	13		21 22 23	

Chapter 2 Resource Charts

	A3: When is Air Resistance Important ?
R6: On Being the Right Size	
T6: Derivation of $d = v_i t + \frac{1}{2} at^2$	A4: Measuring your reaction time
	A5: Falling weights
L1: Acceleration due to gravity – Method I L2: Acceleration due to gravity – Method II	
T6: Derivation of $d = v_i t + \frac{1}{2} at^2$	

E5* A Seventeenth-century Experiment
or
E6* A Twentieth-century version of Galileo's Experiment
E7 Measuring Acceleration of Gravity

A series of five alternative experiments. The first two, involving motion down an inclined plane, simulate Galileo's experiment and are starred. The remaining three are measurements of a_g; if one of these cannot be done in the laboratory it may be done as a teacher-group experiment.

For all five experiments text Secs. 2.5 to 2.9 are necessary background. The modern equivalent of Galileo's experiment uses an air track as an inclined plane and a strobed Polaroid photograph for data.

a_g from a film loop requires the use of a clock with a sweep second hand to measure the motion of a falling ball in the film loop Acceleration Due to Gravity.

a_g from a pendulum requires a metal ball (or other symmetrical mass with an easily located center) suspended by a thread. Its back-and-forth motion (timed with a clock with a sweep second hand), and the length of the pendulum (meter stick needed) lead to a value of a_g when substituted into a simple formula.

a_g from direct fall requires the use of a tuning fork, which is used to ink a wavy line onto a strip of ticker tape dragged past it by a falling weight. From the increasing length of each "wave" on the tape a_g can be calculated. This experiment is direct but more difficult than the others both in procedure and in computation.

Major equipment for seventeenth-century experiment:

Grooved incline about 6 feet long
Supporting ringstands
Ball to roll in groove
Water clock*

Major equipment for twentieth-century version:

Air track and glider*
Polaroid camera*
Rotating disc strobe*
Light source*
Blower for air track*

Chapter 3 Schedule Blocks

Each block represents one day of classroom activity and implies a 50-minute period. The words in each block indicate only the basic material under consideration.

Chapter 3: The Birth of Dynamics – Newton Explains Motion

Read 3.1, 3.2, 3.3, 3.4

Force and Mass
Aristotle's "why" of motion

Read 3.5, 3.6

Forces in equilibrium and Newton I

Read 3.7

Newton II
Mass, weight and gravitation

Read Experiment

Newton's second law

Analyze Lab

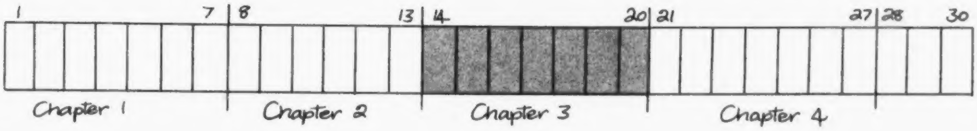
Post-Lab and/or problem seminar

Read 3.9, 3.10, 3.11

Newton III and the unity of the three laws

Review

Test



Chapter 3 Resource Charts

	Easy		Hard	
3.1: The concept of mass and force				
3.2: About vectors				D7: Addition of vectors D8: Direction of $\vec{v}$ and $\vec{a}$ D9: Direction of $\vec{v}$ and $\vec{a}$ (airtrack) D10: Non-commutativity (rotation)
3.3: Explanation and the laws of motion	1			
3.4: The Aristotelian explanation of motion	2			
3.5: Forces in equilibrium		3		
3.6: Newton's first law of motion	4 5	6 7		D11: Newton's first law
3.7: Newton's second law of motion	8 11	9 10		E8: Newton's second law D12: Newton's laws (airtrack) D13: Effects of friction on $\vec{a}$
3.8: Mass, weight and gravitation		15 17 18	12	E9: Inertial and gravitational mass D14: Demonstrations with rockets D15: Making an inertial balance
3.9: Newton's third law		19		D16: Action – Reaction (rope) No. 1 D17: Action – Reaction (rope) No. 2 D18: Reaction forces of a wall
3.10: Using Newton's laws of motion		13	16 14	D19: Newton's third law D20: Action – Reaction (car) D21: Action – Reaction (nail) D22: Action – Reaction (jumping)
3.11: Nature's basic forces				

Chapter 3 Resource Charts

L3: Vector addition I – Velocity of a boat
P4: Vectors

P4: Vectors

L4: A Matter of relative motion (qualitative)
R12: Newton's Laws of Dynamics
The Laws of Motion and Proposition One (R10 in Reader 2)
F2: — PSSC — Inertia

A6: Newton's first law

A7: Newton's second law

A8: Accelerometers

T8: The Tractor – Log Paradox

R15: The Scientific Revolution
R16: How the Scientific Revolution of the 17th Century
Affected other Branches of Thought

E8 Newton's Second Law

A dynamics cart of known mass is accelerated by means of a falling weight. The acceleration is measured from a stroboscope photograph or accelerometer. The accelerating force F is compared with the resulting ma.

A discussion of experimental errors is a very important part of this experiment that is assumed in future work.

Students should have studied text Sec. 3.7 before the experiment.

Major equipment:

Dynamics cart*
Blinky*
Spring scale taped to cart
Table-corner pulley
(Hooked) weights and string
(Polaroid camera*
either (Rotating disc stroboscope*
(Light source*
or Accelerometer*

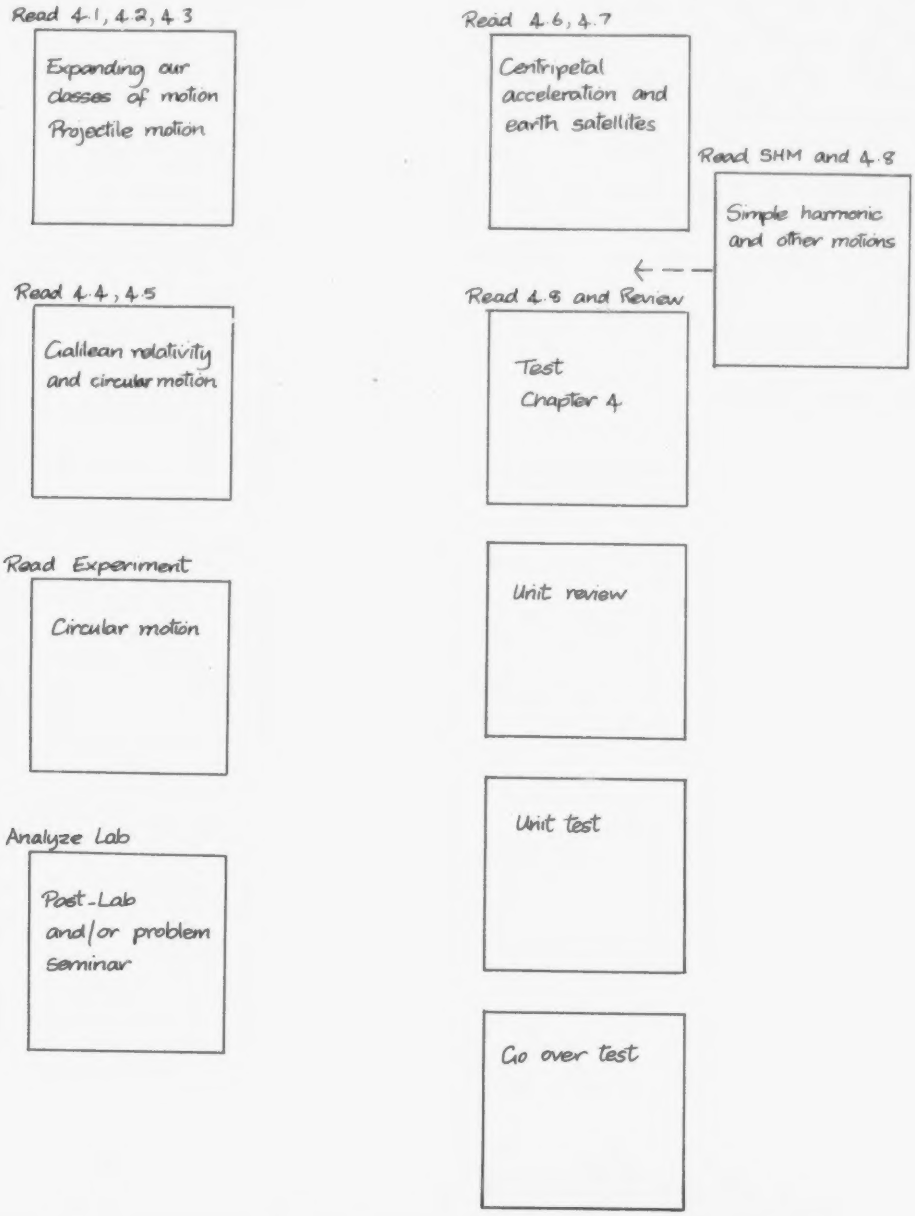
E9 Inertial Gravitational Mass

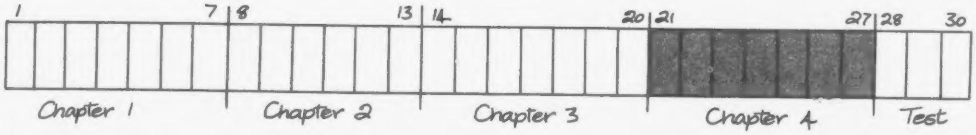
The experiment uses a simple inertial balance and ordinary weighing to show the proportionality of inertial and gravitational mass. The operation of the inertial balance is shown to be independent of gravity.

Chapter 4 Schedule Blocks

Each block represents one day of classroom activity and implies a 50-minute period. The words in each block indicate only the basic material under consideration.

Chapter 4: Understanding Motion


Read 4.1, 4.2, 4.3
Expanding our classes of motion
Projectile motion
Read 4.4, 4.5
Galilean relativity and circular motion
Read Experiment
Circular motion
Analyze Lab
Post-Lab and/or problem seminar
Read 4.6, 4.7
Centripetal acceleration and earth satellites
Read SHM and 4.8
Simple harmonic and other motions
Read 4.8 and Review
Test
Chapter 4
Unit review
Unit test
Go over test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Chapter 1
Chapter 2
Chapter 3
Chapter 4
Test

Chapter 4 Resource Charts

	Easy		Hard	
4.1: A trip to the moon	1 2			
4.2: Projectile motion	3 4			
4.3: What is the path of a projectile?		8		E10: Trajectories
4.4: Galilean relativity	5 6 7 10			D23: Frames of reference D24: Inertial vs. non-inertial reference frames
4.5: Circular motion	9 11 12			D25: Uniform circular motion E11: Circular motion I E12: Circular motion II
4.6: Centripetal acceleration		15 20 21 23	19	
4.7: Motion of earth satellites	13 14	22 24		D26: Simple harmonic motion D27: Simple harmonic motion (airtrack) (See also T.G. Unit 3. Chapter 12)
4.8: What about other motions?				

Chapter 4 Resource Charts

T9: Projectile motion R11: Galileo's Discussion of Projectile Motion	A10: Speed of a stream of water A9: Projectile motion demonstrations A11: Photographing projectile motion
T9: Projectile motion T10: Path of a projectile F3: PSSC - Freefall and projectile motion	
L4: A matter of relative motion L5: Galilean relativity 1 (ball dropped from ship) L6: Galilean relativity 2 (object dropped from aircraft) L7: Galilean relativity 3 (projectile fired vertically) F4: PSSC - Frames of reference	A12: Motion in a rotating reference frame
R13: The Dynamics of a Golf Club	A14: Penny and coat hanger
T11: Centripetal acceleration — graphical F5: PSSC — Vector kinematics F5: PSSC - Vector kinematics R12: Newton's Laws of Dynamics L8: Analysis of a hurdle race I L9: Analysis of a hurdle race II	A15: Harmonograms A13: Photographic analysis of walking
R17: Rigid Body R18: The Vision of Our Age R19: Chart the Future	

E10 Trajectories

In Trajectories-I a launching ramp and impact board provide a means for obtaining a direct record of the trajectory of a steel ball. Analysis of the record illustrates the independence of the horizontal and vertical motion.

In Trajectories-II a ball rolling at measured speed rolls off the table and hits the floor. The student's task is to use the equations for trajectory motion to predict the impact point. Successful prediction verifies the equations and the assumptions governing their use.

Major equipment: Trajectories-I

Trajectory plotting equipment*
Onion skin paper
Carbon paper
Steel ball*
Graph paper

Major equipment: Trajectories-II

Steel ball
Meter stick
Clock with sweep second hand
(preferably stopwatch)

E11, E12 Circular Motion

Done as a laboratory exercise or as a teacher-group experiment, either version of Circular motion builds on earlier understanding of Newton's second law and leads into the subsequent study of satellite motion in Unit 2.

Circular motion I assumes an acquaintance with text Secs. 4.5 and 4.6. A weight is put at increasing distances from the center of a rotating turntable until it slips. From its position when it slips $F = mv^2/R$ can be verified.

Circular motion II assumes no acquaintance with Secs. 4.5 and 4.6. The relationship $F = mv^2/R$ is deduced from tension and length of a string constraining a known weight in a circular path when whirled around the experimenter's head.

Major equipment: Circular motion I

Turntable with large masonite top*
Weights
Spring scale and string

Major equipment: Circular motion II

2 or 3 medium-sized rubber stoppers,
preferably with holes
Spring scale
Meter stick
String
Medicine dropper (for bearing)
Clock with sweep second hand

Brief Answers to Chapter 1 Study Guide

1.2 (a) $v = 6.0$ cm/sec
(b) $s = 15$ mi
(c) $t = 0.25$ min
(d) $\underline{v} = 3$ m/sec
(e) $\overline{v} = 40$ mi/hr
(f) $v = 40$ mi/hr
$s = 120$ mi
(g) $\Delta t = 5.5$ sec
(h) $\Delta s = 8.8$ m

1.3 $\overline{v} = 1.998$ mi/hr

1.4 $s = 2.2 \times 10^3$ mi

1.5 $t = 8.5$ yr

1.6 (a) $\overline{v} = 1.7$ m/sec
(b) $\overline{v} = 3.0$ m/sec

1.7 discussion

1.8 discussion

1.9 discussion

1.10 discussion

1.11 $\Delta y/\Delta x = 0.5$
$\Delta y/\Delta x = 1.0$
$\Delta y/\Delta x = 1.5$
$\Delta y/\Delta x = 2.0$

1.12 $v = 315{,}000$ in/sec

1.14 $s = 25.6$ m

1.15 (a) from 1 to 4.5 sec
$v = 1.3$ m/sec
(b) from 6 to 10 sec
(c) $v = 0.17$ m/sec
(c) $v = 0.74$ m/sec
(d) $v = 0.79$ m/sec
(e) $s = 0.42$ m

1.16 discussion

1.17 $v = 40$ mi/hr

1.18 (a) 14.05 m/sec
(b) 6.3 m/sec^2
1.19 discussion

Solutions to Chapter 1 Study Guide

STUDY GUIDE SOLUTIONS

1.2 Some practice problems:

Problem	Situation	Find
a	Speed uniform, distance = 72 cm, time = 12 sec	Speed
b	Speed uniform at 45 mph	Distance traveled in 20 min
c	Speed uniform at 36 ft/min	Time to move 9.0 ft
d	$s_1 = 0$, $s_2 = 15$ m, $s_3 = 30$ m $t_1 = 0$, $t_2 = 5.0$ sec, $t_3 = 10$ sec	Speed and position at 8.0 sec
e	You drive 240 mi in 6.0 hr	Average speed
f	Same	Speed and position after 3.0 hr
g	Average speed = 76 cm/sec computed over a distance of 418 cm	Time taken
h	Average speed = 44 m/sec computed over time interval of 0.20 sec	Distance moved

----

(a) $v = \frac{s}{t}$

$s = 72$ cm

$t = 12$ sec

$$v = \frac{72\text{ cm}}{12\text{ sec}}$$

$$\underline{v = 6.0\ \frac{\text{cm}}{\text{sec}}}$$

(b) $s = vt$

$v = 45\ \frac{\text{mi}}{\text{hr}}$

$$t = 20\text{ min} = \frac{20\text{ min}}{60\ \frac{\text{min}}{\text{hr}}}$$

$t = 0.33$ hr

$$s = 45\ \frac{\text{mi}}{\text{hr}} \times .33\text{ hr}$$

$$\underline{s = 15\text{ m}}$$

(c) $s = vt$

$t = \frac{s}{v}$

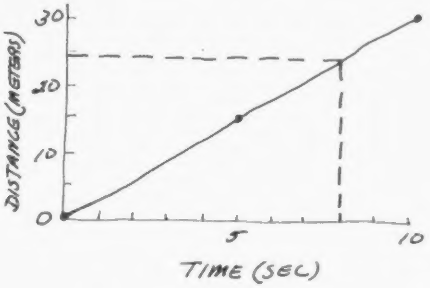
$s = 9.0$ ft

$v = 36\ \frac{\text{ft}}{\text{min}}$

$$t = \frac{9.0\text{ ft}}{36\ \frac{\text{ft}}{\text{min}}}$$

$$\underline{t = 0.25\text{ min}}$$

(d) In this problem, we need to interpolate between data points. At 8 seconds.



$v = \frac{s}{t}$

$s = 24$ m

$t = 8$ sec

$$v = \frac{24\text{ m}}{8\text{ sec}}$$

$$\underline{v = 3\ \frac{\text{m}}{\text{sec}}}$$

(e) $\bar{v} = \frac{\Delta s}{\Delta t}$

$\Delta s = 240$ mi

$\Delta t = 6.0$ hr

$\bar{v} = \frac{240 \text{ mi}}{6.0 \text{ hr}}$

$\underline{\bar{v} = 40 \frac{\text{mi}}{\text{hr}}}$

(f) Here, we must assume uniform speed. From problem (e), $\bar{v} = 40 \frac{\text{mi}}{\text{hr}}$

$s = \bar{v}t$

$t = 3.0 \text{ hr}$

$s = 40 \frac{\text{mi}}{\text{hr}} \times 3.0 \text{ hr}$

$\underline{s = 120 \text{ mi}}$

(g) $\bar{v} = \frac{\Delta s}{\Delta t}$

$\Delta s = 418 \text{ cm}$

$\bar{v} = 76 \frac{\text{cm}}{\text{sec}}$

$\Delta t = \frac{418 \text{ cm}}{76 \frac{\text{cm}}{\text{sec}}}$

$\underline{\Delta t = 5.5 \text{ sec}}$

(h) $\bar{v} = \frac{\Delta s}{\Delta t}$

$\Delta s = \bar{v}\Delta t$

$\bar{v} = 44 \frac{\text{m}}{\text{sec}}$

$\Delta t = 0.20 \text{ sec}$

$\Delta s = 44 \frac{\text{m}}{\text{sec}} \times 0.20 \text{ sec}$

$\underline{\Delta s = 8.8 \text{ m}}$

1.3 If you traveled one mile at a speed of 1000 miles per hour and another mile at a speed of 1 mile per hour your average speed would not be $\frac{1000 + 1}{2}$ mph or 500.5 mph.

What would be your average speed?

$\bar{v} = \frac{\Delta s}{\Delta t}$

$\Delta s = 1 \text{ mi} + 1 \text{ mi} = 2 \text{ mi}$

Δt = time for first mile, t_1, plus time for second mile, t_2.

$t_1 = \frac{s}{v} = \frac{1 \text{ mi}}{1000 \frac{\text{mi}}{\text{hr}}} = 0.001 \text{ hr}$

$t_2 = \frac{1 \text{ mi}}{1 \frac{\text{mi}}{\text{hr}}} = 1 \text{ hr}$

$\Delta t = t_1 + t_2 = 1.001 \text{ hr}$

$\bar{v} = \frac{2 \text{ mi}}{1.001 \text{ hr}} = \underline{1.998 \frac{\text{mi}}{\text{hr}}}$

1.4 A tsunami (incorrectly called "tidal wave") caused by an earthquake occurring near Alaska in 1946 consisted of several sea waves which traveled at the average speed of 490 miles/hour. The first of the waves reached Hawaii four hours and 34 minutes after the earthquake occurred. From these data, calculate how far the origin of the tsunami was from Hawaii.

$$s = \bar{v}t$$

$$\bar{v} = 490\ \frac{\text{mi}}{\text{hr}}$$

$$t = 4\text{ hr }34\text{ min} - 4.57\text{ hr}$$

$$s = 490\ \frac{\text{mi}}{\text{hr}} \times 4.57\text{ hr}$$

$$\underline{s = 2.2 \times 10^3\text{ mi}}$$

1.5 Light and radio waves travel through a vacuum in a straight line at a speed of nearly 3×10^8 m/sec. The nearest star, Alpha Centauri, is 4.06×10^{16} m distant from us. If this star possesses planets on which highly intelligent beings live, how soon could we expect to receive a reply after sending them a radio or light signal strong enough to be received there?

Assuming an instant reply to our message, the wave would have to travel twice the distance to Alpha Centauri.

$$s = vt$$

$$t = \frac{s}{v}$$

$$s = 2 \times 4.06 \times 10^{16}\text{ m} = 8.12 \times 10^{16}\text{ m}$$

$$v = 3 \times 10^8\ \frac{\text{m}}{\text{sec}}$$

To make this figure more meaningful we should convert it into years.

$$365\ \frac{\text{day}}{\text{yr}} \times 24\ \frac{\text{hr}}{\text{day}} \times 60\ \frac{\text{min}}{\text{hr}} \times 60\ \frac{\text{sec}}{\text{min}} = 3.18 \times 10^7\ \frac{\text{sec}}{\text{yr}}$$

$$\underline{\frac{2.7 \times 10^8\text{ sec}}{3.18 \times 10^7\ \frac{\text{sec}}{\text{yr}}} = 8.5\text{ years}}$$

1.6 What is your average speed in the following cases?

(a) You run 100 m at a speed of 5.0 m/sec and then you walk 100 m at a speed of 1.0 m/sec.

(b) You run for 100 sec at a speed of 5.0 m/sec and then you walk for 100 sec at a speed of 1.0 m/sec.

(a) First we need to find the total time taken.

$$v = \frac{s}{t}$$

$$t = \frac{s}{v}$$

$$t = t_1 + t_2$$

$$t = \frac{s_1}{v_1} + \frac{s_2}{v_2}$$

$$t = \frac{100\text{ m}}{5.0\ \frac{\text{m}}{\text{sec}}} + \frac{100\text{ m}}{1.0\ \frac{\text{m}}{\text{sec}}} = 120\text{ sec}$$

The average speed for the total trip is

$$\bar{v} = \frac{\Delta s}{\Delta t}$$

$$\bar{v} = \frac{2 \times 100\text{ m}}{120\text{ sec}}$$

$$\underline{\bar{v} = 1.7\ \frac{\text{m}}{\text{sec}}}$$

(b) $\bar{v} = \frac{\Delta s}{\Delta t}$

but $\Delta s = \Delta s_1 + \Delta s_2$

$$\Delta s = v_1 \Delta t_1 + v_2 \Delta t_2$$

$$\Delta s = 5.0\ \frac{\text{m}}{\text{sec}} \times 100\text{ sec} + 1.0\ \frac{\text{m}}{\text{sec}} \times 100\text{ sec}$$

$$\Delta s = 600\text{ m}$$

$$\bar{v} = \frac{600\text{ m}}{200\text{ sec}}$$

$$\bar{v} = 3.0\ \frac{\text{m}}{\text{sec}}$$

Note that in one case the average speed is the average of the individual speeds; in another it is not. Why?

1.7 Design some experiments which will enable you to make estimates of the average speeds for some of the following objects in motion.

(a) baseball heaved from outfield to home plate
(b) the wind
(c) a cloud
(d) a raindrop (Do all drops have different speeds?)
(e) hand moving back and forth as fast as possible
(f) the tip of a baseball bat
(g) walking on level ground, up stairs, down stairs
(h) a bird flying
(i) an ant walking
(j) a camera shutter opening and closing

(a) Compare speed of ball with speed of runner from third base. Times about the same. A runner does 100 yards in 10 seconds or 30 feet per second. Distance to outfield 300 feet. Speed of ball 100 feet per second.

(b) Make a cup anemometer. Time speed with strobe.

(c) Sight on edge of cloud. Get time to go certain angle. Assume distance of cloud two or three miles.

(d) Get trace on car window. Slope gives comparative speed of car and drop. Large drops off a roof follow falling body laws. Drops in fine mist depend on viscosity of air. Compare with Millikan's oil drop experiment.

(e) Use fingers as a strobe to stop light flashing at known rate.

(f) Speed of bat same as ball. Get speed to give range to center field bleachers.

(g) Time with stop watch. Repeat several times.

(h) Scare a pigeon from a roof to a neighboring tree.

(i) Confine ant to a definite path.

(j) Note length of trace of fast moving bright light as photographed.

1.8 What problems arise when you attempt to measure the speed of light? Can you design an experiment to measure the speed of light?

Short time intervals on account of high speeds. Receive a light pulse reflected from a known distance to excite a photocell. Amplify output of cell to excite the lamp giving light pulse. Measure high frequency of pulses. The light beam couples the lamp to the cell. If the action of lamp and cell takes appreciable time, note change in frequency as reflecting mirror is moved a known distance.

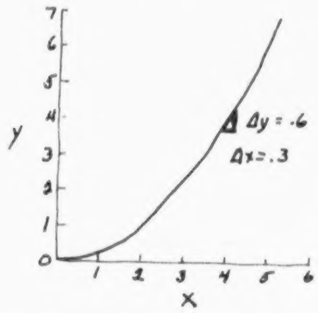
1.9 Sometime when you are a passenger in an automobile compare the speed as read from the speedometer to the speed calculated from $\frac{\Delta s}{\Delta t}$. Explain any differences.

Large differences will probably be due to long periods in slow moving traffic, or intervals of zero speed such as when waiting for red lights. Some speedometers tend to read high. Also faulty calibration between odometer and speedometer might be a factor. Where the highway department has marked "measured miles" these last two possibilities can be checked.

1.10 An automobile speedometer is a small current generator driven by a flexible cable run off the drive shaft. The current produced increases with the rate at which the generator is turned by the rear axle. The speedometer needle indicates the current. Until the speedometer is calibrated it can only indicate changes in speed, but not actual speeds in miles per hour. How would you calibrate the speedometer in your car if the company had forgotten to do the job? If you replaced your 24" diameter rear wheels with 28" diameter wheels, what would your actual speed be if your speedometer read 50 mph? Would your speedometer read too high or too low if you loaded down the rear end of your car and had the tire pressure too low? What effect does the speedometer have on the speed of the car? Can you invent a speedometer that has no effect on the motion of the car?

Time your car over measured mile at various constant speeds. Try to hold the speedometer at one spot on dial during a run. Compute speed and mark dial. You go $\frac{28}{24}$ or $\frac{7}{6}$ more per turn. Therefore your speed is 58.3 mph. With reduced radius of wheel the speedometer reads too high. Practically no effect on the speed. It takes little power. You might design one acting on wind pressure which the car must overcome anyway. It would have a streamlined intake.

1.11 Take a look at the graph of y versus x shown below:



Notice that in this graph the steepness increases as x increases. One way to indicate the steepness of the graph at a point is by means of the "slope." The numerical value of the slope at a point P is obtained by the following procedure, which is diagramed above. Move a short distance along the graph from point A to point B, which are on the curve and lie on either side of point P. Measure the change in y, (Δy) in going from A to B. In this example $\Delta y = .6$. Measure the corresponding change in x, (Δx) in going from A to B. Δx here is .3. The slope is defined as the ratio of Δy to Δx.

$$\text{Slope} = \frac{\Delta y}{\Delta x}\ .$$

In the example

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{.6}{.3} = 2.$$

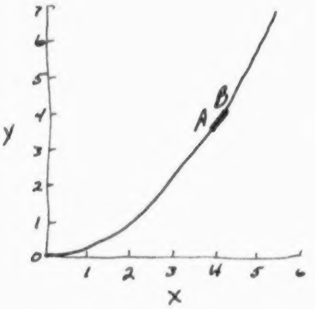
Now there are three important questions concerned with slopes that we must answer.

Q. What are the dimensions or units for the slope?

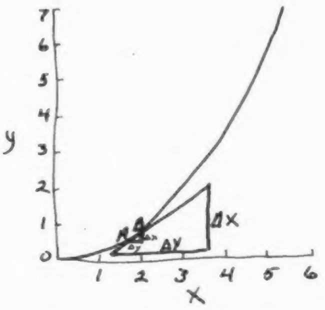
A. The dimensions are just those of y/x. For example, if y represents a distance in meters and x represents a time in seconds then the units for slope will be meters/seconds or meters per second.

Q. In practice how close do A and B have to be to point P? (Close is not a very precise adjective. New York is close to Philadelphia if you are traveling by jet. If you are walking it is not close.)

A. Choose A and B near enough to point P so that the line connecting A and B lies along the curve at point P. For example:



Q. Suppose A and B are so close together that you cannot read Δx or Δy from your graph. What does one do to calculate the slope?



A. Extend line AB as it is shown in the figure and compute its slope. Notice that the small triangle is similar to the large triangle and that $\frac{\Delta Y}{\Delta X} = \frac{\Delta y}{\Delta x}$.

Determine the slopes of this graph of y versus x at x = 1, 2, 3 and 4.

at x = 1 $\quad \frac{\Delta y}{\Delta x} = 0.5$

at x = 2 $\quad \frac{\Delta y}{\Delta x} = 1.0$

at x = 3 $\quad \frac{\Delta y}{\Delta x} = 1.5$

at x = 4 $\quad \frac{\Delta y}{\Delta x} = 2.0$

Expect answers to vary due to error involved in individual estimation.

1.12 The electron beam in a TV set sweeps out a complete picture in 1/30th of a second and each picture is composed of 525 lines. If the width of the screen is 20 inches, what is the speed of that beam over the surface of the screen?

$$v = \frac{s}{t} = \frac{525 \times 20}{1/30} = 315{,}000\ \frac{\text{in}}{\text{sec}}$$

approximately 8 kilometer/sec or 5 miles/sec.

1.14 World's 400-meter swimming records for men and women. Ages are in parentheses:

1926	4:57.0 5:53.2	Weissmuller (18) Gertrude Ederle (17)
1936	4:46.4 5:28.5	Syozo Makino (17) Helene Madison (18)
1946	4:46.4 5:00.1	Makino (17) Hveger (18)
1956	4:33.3 4:47.2	Hironoshin Furuhashi (23) Crapp (18)
1966	4:11.1 4:38.0	Frank Weigand (23) Martha Randall (18)

By about how many meters would Martha Randall have beaten Johnny Weissmuller if they had raced each other? Could you predict the 1976 world's record for the 400-meter race by extrapolating the graph of world records vs. dates up to the year 1976?

By extrapolation quite uncertain.

For men 4:00
For women 4:30 in 1976.

In making graph use as ordinates for women, seconds above 4:30, for men, seconds above 4:00.

Martha wins over Weissmuller by 19 sec. Weissmuller's speed is $\frac{400 \text{ meters}}{297 \text{ sec}}$.

$$s = vt = \frac{400 \times 19}{297} = 25.6 \text{ meters.}$$

1.15 Detailed analysis of a stroboscopic photograph of a rolling ball yielded information which was plotted on this graph (Fig. 1.35). By placing your ruler tangent to the curve at appropriate points estimate the following:

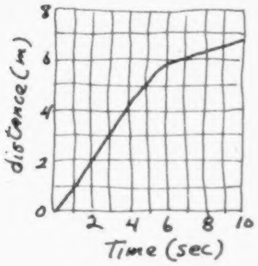
(a) At what moment or interval was the speed greatest? What was the value of the speed at that time?

(b) At what moment or interval was the speed least? What was it at that time?

(c) What was the speed at time 5.0 sec?

(d) What was the speed at time 0.5 sec?

(e) How far did the ball move from time 7.0 sec to 9.5 sec?



(a) The speed is greatest in the interval from 1 to 4.5 seconds.

$v = \frac{\Delta s}{\Delta t}$

$\Delta s = 5.2 \text{ m} - 0.8 \text{ m} = 4.4 \text{ m}$
$\Delta t = 4.5 \text{ sec} - 1 \text{ sec} = 3.5 \text{ sec}$

$v = \frac{4.4 \text{ m}}{3.5 \text{ sec}}$

$$\underline{v = 1.3 \ \frac{\text{m}}{\text{sec}}}$$

(b) Speed is least in the interval from 6 to 10 seconds.

$v = \frac{\Delta s}{\Delta t}$

$\Delta s = 6.7 \text{ m} - 5.0 \text{ m} = 1.7 \text{ m}$
$\Delta t = 10 \text{ sec} - 0 \text{ sec} = 10 \text{ sec}$

$v = \frac{1.7 \text{ m}}{10 \text{ sec}}$

$$\underline{v = 0.17 \ \frac{\text{m}}{\text{sec}}}$$

(c) $v = \frac{\Delta s}{\Delta t}$

$v = \frac{6.1 \text{ m}}{8.2 \text{ sec}}$

$$\underline{v = 0.74 \ \frac{\text{m}}{\text{sec}}}$$

(d) $v = \frac{s}{t} = \frac{7.7 \text{ m}}{9.7 \text{ sec}}$

$$\underline{v = 0.79 \ \frac{\text{m}}{\text{sec}}}$$

(e) During this interval the velocity was constant at about 0.17 m/sec [from part (a)]

$s = vt$
$s = 0.17 \ \frac{\text{m}}{\text{sec}} \times 1.5 \text{ sec}$

$$\underline{s = 0.25 \text{ m}}$$

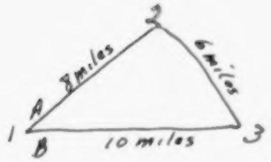
This can be checked by noting on the graph that the ball moved from about 6.3 to 6.7 meters, a distance of approximately 0.4 m. Whic method would you think is more precise?

1.16 Suppose you must measure the instantaneous speed of a bullet as it leaves the barrel of a rifle. Explain how you would do this.

Fire bullet through two rotating thin paper discs spaced a short known distance apart and rotating at a high known speed. The bullet hole in the second disc will be displaced a certain angle relative to that in the first. The fraction this angle is of a whole revolution times the period of one revolution gives the time for the bullet to travel between discs, from which the speed may be computed by $v = \frac{s}{t}$.

An optional method is to use a ballistic pendulum using the law of conservation of momentum.

1.17



Car A and car B leave point 1 simultaneously and both travel at the same speed. Car A moves from 1 to 2 to 3 while car B moves from 1 to 3 directly. If B arrives at point 3 six minutes before A arrives, what was the speed of either car?

Speed does not take into account direction. Let t_A and t_B be the times of the two cars and v their common speed.

Then $v = \frac{s}{t} = \frac{6 + 8}{t_A} = \frac{10}{t_B}$.

But $t_A = t_B + 6$

Substituting

$$\frac{6 + 8}{t_B + 6} = \frac{10}{t_B}$$

or $14t_B = 10t_B + 60$

$t_B = 15$ minutes

$v = \frac{10}{15} = \frac{2}{3}$ miles per minute or 40 mph.

1.18 The data below show the instantaneous speed in a test run of a Corvette car, starting from rest. Plot the speed-versus-time graph, and derive and plot the acceleration-time graph.

a) What is the speed at t = 2.5 sec?
b) What is the maximum acceleration?

Time (sec)	Speed (m/sec)
0.0	0.0
1.0	6.3
2.0	11.6
3.0	16.5
4.0	20.5
5.0	24.1
6.0	27.3
7.0	29.5
8.0	31.3
9.0	33.1
10.0	34.9

(a) The speed at t = 2.5 sec is $v = (11.6+16.5)/2 = 14.05$ m/sec

(b) From the data given, the maximum acceleration would appear to be $(6.3-0.0)/1.0 = 6.3$ m/sec^2 for the first time interval. Graphical interpolation might give a slightly different result.

1.19 Discuss the motion of the animals in the following photographs.

The student may be interested in studying the constancy of the velocity in the various sequences: For example: does the vertical and horizontal velocity of the bird vary with its wingbeat? Draw a velocity vs time graph for the leopard. Estimate what percentage of the time the horse is completely off the ground.

Brief Answers to Chapter 2 Study Guide

2.1 discussion

2.2 discussion

2.3 discussion

2.4 discussion

2.5 discussion

2.6 proof

2.7 proof

2.8 (a) true
(b) true
(c) false
(d) true

2.9 (a) true
(b) true
(c) true
(d) true

2.10 table

2.11 discussion

2.12 discussion

2.13 discussion

2.14 discussion

2.15 discussion

2.16 (a) $s = -5.0$ m
(b) $v = -10$ m/sec
(c) $s = -15$ m

2.17 (a) $v = 10$ m/sec
(b) $s = 15$ m
(c) $t = 2$ sec
(d) $s = 20$ m
(e) $v = -20$ m/sec

2.18 (a) $v_f = 20$ m/sec
(b) $v_f = -20$ m/sec
(c) $t = 4$ sec
(d) $s = 80$ m
(e) $v_f = 0$ m/sec
(f) $v_f = -40$ m/sec

2.19 (a) $a = -2$ m/sec^2
(b) $\bar{v} = 2$ m/sec
(c) $v_f = 2$ m/sec
(d) $s = 4$ m
(e) $v_f = -2$ m/sec
(f) $t = 4$ sec

2.20 (a) $\bar{a} = 57$ m/sec^2
(b) $s = 710$ m
(c) $\bar{a} = -190$ m/sec^2

2.21 proof

2.22 proof

2.23 (a) $a = 4.8$ welfs/surg^2
(b) $a = 11$ m/sec^2

Solutions to Chapter 2 Study Guide

2.1 List the steps by which Galileo progressed from his first definition of uniformly accelerated motion to his final confirmation that this definition is useful in describing the motion of a freely falling body. Identify each step as a hypothesis, deduction, observation, or computation, etc. What limitations and idealizations appear in the argument?

The steps in Galileo's investigation may be identified as follows:

a) Definition: Define uniform acceleration as constant increase in velocity with time.

b) Hypothesis: Freely falling bodies are uniformly accelerated, as defined above.

c) Deduction: $\frac{v}{t} = a = \text{constant}$ for bodies falling from rest.

d) Deduction: $\frac{d}{t^2} = \frac{a}{2} = \text{constant}$ for bodies falling from rest.

e) Deduction: $\frac{d}{t^2} = \frac{a}{2} = \text{constant}$ for balls rolling down an inclined plane.

f) Observation: (e) is verified by experiment.

g) Conclusion: (b) is verified by the above process.

The argument was limited by Galileo's ability to measure time intervals accurately and by his idealization that rolling motion was a slowed-down falling motion, thus ignoring the rotational motion of the ball about its center.

2.2 Which of the following statements best summarizes the work of Galileo on free fall? (Be prepared to defend your choice.) Galileo:

a) proved that all objects fall at exactly the same speed regardless of their weight.

b) proved that for any one freely falling object, the ratio: d/t^2 is constant for any distance.

c) demonstrated conclusively that an object rolling down a smooth incline accelerates in the same way as (although more slowly than) the same object falling freely.

d) used logic and experimentation to verify indirectly his assertion that the speed of a freely falling object at any pointdepends only upon, and is proportional to, the time elapsed.

e) made it clear that until a vacuum could be produced, it would not be possible to settle the free-fall question once and for all.

Answer (d) is the best. Galileo did not prove (a). He concluded it, using as a basis his experimental results on rolling balls. (b) is incorrect because Galileo did not prove this for a freely falling object. (c) Galileo's connection between rolling and freely falling bodies was logical rather than empirical. Although Galileo suspected that air resistance played a part in his experiments, it had a relatively small effect and the problem could still be attacked with mathematics, logic, and experimentation.

2.3 Write a short statement (not more than two or three sentences) summarizing Galileo's work on free fall better than any of those in 2.2 above.

By assuming that the speed of a body increases uniformly with time, Galileo was able to show mathematically that d/t^2 should be constant. He found experimentally that this was the case for a rolling ball. Since d/t^2 continued to be aconstant for steeper angles, Galileo concluded that it would also be constant in the case of vertical free fall.

2.4 As Director of Research in your class, you receive the following research proposals from physics students wishing to improve upon Galileo's free-fall experiment. Would you recommend support for any of them? If you reject a proposal, you should make it clear why you do so.

a) Historians believe that Galileo never dropped objects from the Leaning Tower of Pisa. Too bad! Such an experiment is more direct and more fun than inclined plane experiments, and of course, now that accurate stopwatches are available, it can be carried out much better than in Galileo's time. The experiment involves dropping, one by one, different size spheres made of copper, steel, and glass from the top of the Leaning Tower and finding how long it takes each one to reach the ground. Knowing d (the height of the tower) and time of fall t, I will substitute in the equation $d = \frac{1}{2}at^2$ to see if the acceleration a has the same value for each sphere.

b) A shotput will be dropped from the roof of a 4-story building. As the shotput falls, it passes a window at each story. At each window there will be a student who starts his stopwatch upon hearing a signal that the shot has been released, and stops the watch as the shot passes his window. Also, each student records the speed of the shot. From his own data, each student will compute the ratio v/t. All four students should obtain the same numerical value of the ratio.

c) Galileo's inclined planes "dilute" motion all right, but the trouble is that there is no reason to suppose that a ball rolling down a board is behaving like a ball falling straight downward. A better way to accomplish this is to use light, fluffy, cotton balls. These will not drop as rapidly as metal spheres, and therefore it would be possible to measure the time of the fall t for different distances d. The ratio d/t^2 could be determined for different distances to see if it remained constant. The compactness of the cotton ball could then be changed to see if a different value was obtained for the ratio.

----

a) The proposal is to determine whether different masses fall with the same acceleration. This experiment involves a direct measurement of the ratio d/t^2. However, variations in t for the same event will probably be relatively large. A "null method" would be more sensitive by revealing which of two bodies mass reaches the ground at very nearly the same time.

b) This project looks interesting and will allow us to determine whether d/t^2 is constant for different values of d. However, the proposal states that each student will obtain instantaneous speed of the ball as it passes his window. The procedure is not outlined and may prove difficult or impossible to do.

c) The effect of air resistance on the cotton balls is going to be appreciable. If this is the case, we cannot expect the balls to have a constant acceleration. The experiment might provide interesting information, but it would not be pertinent to Galileo's problem. However, a student might justify accepting the proposal on the basis that the effect of air resistance might not be known until someone had performed this experiment.

2.5 Consider Aristotle's statement "A given weight moves [falls] a given distance in a given time; a weight which is as great and more moves the same distance in less time, the times being in inverse proportion to the weights. For instance, if one weight is twice another, it will take half as long over a given movement." [De Caelo]

Indicate how Simplicio and Salviati would probably have interpreted each of the following:

a) A two-pound rock falls from a cliff and, while dropping, breaks into two equal pieces.

b) A hundred-pound rock is dropped at the same time as one hundred one-pound pieces of the same type of rock.

c) A hundred one-pound pieces of rock, falling from a height, drop into a loosely held sack which pulls loose and falls. All the rocks are in the sack and continue falling while contained by the sack.

----

a) Simp.: Both pieces slow down to 1/2 speed and fall together, taking twice the time that the 2 lb. rock would have taken to fall the remaining distance.

Salv: Both pieces continue to fall at same rate as before fracture and strike the ground at the same time as the 2 lb. rock would have.

b) Simp: The single rock would reach bottom in 1/100 the time as the 100 one-pound pieces.

Salv: They would fall at the same rate.

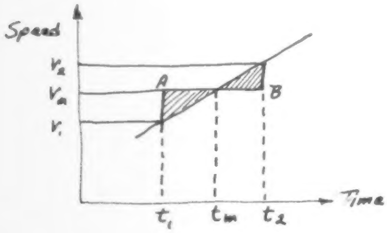
c) Simp: The sack containing the 100 rocks would speed up to fall the remaining distance in 1/100 the time required by uncaptured rocks.

Salv: The sack would reach the bottom in the same time as that taken by the separate rocks.

2.6 A good deal of work preceded that of Galileo on the topic of motion. In the period (1280-1340) mathematicians at Merton College, Oxford, carefully considered quantities that change with the passage of time. One result that had profound influence was a general theorem known as the "Merton Theorem" or "Mean Speed Rule."

This theorem might be restated in our language and applied to uniform acceleration as follows: the distance an object goes during some time while its speed is changing uniformly is the same distance it would go if it went at the average speed the whole time.

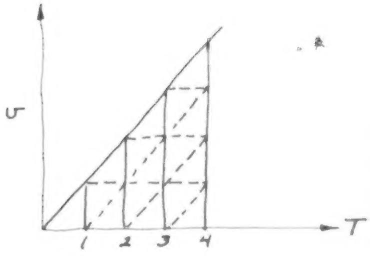
Using a graph, and techniques of algebra and geometry, construct a proof of the "Merton Rule."



A quick geometrical proof results from our knowledge that distance is measured by the area under the v-t curve. Since $v_1 - v_m = v_m - v_2$ and $t_1 - t_m = t_m - t_2$, the shaded areas of the graph are equal in area and the upper can be inserted in place of the lower. This forms a rectangle A B T_2 T_1 of area = area of the original trapezoid. This rectangle is of height v_m, equivalent to saying $\Delta s = v_m \Delta t$, our definition of average velocity.

2.7 In the Two New Sciences Galileo states, "...for so far as I know, no one has yet pointed out that the distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity (namely 1:3:5:7...)...."

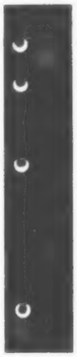
The area beneath a speed-time graph represents the distance traveled during some time interval. Using that idea, give a proof that the distances an object falls in successive equal time intervals will be in such a ratio.



Students may see that each successive trapezoid may be broken up into a number of congruent triangles which may be then added up as 1:3:5:7. They may also notice that the total area is in the ratio 1:4:9:16, or is proportional to T^2.

This problem may also be done by computing the area of each trapezoid as $\frac{B(h_1 + h_2)}{2}$ using arbitrary units, but the purely geometric approach seems preferable.

2.8 Indicate whether the following statements are true or false when applied to the strobe photo at the right:



a) The speed of the ball is greater at the bottom than at the top.

b) The direction of the acceleration is vertically downward.

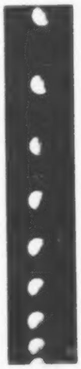
c) This could be a freely falling object.

d) This could be a ball thrown straight upward.

----

a) true
b) true
c) false
c) true (if air resistance is present)

2.9 Apply the same statements to the photo at the right, once again indicating whether each statement is true or false.



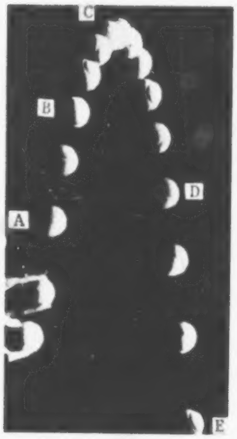
----

a) true
b) true
c) true
d) true

2.10 These last two questions raise the issue of direction. The photograph in the figure below is of a ball thrown upward, yet its acceleration is downward. The acceleration due to gravity may appear as the slowing down of an upward moving object, or as the speeding up of a downward moving one. To keep these matters straight, a plus and minus sign convention is adopted. Such a convention is merely an arbitrary but consistent set of rules.

The main rule we adopt is: up is the positive direction. It follows that the acceleration due to free fall g always takes the negative sign; distances above the point of release are positive, those below it negative; and the speed of an object moving upward is positive, downward negative.

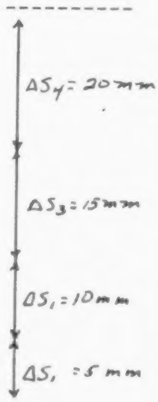
The figure below is a photo of the path that a ball might take if you threw it up and then let it fall to the ground rather than catching it when it reached your hand again. To assure yourself that you understand the sign convention stated above, complete the table below.



Position	s	v	a
A	+	+	-
B	+	+	-
C	+	0	-
D	+	-	-
E	-	-	-

If there are any or no arguments about -s in part E, it will provide a good oportunity to discuss frames of reference again.

2.11 Draw a set of points (as in a strobe photo) to show the successive positions of an object that had a positive acceleration upward. Can you think of any way to produce such an event physically?

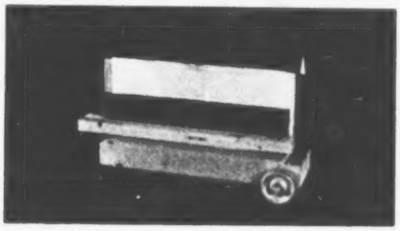


The spacing between successive points, Δs, must differ by a constant amount for constant acceleration.

$$\Delta s_4 - \Delta s_3 = \Delta s_3 - \Delta s_2 = \Delta s_2 - \Delta s_1$$

This might be accomplished by pulling the object upwards with a constant pull.

2.12 The instrument shown is called a liquid surface accelerometer. Whenever the accelerometer experiences an acceleration in a direction parallel to its long dimension, the surface of the liquid does not remain level. If your laboratory has one of these instruments and an air track, design an experiment in which acceleration remains constant but speed changes.

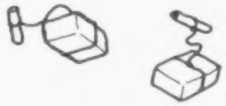


Such an experiment might consist of attaching a weight over a pulley so that it pulls on the a cart carrying the accelerometer. Photographs can be taken and the slope of the surface compared for different times during a "run", and for runs with different weights attached.

2.13 Drop sheets of paper with various degrees of "crumpling." Can you crumple a sheet of paper tight enough that it will fall at the same rate as a tennis ball?

If dropped from over your head, you will probably be able to distinguish between times of arrival at the floor of a tennis ball and a tightly crumpled ball of newspaper having the same size.

2.14 Tie two objects (of greatly different weight) together with a piece of string. Drop the combination with different orientations of objects. Watch the string. In a few sentences summarize your results.



If the objects are of comparable density, they will fall with the same acceleration and the string will hang limp between them. It will be similar to the limp umbilical cord which attached Astronaut White to his space capsule for the same reason: again, both have the same acceleration. In an extended situation where there is appreciable air resistance, an object with a greater cross sectional area per unit mass will be retarded more, accelerate more slowly, and cause the string to become taut.

2.15 In these first two chapters we have been concerned with motion in a straight line. We have dealt with distance, time, speed and acceleration, and with the relationships between them. Surprisingly, most of the results of our discussion can be summarized in the three equations listed below.

$$v_{av} = \frac{\Delta d}{\Delta t} \qquad a_{av} = \frac{\Delta v}{\Delta t} \qquad d = \tfrac{1}{2}at^2$$

The last of these equations applies only to those cases where the acceleration is constant. Because these three equations are so useful, they are worth remembering.

a) State each of the three equations in words.

b) Which of the equations can be applied only to objects starting from rest?

c) Make up a simple problem to demonstrate the use of each equation. For example: How long will it take a jet plane to travel 3200 miles if it averages 400 mi/hr? Also work out the solution just to be sure the problem can be solved.

----------

a) The average speed is equal to the distance interval traveled divided by the time interval over which the distance was measured.

The average acceleration is equal to a change in velocity divided by the time interval over which the velocity change was measured.

The distance traveled by a uniformly accelerating object is equal to one-half the acceleration measured from the start multiplied by the square of the time since the start.

b) $s = \tfrac{1}{2}at^2$ must be used only with objects starting from rest.

c) 8 hours

2.16 Memorizing equations will not save you from having to think your way through a problem. You must decide if, when and how to use equations. This means analyzing the problem to make certain you understand what information is given and what is to be found. Test yourself on the following problem. Assume that the acceleration due to gravity is 10 m/sec^2.

Problem: A stone is dropped from rest from the top of a high cliff.

a) How far has it fallen after 1 second?

b) What is the stone's speed after 1 second of fall?

c) How far does the stone fall during the second second? (That is, from the end of the first second to the end of the second second.)

----------

a) $s = \tfrac{1}{2}at^2$

$$a = -10\ \frac{\text{m}}{\text{sec}^2}$$

$$t = 1.0\ \text{sec}$$

$$s = \tfrac{1}{2} \times -10\ \frac{\text{m}}{\text{sec}^2}\ (1.0\ \text{sec})^2$$

$$\underline{s = -5.0\ \text{m}}$$

b) $v = at$

$$a = -10\ \frac{\text{m}}{\text{sec}^2}$$

$$t = 1.0\ \text{sec}$$

$$v = -10\ \frac{\text{m}}{\text{sec}^2} \times 1.0\ \text{sec}$$

$$\underline{v = -10\ \frac{\text{m}}{\text{sec}}}$$

c) $s = \tfrac{1}{2}at_2{}^2 - \tfrac{1}{2}at_1{}^2$

$$s = \tfrac{1}{2}a(t_2{}^2 - t_1{}^2)$$

$$a = -10\ \frac{\text{m}}{\text{sec}^2}$$

$$t_2 = 2.0\ \text{sec}$$

$$t_1 = 1.0\ \text{sec}$$

$$s = \tfrac{1}{2} \times -10\ \frac{\text{m}}{\text{sec}^2}\ (4.0\ \text{sec}^2 - 1.0\ \text{sec}^2)$$

$$\underline{s = -15\ \text{m}}$$

2.17 Think you have it now? Test yourself once more. If you have no trouble with this, you may wish to try problem 2.18, 2.19, or 2.20.

Problem: An object is thrown straight upward with an initial velocity of 20 m/sec.

a) What is its speed after 1.0 sec?

b) How far did it go in this first second?

c) How long did the object take to reach its maximum height?

d) How high is this maximum height?

e) What is its final speed just before impact?

----

a) $v_f = v_i + at$

$$v_i = 20\ \frac{m}{sec}$$

$$a = -10\ \frac{m}{sec^2}$$

$$t = 1.0\ sec$$

$$v = 20\ \frac{m}{sec} - 10\ \frac{m}{sec^2} \times 1\ sec$$

$$\underline{v = 10\ \frac{m}{sec}}$$

b) $s = \bar{v}t$

$$\bar{v} = \frac{v_i + v_f}{2}$$

$$s = \left(\frac{v_i + v_f}{2}\right) \times t$$

$$v_i = 20\ \frac{m}{sec}$$

$$v_f = 10\ \frac{m}{sec}$$

$$t = 1\ sec$$

$$s = \left(\frac{20\ \frac{m}{sec} + 10\ \frac{m}{sec}}{2}\right) \times 1\ sec$$

$$\underline{s = 15\ m}$$

c) $v_f = v_i + at$

$$at = v_f - v_i$$

$$t = \frac{v_f - v_i}{a}$$

$$v_f = 0\ \frac{m}{sec}$$

$$v_i = 20\ \frac{m}{sec}$$

$$a = -10\ \frac{m}{sec^2}$$

$$t = \frac{-20\ \frac{m}{sec}}{-10\ \frac{m}{sec^2}}$$

$$\underline{t = 2\ sec}$$

d) $s = \bar{v}t$

$$s = \left(\frac{v_f + v_i}{2}\right)t$$

$$v_f = 0\ \frac{m}{sec}$$

$$v_i = 20\ \frac{m}{sec}$$

$$t = 2\ sec$$

$$s = \left(\frac{0\ \frac{m}{sec} + 20\ \frac{m}{sec}}{2}\right) \times 2\ sec$$

$$\underline{s = 20\ m}$$

e) Since the object falls the same distance it rose and undergoes the same acceleration, we can immediately say that the downward trip will be similar to the upward one. Then it will take the same time and Δv will be the same. We can also show mathematically:

$$s = \tfrac{1}{2}at^2$$

The time to fall a distance s is

$$t = \left(\frac{2s}{a}\right)^{\frac{1}{2}}$$

The velocity to which the object will accelerate in this time is

$$v = at = a\left(\frac{2s}{a}\right)^{\frac{1}{2}} = (2as)^{\frac{1}{2}}$$

$$a = -10\ \frac{m}{sec^2}$$

$s = -20$ m (s is negative because the object is going in a negative direction)

$$v = (2 \times -10\ \frac{m}{sec^2} \times -20\ m)^{\frac{1}{2}}$$

$$v = \pm\ 20\ \frac{m}{sec}$$

$$\underline{v = -20\ \frac{m}{sec}}$$

(The negative root is the one with meaning in this situation.)

2.18 A batter hits a pop fly that travels straight upwards. The ball leaves his bat with an initial speed of 40 m/sec.

a) What is the speed of the ball at the end of 2 seconds?

b) What is its speed at the end of 6 seconds?

c) When does the ball reach its highest point?

d) How high is this highest point?

e) What is the speed of the ball at the end of 10 seconds? (Graph this series of speeds.)

f) What is its speed when caught by the catcher?

----

a) $v_f = v_i + at$

$$v_i = 40\ \frac{m}{sec}$$

$$a = -10\ \frac{m}{sec^2}$$

$$t = 2\ sec$$

$$v_f = 40\ \frac{m}{sec} - 10\ \frac{m}{sec^2} \times 2\ sec$$

$$\underline{v_f = -20\ \frac{m}{sec}}$$

b) $v_f = v_i + at$

$$t = 6\ sec$$

$$v_f = 40\ \frac{m}{sec} - 10\ \frac{m}{sec^2} \times 6\ sec$$

$$\underline{v_f = -20\ \frac{m}{sec}}$$

c) The ball reaches its highest point when $v_f = 0$.

$$v_f = v_i + at$$

$$t = \frac{v_f - v_i}{a}$$

$$v_f = 0\ \frac{m}{sec}$$

$$v_i = 40\ \frac{m}{sec}$$

$$a = -10\ \frac{m}{sec^2}$$

$$t = \frac{0\ \frac{m}{sec} - 40\ \frac{m}{sec}}{-10\ \frac{m}{sec^2}}$$

$$\underline{t = 4\ sec}$$

d) From problem (c) the highest point occurs at 4 sec. Although the problem is most rigorously solved by using the relationship

$$s = v_i t + \tfrac{1}{2}at^2,$$

it is more easily approached by realizing that the ball will travel the same distance in going straight up for four seconds, while it goes from $40\ \frac{m}{sec}$ to $0\ \frac{m}{sec}$ as it will in falling down for four seconds from the top of its rise. This means

$s = \tfrac{1}{2}at^2$

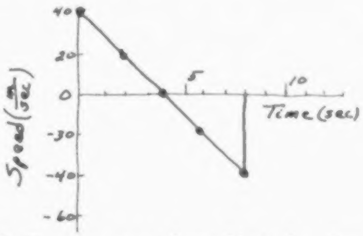
$$a = -10\ \frac{m}{sec^2}$$

$$t = 4\ sec$$

$$s = \tfrac{1}{2} \times -10\ \frac{m}{sec^2} \times (4\ sec)^2$$

$$\underline{s = -80\ m}$$

e) The speed is zero, since by symmetry the ball must reach the ground (and hence come to a stop) in just 8 sec.



f) We can say immediately from the symmetry of the problem that the speed will be the same magnitude when it gets back to the ground as it was when it left. This may be proved in the following manner:

$$v_f = v_i + at$$

The time t is the time it takes the distance, s, to return to zero.

$$s = v_i t + \tfrac{1}{2}at^2$$

letting s = 0 and solving for t,

$$t(v_i + \tfrac{1}{2}at) = 0$$

$$v_i + \tfrac{1}{2}at = 0$$

$$t = \frac{-2v_i}{a}$$

(The root t = 0 describes the initial conditions)

Substituting

$$v_f = v_i + a\left(\frac{-2v_i}{a}\right) = -v_i$$

Since $v_i = 40\ \frac{m}{sec}$,

$$\underline{v_f = -40\ \frac{m}{sec}}$$

2.19 A ball starts up an inclined plane with a speed of 4 m/sec, and comes to a half after 2 seconds.

a) What acceleration does the ball experience?

b) What is the average speed of the ball during this interval?

c) What is the ball's speed after 1 second?

d) How far up the slope will the ball travel?

e) What will be the speed of the ball 3 seconds after starting up the slope?

f) What is the total time for a round trip to the top and back to the start?

----

a) $a = \frac{\Delta v}{\Delta t}$

$$\Delta v = v_f - v_i = 0\ \frac{m}{sec} - 4\ \frac{m}{sec}$$

$$\Delta v = -4\ \frac{m}{sec}$$

$$\Delta t = 2\ sec$$

$$a = \frac{-4\ \frac{m}{sec}}{2\ sec}$$

$$\underline{a = -2\ \frac{m}{sec^2}}$$

b) $\bar{v} = \frac{v_i + v_f}{2}$

$$\bar{v} = \frac{4\ \frac{m}{sec} + 0\ \frac{m}{sec}}{2}$$

$$\underline{\bar{v} = 2\ \frac{m}{sec}}$$

c) $v_f = v_i + at$

$$v_i = 4\ \frac{m}{sec}$$

$$a = -2\ \frac{m}{sec^2}$$

$$v_f = 4\ \frac{m}{sec} - 2\ \frac{m}{sec} \times 1\ sec$$

$$\underline{v_f = 2\ \frac{m}{sec}}$$

d) $s = v_i t + \frac{1}{2}at^2$

$$v_i = 4\ \frac{m}{sec}$$

$$a = -2\ \frac{m}{sec^2}$$

$$t = 2\ sec$$

$$s = 4\ \frac{m}{sec} \times 2\ sec - \frac{2}{2}\ \frac{m}{sec^2}\ (2\ sec)$$

$$\underline{s = 4\ m}$$

Note: This could also be calculated by using average velocity and time.

e) $v_f = v_i + at$

$$t = 3\ sec$$

$$v_f = 4\ \frac{m}{sec} - 2\ \frac{m}{sec} \times 3\ sec$$

$$\underline{v_f = -2\ \frac{m}{sec}}$$

f) The total time for a round trip up the slope is twice the time for a one way trip. (See problem 2.18 (f) for the proof.)

$$t_2 = 2t_1 = 2 \times 2\ sec$$

$$\underline{t_2 = 4\ sec}$$

2.20 Lt. Col. John L. Stapp achieved a speed of 632 mph (284 m/sec) in an experimental rocket sled at the Holloman Air Base Development Center, Alamogordo, New Mexico, on March 19, 1954. Running on rails and propelled by nine rockets, the sled reached its top speed within 5 seconds. Stapp survived a maximum acceleration of 22 g's in slowing to rest during a time interval of 1½ seconds.

a) Find the average acceleration in reaching maximum speed.

b) How far did the sled travel before attaining maximum speed?

c) Find the average acceleration while stopping.

----

a) $\bar{a} = \frac{\Delta v}{\Delta t}$

$$\Delta v = 284\ \frac{m}{sec}$$

$$\Delta t = 5.0\ sec$$

$$\bar{a} = \frac{284\ \frac{m}{sec}}{5\ sec}$$

$$\underline{\bar{a} = 57\ \frac{m}{sec^2}}$$

b) $s = \frac{1}{2}at^2$

$$a = 57\ \frac{\text{m}}{\text{sec}^2}$$

$$t = 5.0\ \text{sec}$$

$$s = \frac{1}{2} \times 57\ \frac{\text{m}}{\text{sec}^2}\ (5.0\ \text{sec})^2$$

$$\underline{s = 710\ \text{m}}$$

c) $\bar{a} = \frac{\Delta v}{\Delta t}$

$$\Delta v = v_f - v_i = 0\ \frac{\text{m}}{\text{sec}} - 284\ \frac{\text{m}}{\text{sec}}$$

$$\Delta t = 1.5\ \text{sec}$$

$$\bar{a} = \frac{284\ \frac{\text{m}}{\text{sec}}}{1.5\ \text{sec}}$$

$$\underline{\bar{a} = -190\ \frac{\text{m}}{\text{sec}^2}}$$

Since 10 m/sec is approximately the acceleration of gravity, this means Col. Stapp was subject to an average acceleration 19 times gravitational acceleration. (The fact that he was subjected to 22 g's indicates that the actual acceleration was not constant.)

2.21 Sometimes it is helpful to have a special equation relating certain variables. For example, initial and final speed, distance, and acceleration are related by the equation

$$v_f^2 = v_i^2 + 2ad.$$

Try to derive this equation from some others you are familiar with.

We know that

$$s = \bar{v}t \text{ or}$$

$$s = \left(\frac{v_f + v_i}{2}\right) \times t.$$

We also know that

$$v_f = v_i + at \text{ or}$$

$$t = \frac{v_f - v_i}{a}.$$

Substitute the expression for t into the equation (1),

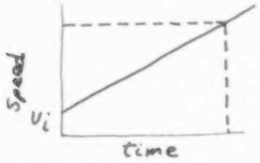
$$s = \left(\frac{v_f + v_i}{2}\right)\left(\frac{v_f - v_i}{a}\right) = \frac{v_f^2 - v_i^2}{2a}$$

$$v_f^2 - v_i^2 = 2as$$

$$v_f^2 = v_i^2 + 2as$$

2.22 Use the graph below, and the idea that the area under a curve in a speed-time graph gives a value for the distance traveled, to derive the equation

$$d = v_i t + \frac{1}{2}at^2$$



$$\text{area} = s = \frac{1}{2}(b_1 + b_2)h$$

$$b_1 = v_i$$

$$b_2 = v_f = v_i + at$$

$$h = t$$

$$s = \frac{1}{2}(v_i + v_i + at)t$$

$$s = \frac{1}{2} \times 2v_i t + \frac{1}{2}at^2$$

$$s = v_i t + \frac{1}{2}at^2$$

2.23 A student on the planet Arret in another solar system dropped an object in order to determine the acceleration due to gravity. The following data are recorded (in local units):

Time (in surgs)	Distance (in welfs)
0.0	0.00
0.5	0.54
1.0	2.15
1.5	4.84
2.0	8.60
2.2	10.41
2.4	12.39
2.6	14.54
2.8	16.86
3.0	19.33

a) What is the acceleration due to gravity on the planet Arret, expressed in welfs/surg²?

b) A visitor from Earth finds that one welf is equal to about 6.33 cm and that one surg is equivalent to 0.167 sec. What would this tell us about Arret?

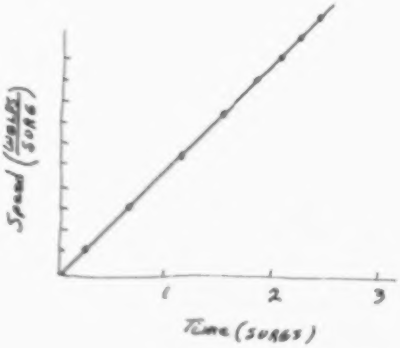
----

a) We can find the acceleration due to gravity on Arret by constructing a graph of speed vs time for the freely falling body and finding its slope. To do this we could first construct a distance vs time graph and measure the slope of the curve at various points, or we can approximate the velocity at different points by finding the average velocity between two points on either side of the desired points. This can be done with a table. Note that the average velocity between points A and B approximates the instantaneous speed at the time

$$\frac{A + B}{2} = 0.25 \text{ sec.}$$

Also note that the time interval in the given data changes from 0.5 sec to 0.2 sec.

Position	Time (surgs)	s (welfs)	Interval	Δs	$\bar{v}$ $\frac{\text{welfs}}{\text{surg}}$
A	0.0	0.00			
			AB	0.54	1.0
B	0.5	0.54			
			BC	1.61	3.2
C	1.0	2.15			
			CD	2.69	5.4
D	1.5	4.84			
			DE	3.76	7.5
E	2.0	8.60			
			EF	1.81	9.0
F	2.2	10.41			
			FG	1.98	10.0
G	2.4	12.39			
			GH	2.15	11.0
H	2.6	14.54			
			HI	2.32	12.0
I	2.8	16.86			
			IJ	2.47	12.0
J	3.0	19.33			



$$a = \frac{\Delta v}{\Delta t} = \frac{12\ \frac{\text{welfs}}{\text{surg}}}{2.8\ \text{surg}}$$

$$a = 4.3\ \frac{\text{welfs}}{\text{surg}^2}$$

b) 1 welf = 0.0633 m

1 surg = 0.167 sec

$$1\ \frac{\text{welf}}{\text{surg}^2} = \frac{1\ \text{welf}\ (0.0633\ \frac{\text{m}}{\text{welf}}}{1\ \text{surg}^2\ (0.167\ \frac{\text{sec}}{\text{surf}}}{}^{2} =$$

$$2.27\ \frac{\text{m}}{\text{sec}^2}$$

$$4.3\ \frac{\text{welf}}{\text{surg}^2} \times 2.27\ \frac{\frac{\text{m}}{\text{sec}^2}}{\frac{\text{welf}}{\text{surg}^2}} = 9.8\ \frac{\text{m}}{\text{sec}^2}$$

The acceleration due to gravity on Arret is 9.8 m/sec^2, slightly greater than on earth.

Brief Answers to Chapter 3 Study Guide

3.2 discussion

3.3 (a) yes
(b) 4.4 units West

3.4 (a) discussion
(b) discussion

3.5 discussion

3.6 discussion

3.7 discussion

3.8 6/1

3.9 discussion

3.10 discussion

3.11 table

3.12 discussion

3.13 $m = 2.0$ kg

3.14 (a) $a = 2.0 \times 10^2$ m/sec^2
$v_{max} = 7.8 \times 10^2$ m/sec
(b) a varied
(c) $\bar{a} = 2.4 \times 10^2$ m/sec^2

3.15 discussion

3.16 (a) $F = 860$ N
(b) $F = 750$ N
(c) $F = 640$ N
(d) The same values as (a), (b), and (c).
(e) discussion

3.17 (a) the same mass
(b) F(Paris) = 9.809 N
(c) F(Washington) = 9.801 N

3.18 (a) diagram
(b) 1.7×10^{-24} m/sec^2
(c) $a_b/a_c = 6 \times 10^{24}$

3.19 discussion

Solutions to Chapter 3 Study Guide

3.2 The Aristotelian explanation of motion should not be dismissed lightly. Great intellects of the Renaissance period such as Leonardo da Vinci, who, among other things, designed artillery for launching projectiles, apparently did not callenge the Aristotelian explanation. One reason for the longevity of Aristotle's ideas is that they are so closely aligned with our common-sense ideas. In what ways do your common-sense notions of motion agree with Aristotle's?

The four elements fire, air, water, earth seem to seek their natural places. The heavier body the more earthlike should naturally fall faster than a lighter one. All bodies slow down and stop in time, why should one agree with Newton's first law of motion.

3.3 Three children, Karen, Keith and Sarah are each pulling on the same toy.

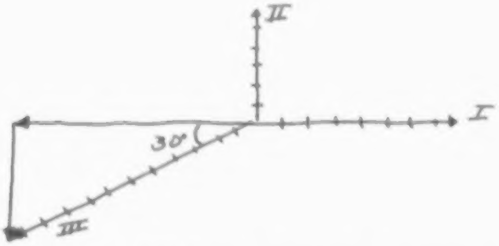
Karen pulls toward the east with a force of 8 units.

Sarah pulls toward the north with a force of 6 units.

Keith pulls in a direction 30° south of west with a force of 12 units.

a) Is there a net (i.e., unbalanced) force on the toy?

b) If there is a net force, what is its magnitude and direction?

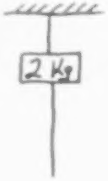


Vector III has two components, one west and one south (in red), of magnitude 12 cos 30 = 6√3 and 6 respectively.

There is no unbalanced force in the North-South direction. In the East-West direction there is a force 6√3 - 6 units West.

$$6\sqrt{3} - 6 = 6(\sqrt{3} - 1) = 6(.732) = 4.392 \text{ units West}$$

3.4 A 2 kg mass is suspended by a string. A second string is tied to the bottom of the mass.



a) If the bottom string is pulled with a sudden jerk, the bottom string breaks.

b) If the bottom string is pulled with a steady pull, the top string breaks.

Explain.

(Try this experiment for yourself. You might tie thread to a brick, or a hammer, or a pipe wrench.)

----

a) The sudden jerk is enough to break the string, but before the string above the mass can break the mass must descend enough to stretch this string to its breaking point. Although the force of the jerk accelerates the mass downward it does last long enough for the mass to move the critical distance. This is a problem in dynamics.

b) A steady pull produces no accelerations. The stress on the string above the mass equals this pull plus the weight of the mass. Therefore it breaks first. This is a problem in statics.

3.5 In terms of Newton's first law, explain:

(a) why people in a moving car lurch forward when the car suddenly decelerates;

(b) what happens to the passengers of a car that makes a sharp and quick turn;

(c) why, when a coin is put on a phonograph turntable and the motor is started, the coin flies off when the turntable reaches a certain speed. Why doesn't it fly off before?

(a) The brakes slow the car but not the passenger, since he is not rigidly attached to the car. His inertia causes his forward motion to continue unchanged momentarily while that of the car is reduced.

(b) The explanation is similar to that in part (a). Velocity is a vector quantity. When the force of the tires against the road changes the direction of the car, it fails to change immediately the direction of the passenger. He continues in the original direction until the force of the seat on his body changes his direction of motion.

(c) The centripetal force needed to hold the coin in "orbit" increases as the rotation rate of the turntable increases. The frictional force which links the coin to the turntable remains constant. When the frequency of the turntable rotation has increased to the point where the centripetal force equals the frictional force, a further increase in the centripetal force required to keep the coin in its "orbit" cannot be provided by friction. The coin will then slip toward the rim of the turntable.

3.6 (a) You exert a force on a box, but it does not move. How would you explain this? How might an Aristotelian explain it?

(b) Suppose now that you exert a greater force, and the box moves. Explain this from your point of view and from an Aristotelian point of view.

(a) We would say that the force is not large enough to overcome the force of starting friction. An Aristotelian would say that rest is a natural state and that any force of motion requires a force. He would probably say that the force is not enough to change the box from its natural state.

(b) We would say that the applied force is now greater than the force of friction, resulting in an unbalanced force. Consequently, the box accelerates according to Newton's Second Law. An Aristotelian probably would have maintained that the force producing the motion was now great enough to displace the box from its natural state. However, the Aristotelians had no clear concept of acceleration.

3.7 Assume that the floor of a laboratory could be made perfectly horizontal and perfectly smooth. A block of wood is placed on the floor and given a small push. Predict the way in which the block will move. How would this motion differ if the whole laboratory were moving with constant velocity during the experiment. How would it differ it the whole laboratory were accelerating along a straight line? If the block were seen to move in a curved path along the floor, how would you explain this?

If there is no friction, a block given a brief push will move with uniform velocity. If the laboratory is itself in uniform motion, the block will still move with a constant velocity $\vec{V}_B$ (relative to the laboratory). If the laboratory is moving relative to the earth with a constant velocity $\vec{V}_L$, the block's motion relative to the earth, $\vec{V}_R$, is the vector sum $(\vec{V}_B + \vec{V}_L)$. Since neither of these vectors change, their sum is also constant.

3.8 A body is being accelerated by an unbalanced force. If the magnitude of the net force is doubled and the mass of the body is reduced to one-third of the original value, what will be the ratio of the second acceleration to the first?

$$F = ma$$

$$a = \frac{F}{m}$$

$$\frac{a_2}{a_1} = \frac{\frac{F_2}{m_2}}{\frac{F_1}{m_1}} = \frac{F_2 \times m_1}{m_2 \times F_1}$$

$$\frac{a_2}{a_1} = \frac{(2) \times (1)}{(\frac{1}{3}) \times (1)}$$

$$\frac{a_2}{a_1} = \frac{6}{1}$$

3.9 Hooke's Law says that the force exerted by a stretched or compressed spring is directly proportional to the amount of the compression or extension. As Robert Hooke put it in announcing his discovery:

> ...the power of any spring is in the same proportion with the tension thereof: that is, if one power stretch or bend it one space, two will bend it two, three will bend it three, and so forward. Now as the theory is very short, so the way of trying it is very easie.

If Hooke says it's "easie," then it might well be so. You can probably think immediately of how to test this law using springs and weights. Try designing such an experiment; then after checking with your teacher, carry it out.

Hooke's experiment is described in his own words in W. F. Magie, A Source Book in Physics, McGraw-Hill, 1935.

A simple experiment could be set up by taking a spring, hanging different masses from it, and noting the extension of the spring for each mass. Since the force of gravity on each weight can be calculated, we can plot force vs. extension. If Hooke is correct, the points will lie along a straight line.

Modern usage requires the substitution of the word force where Hooke used power. Today, power has a different meaning.

3.10 If you have dynamics carts available, here is one way of doing an experiment to test the inverse proportionality between acceleration and mass:

(a) Add load blocks to one or the other of two carts until the carts balance when placed on opposite platforms of a laboratory balance. Balance a third cart with one of the first pair. Each cart now has mass m. (State two main assumptions involved here.)

(a) Accelerate one cart on a level surface using the rubber-band technique; that is, pull the car with the rubber band keeping it stretched a constant amount. Any other method can also be used that will assure you that, within reason, the same force is being applied each time. Record the position of the cart at equal time intervals by means of stroboscopic photography.

(c) Repeat the last step in all details, but use two carts hooked together. Repeat again using all three carts hooked together. In all three cases it is crucial that the applied force be essentially the same.

(d) Determine the value of acceleration for masses of m (1 cart), 2m (2 carts), and 3m (3 carts).

(e) Prepare a graph of $\underline{a}$ vs. $\underline{m}$, of $\underline{a}$ vs. $\frac{1}{m}$, and of $\frac{1}{a}$ vs. $\underline{m}$. Comment on your results.

This problem concerns a suggested experiment to test the relation between acceleration and mass.

Two of the assumptions involved are: (1) if two carts, placed on an equal-arm balance, exert the same force, than the masses of the carts are equal; (2) if one of these carts balances a third cart, then all three carts have the same mass. Another, more fundamental, assumption is hidden here. By using an equal-arm balance, we have equated the gravitational masses of the objects; from this we have assumed that their inertial masses are also equal. It would be advisable not to delve into the latter assumption unless you are prepared to spend quite a lot of time.

If the experiment is performed carefully, the graph of $\underline{a}$ vs m will be a curve, while the graphs of $\underline{a}$ vs $1/m$ and $1/a$ vs m will be straight lines. This means that $\underline{a}$ is proportional to $1/m$.

A more detailed experiment using dynamics carts pulled by rubber bands is described in PSSC labs III-2 and III-3.

3

3.11 Complete this table:

(a)	1.0 N	1.0 kg	1.0 m/sec^2
(b)	24.0 N	2.0 kg	12.0 m/sec^2
(c)	N	3.0 kg	8.0 m/sec^2
(d)	N	74.0 kg	0.2 m/sec^2
(e)	N	0.0066 kg	130.0 m/sec^2
(f)	72.0 N	kg	8.0 m/sec^2
(g)	3.6 N	kg	12.0 m/sec^2
(h)	1.3 N	kg	6.4 m/sec^2
(i)	30.0 N	10.0 kg	m/sec^2
(j)	0.5 N	0.20 kg	m/sec^2
(k)	120.0 N	48.0 kg	m/sec^2

Complete this table:

	Force	Mass	Acceleration
(a)	1.0 N	1.0 kg	1.0 m/sec^2
(b)	24.0 N	2.0 kg	12.0 m/sec^2
(c)	24.0 N	3.0 kg	8.0 m/sec^2
(d)	14.8 N	74.0 kg	0.2 m/sec^2
(e)	0.86N	0.0066 kg	130.0 m/sec^2
(f)	72.0 N	9.0 kg	8.0 m/sec^2
(g)	3.6 N	0.30 kg	12.0 m/sec^2
(h)	1.3 N	0.20 kg	6.4 m/sec^2
(i)	30.0 N	10.0 kg	3.00m/sec^2
(j)	0.5 N	0.20 kg	2.5 m/sec^2
(k)	120.0 N	48.0 kg	2.50m/sec^2

Note: The number of significant digits in the answers above are limited to the maximum present in the least of the other factors, (e.g. (d) and (g)).

3.12 Recount in detail what steps you must take (in idealized experimentation) to determine the unknown mass m (in kilograms) of a certain object if you are given nothing but a frictionless horizontal plane, a 1-kg standard, an uncalibrated spring balance, a meter stick, and a stopwatch.

To determine the unknown mass we must first calibrate the spring balance. This may be done by accelerating the 1 kg standard with a constant force indicated on the spring balance. The time to cover a measured distance from rest can be determined and the acceleration calculated:

$$s = \tfrac{1}{2}at^2, \quad a = \frac{2s}{t^2}.$$

From the known values of m and a, F can be calculated using Newton's second law, F = ma.

The unknown mass can then be accelerated with this same force and its acceleration measured. If values for F and a are substituted into F = ma, the unknown mass can be calculated.

As the same force is used each time, it is not necessary to compute the value of the force to find the mass:

$$F_1 = F_2$$

$$m_1a_1 = m_2a_2, \quad \text{and}$$

$$m_2 = \frac{m_1a_1}{a_2}$$

3.13 A certain block is dragged with constant velocity along a rough horizontal table top, by means of a spring balance horizontally attached to it which reads 0.40 N, no matter what the velocity happens to be. This means that the retarding frictional force between block and table is 0.40 N and not dependent on the speed. When the block is given a constant acceleration of 0.85 m/sec^2, the balance is found to read 2.1 N. Compute the mass of the block.

Since the balance reading is 0.40 N when the block is dragged at constant velocity, this must be the frictional force. The net force is the applied force less the frictional force.

$$F_{net} = ma$$

$$m = \frac{F_{net}}{a}$$

$$F_{net} = F_{applied} - F_{friction}$$

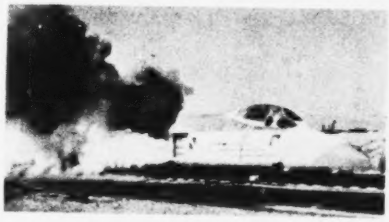
$$F_{net} = 2.1\ N - 0.40\ N = 1.7\ N$$

$$a = 0.85\ \frac{m}{sec^2}$$

$$m = \frac{1.7\ \frac{kg\ m}{sec^2}}{0.85\ \frac{m}{sec^2}}$$

$$\underline{m = 2.0\ kg}$$

3.14 A sled has a mass of 4440 kg and is propelled by a solid propellant rocket motor of 890,000 N thrust which burns for 3.9 seconds.



(a) What is this sled's average acceleration and maximum speed?

(b) The data source states that this sled has a maximum acceleration of 30 a. How can that be, considering the data given?

(c) If the sled travels a distance of 1530 m while attaining a top speed of 860 m/sec (how did it attain that high a speed?!), what is its average acceleration?

(a) $a = \frac{F}{m}$

$$F = 8.9 \times 10^5\ N$$

$$m = 4.44 \times 10^3\ kg$$

$$\bar{a} = \frac{8.9 \times 10^5\ \frac{kg\text{-}m}{sec^2}}{4.44 \times 10^3\ kg}$$

$$\underline{\bar{a} = 2.0 \times 10^2\ \frac{m}{sec^2}}$$

$$v = at$$

$$t = 3.9\ sec$$

$$v = 2.0 \times 10^2\ \frac{m}{sec^2} \times 3.9\ sec$$

$$\underline{v = 7.8 \times 10^2\ \frac{m}{sec}}$$

(b) $200\ \frac{m\to}{sec}$ is about 20 g. Since the maximum acceleration is 30 g, the acceleration varied with 20 g being the average.

(c) $v^2 = 2as$

$$a = \frac{v^2}{2s}$$

$$v = 860\ \frac{m\to}{sec}$$

$$s = 1530\ m$$

$$a = \frac{(860\ \frac{m\to}{sec})^2}{2 \times 1530\ m}$$

$$\bar{a} = 2.4 \times 10^2\ \frac{m\to}{sec^2}$$

The average acceleration and the maximum speed turn out to be higher than that obtained by using Newton's equation of motion as in (a) above. The discrepancy explained by the fact that the rocket mass is constantly decreasing, and hence it is incorrect to use the initial mass for the whole run.

A good student might like to try to calculate the mass lost during the 3.9 run.

3.15 Discuss the statement that while the mass of an object is the same everywhere, its weight may vary from place to place.

Mass is a measure of the inertia of a body. A certain mass will be accelerated at the same rate by a given force no matter where the mass is located. The mass of a body is therefore the same everywhere. Its weight, however, depends on the force of gravity, which differs from place to place. Force and mass are related by $F = ma_g$ where a_g is the acceleration of a freely falling body.

3.16 A 75 kg man stands in an elevator. What force does the floor exert on him when the elevator

(a) starts moving upward with an acceleration of 1.5 m/sec^2?

(b) When the elevator moves upward with a constant speed of 2.0 m/sec?

(c) When the elevator starts accelerating downward at 1.5 m/sec^2?

(d) If the man were standing on a bathroom (spring) scale during his ride, what readings would be scale have in parts a, b, and c?

(e) Why may one say that his apparent "weight" changes when the elevator accelerates this person?

(a) To accelerate a 75-kg man at 1.5 m/sec^2 requires an unbalanced net force which has a magnitude equal to the product of his mass and acceleration.

$$F_{net} = ma$$

$$m = 75 \text{ kg}$$

$$a = 1.5 \frac{m}{sec^2}$$

$$F_{net} = (75 \text{ kg})(1.5 \frac{m}{sec^2}) = 110 \text{ N}$$

Gravity exerts a constant downward force on the man, his weight F_w. In order that the man experience an upward acceleration, the elevator floor must exert an upward force F_E that is greater than his weight. The net force will equal the excess of F_E over F_w.

$$F_{net} = F_E - F_w$$

$$F_E = F_{net} + F_w$$

$$F_{net} = 110 \text{ N}$$

$$F_w = ma_g$$

$$= 75 \text{ kg} \times 10 \frac{m}{sec}$$

$$= 750 \text{ N}$$

$$F_E = 110 \text{ N} + 750 \text{ N}$$

$$\underline{F_E = 860 \text{ N, upward}}$$

(b) The net force on any body is zero if it moves with constant velocity. Therefore, the elevator floor must exert an upward force F_E equal in magnitude to the man's weight F_w.

$$F_E = F_w = ma_g$$

$$= 75 \text{ kg} \times 10 \text{ m/sec}^2$$

$$\underline{F = 750 \text{ N, upward}}$$

(c) When the man accelerates downward, he must experience a net force downward. In this case, the man's weight F_w must be greater than the upward force F_E exerted by the elevator floor. Again, the net force will equal their difference of F_E and F_w

$$F_{net} = F_w - F_E$$

$$F_E = F_w - F_{net}$$

$$F_{net} = 110 \text{ N (from part a)}$$

$$F_w - 750 \text{ N (from part a)}$$

$$F_L = 750 \text{ N} - 110 \text{ N}$$

$$\underline{F_E = 640 \text{ N}}$$

(d) According to Newton's third law, for every force there is an equal and opposite force. When the elevator floor exerts a certain force on the man, he will in turn exert an equal force (in the opposite direction) on the floor or scale. The bathroom scale would read the values calculated in (a), (b) and (c).

(e) As a result of the different forces in the conditions examined above, it does appear that the man's weight changes, since we are accustomed to associating weight with the force we exert against the floor (or vice versa according to Newton's third law). We should remember, however, that since we defined weight as $F_w = ma_g$, his actual weight does not change. The apparent change was due to his being in an accelerated frame of reference.

3.17 A replica of the standard kilogram is constructed in Paris and then sent to the National Bureau of Standards in Washington. Assuming that this secondary standard is not damaged in transit, what is

(a) its mass in Washington,

(b) its weight in Paris and in Washington. (In Paris, $a_g = 9.81\ \text{m/sec}^2$; in Washington, $a_g = 9.80\ \text{m/sec}^2$.)

(a) The mass will be identical in both places; 1 kg.

(b) $F = ma_g$

$$m = 1.000\ \text{kg}$$

$$a_g\,(\text{Paris}) = 9.81\ \frac{\text{m}}{\text{sec}^2}$$

$$F = 1.000\ \text{kg} \times 9.81\ \frac{\text{m}}{\text{sec}^2}$$

$$\underline{F(\text{Paris}) = 9.81\ \text{N}}$$

$$a_g\,(\text{Washington}) = 9.80\ \frac{\text{m}}{\text{sec}^2}$$

$$F = 1.000\ \text{kg}\quad 9.80\ \frac{\text{m}}{\text{sec}^2}$$

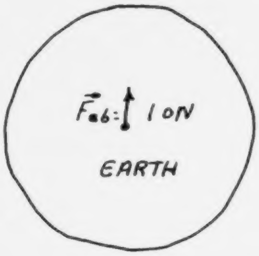
$$\underline{F(\text{Washington}) = 9.80\ \text{N}}$$

3.18 Consider the system consisting of a 1.0 kg ball and the earth. The ball is dropped from a short distance above the ground and falls freely. We can take the mass of the earth to be approximately 6.0×10^{24} kg.

(a) Make a vector diagram illustrating the important forces acting on the members of the system.

(b) Calculate the acceleration of the earth in this interaction.

(c) Find the ratio of the magnitude of the ball's acceleration to that of the earth's acceleration (a_b/a_e).



Since the force exerted on the ball is

$$F = ma_g = 1.0\ \text{kg} \quad 10\ \frac{\text{m}}{\text{sec}^2} = 10\ \text{N},$$

the force exerted on the earth by the ball is equal and opposite, according to Newton's third law.

(b) $a = \frac{F}{m}$

$$F = 10\ \text{N}$$

$$m = 6 \times 10^{24}\ \text{kg}$$

$$a = \frac{10\ \text{N}}{6 \times 10^{24}\ \text{kg}}$$

$$a = 1.7 \times 10^{-24}\ \frac{\text{m}}{\text{sec}^2}$$

(c) $a_b = 10\ \frac{\text{m}}{\text{sec}^2}$

$$a_e = 1.7 \times 10^{-24}\ \frac{\text{m}}{\text{sec}^2}$$

$$\frac{a_b}{a_e} = \frac{10\ \frac{\text{m}}{\text{sec}^2}}{1.7 \times 10^{-24}\ \frac{\text{m}}{\text{sec}^2}}$$

$$\underline{\frac{a_b}{a_e} = 6 \times 10^{24}}$$

3.19 In terms of Newton's third law assess the following statements:

(a) You are standing perfectly still on the ground; therefore you and the earth do not exert equal and opposite forces on each other.

(b) The reason that a jet airplane cannot fly above the atmosphere is that there is no air to push against, as required by the third law.

(c) The mass of object A is 100 times greater than that of object B, but even so the force it (A) exerts on B is no greater than the force of B on it.

(d) C, D, and E are three objects having equal masses; if C and D both push against E at the same time, then E exerts only one-half as much force on C as C does on E.

(a) False. According to Newton's third law, the forces are equal.

(b) False. No air is required for the jet to push against. A backwards force on the burning gases produces a force on the plane's engines in the opposite direction. However, a jet airplane does require an atmosphere to provide the necessary lift on its wings, and oxygen to combine with its fuel.

(c) True. This is an example of Newton's third law.

(d) False. The force exerted by C on E (more properly, the pair ED) is equal to the force exerted by E (actually, the pair ED) on C.

3.20 Consider a tractor pulling a heavy log in a straight line. On the basis of Newton's third law, one might argue that the log pulls back on the tractor just as strongly as the tractor pulls the log. But why, then, does the tractor move?

Think what the tractor must do to bring about its motion. As power is applied, the tracks push backward against the surface of the earth. Some loose earth may be pushed away. The locomotion of objects commonly involves pushing backward, opposite to the direction of motion. But according to the third law, if the treads of the tractor push backward on the surface of the earth, the earth must simultaneously push forward on the treads. Whether or not the tractor moves depends solely on the balance of forces impinging on the tractor: the tractor will accelerate if, and only if, there is an unbalanced force on it. The force of the log on the tractor opposes the motion of the tractor, as does the friction in the moving parts of the tractor and between the tractor and the ground. It is only when the force of the earth on the tractor becomes greater than those retarding forces that it will begin to move. Another way of answering the question about why the tractor moves, is to say that the force it exerts on the ground is greater than it exerts on the log; therefore, the accelerating force of the earth is greater than the retarding force of the log.

Brief Answers to Chapter 4 Study Guide

4.1 discussion

4.2 Answer (e)

4.3 (a) $x = 1.5$ m, $y = 1.25$ m, $d = 1.9$ m at angle 40° below horizontal
(b) $v = 5.7$ m/sec at angle 59° below horizontal.

4.4 $v = 177$ m/sec
no

4.5 (1) True
(2) True
(3) True. Discussion

4.6 discussion

4.7 discussion

4.8 (a) 10.2 meters
(b) 61.2 meters

4.9 discussion

4.10 discussion

4.11 (a) 0.2 sec
(b) 5 cps
(c) 62.8 m/sec

4.12 (a) $T = 1.9$ sec
(b) $f = 32\ \text{min}^{-1}$
(c) $v = 50$ cm/sec
(d) $v = 35$ cm/sec
(e) $v = 0$ cm/sec
(f) $\alpha = 190^{\circ}\ \text{sec}^{-1}$ They are equal
(g) $a_c = 120\ \text{cm/sec}^2$
(h) $ac = 160\ \text{cm/sec}^2$

4.13 (a) Above the surface
(b) Syncom 2
(c) Lunik 3 discussion
(d) Luna 4
(e) 24.34 hr
(f) remain overhead
(g) Midas 3
(h) $9.36\ \text{m/sec}^2$

4.14 (a) v(Venus) = 7.3 A.U./yr
v(Earth) = 6.28 A.U./yr
v(Neptune) = 1.14 A.U./yr
(b) a_c(Venus) = 74 A.U./yr^2
a_c(Earth) = 39.5 A.U./yr^2
a_c(Neptune) = 4.37×10^{-2} A.U./yr^2
(c) a_c is proportional to $\frac{1}{r^2}$

4.15 discussion. no

4.16 discussion

4.17 discussion

4.18 Table

4.19 $a_c(\text{main}) = 1.18 \times 10^4\ \text{m/sec}^2$
$a_c(\text{nose}) = 1.8 \times 10^4\ \text{m/sec}^2$

4.20 $a_1/a_2 = 2/1$

4.21 (a) $a_c = 2 \times 10^{-10}\ \text{m/sec}^2$
(b) $F_s = 4 \times 10^{20}$ N
(c) $F_e = 3.55 \times 10^{22}$ N

4.22 1 (a) independent
(b) T l
(c) independent
2 (a) T m
(b) T 1/ k

4.23 85 min; 17650 mi/hr

4.24 13.6 m/sec^2; 2.71 sec; discussion

4.25 discussion

Solutions to Chapter 4 Study Guide

4.1 Using symbols other than words, give an example of each of these:

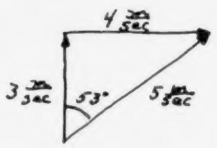
(a) a scalar.
(b) a vector.
(c) the addition of two scalars.
(d) the addition of two vectors.
(e) the addition of three vectors.
(f) the subtraction of one scalar from another.
(g) the subtraction of one vector from another.

(a) Students will give examples of quantities that exhibit only magnitude and not direction, e.g. 59°C, 25 cycles/sec.

(b) $75\ \frac{\text{ft}}{\text{sec}}$ (at 90°) 10 lbs (at 45°)

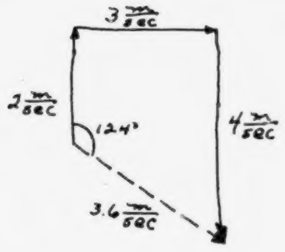
(c) \$1.55 + \$.75 = \$2.30

(d)



$$3\ \frac{\text{m}}{\text{sec}}\ (\text{at } 0^{\circ}) + 4\ \frac{\text{m}}{\text{sec}}\ (\text{at } 90^{\circ}) = 5\ \frac{\text{m}}{\text{sec}}\ (\text{at } 53^{\circ})$$

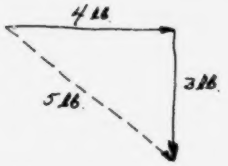
(e)



$$2\ \frac{\text{m}}{\text{sec}}\ (\text{at } 0^{\circ}) + 3\ \frac{\text{m}}{\text{sec}}\ (\text{at } 90^{\circ}) + 4\ \frac{\text{m}}{\text{sec}}\ (\text{at } 180^{\circ}) = 3.6\ \frac{\text{m}}{\text{sec}}\ (\text{at } 124^{\circ})$$

(f) 79°C - 16°C = 63°C

(g)

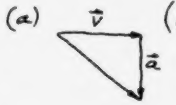


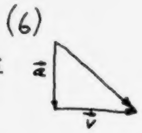
4 lb (at 90°) - 3 lb (at 0°) = 5 lb (at 127°)
Note that the 3-lb vector has been reversed in direction and then added in order to perform the subtraction.

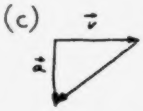
4.2 For a given moving object the velocity and acceleration can be represented by these vectors:

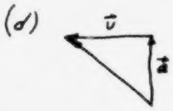
$\vec{v}$ and $\vec{a}$

The sum of these two vectors is:





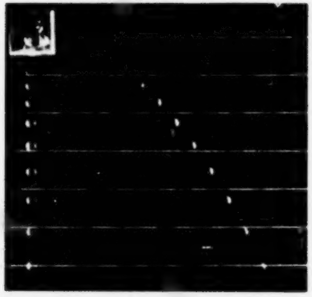




e) They cannot be added.

The answer is (e). They cannot be added because they are different physical quantities.

4.3 A sphere is launched horizontally, as shown below. Suppose the initial speed v_x is 3.0 m/sec. Where is the projectile (displacement), and what is its speed and direction (velocity) 0.5 sec after launching?



4.3 a) $x = 1.5$ m, $y = 1.25$ m, $\vec{d} = 1.9$ m at angle 40° below horizontal

b) $\vec{v} = 5.7$ m/sec at angle 59° below horizontal.

$$x = v_x t = 3 \times .5 = 1.5 \text{ m}$$

$$y = \frac{1}{2}\, at^2 = \frac{1}{2} \times 10 \times .25 = 1.25 \text{ m}$$

$$a = 10 \text{ m/sec}$$

$$\text{displacement} = \sqrt{x^2 + y^2} = \sqrt{2.25 + 1.56} = 1.95 \text{ m}$$

$$\tan 1.25/1.5 = \tan 40^\circ$$

$$v_x = 3 \text{ m/s}$$

$$v_y = at = 10 \times .5 = 5 \text{ m/s}$$

$$v = \sqrt{9 + 25} = 5.84 \text{ m/s}$$

$$\tan \frac{5}{3} = \tan 59^\circ$$

4.4 If a raindrop accelerated at a constant rate of 9.8 m/sec^2 from a cloud 1 mile up what would be its speed just before striking the ground? Does a raindrop accelerate at a constant rate over a 1 mile fall?

$$v^2 = 2\, as$$

$$= 2 \times 9.8 \times 1600 \text{ m}^2/\text{sec}^2$$

$$v = 177 \text{ m/sec.}$$

No. The drop fractionates and the small droplets are slowed up by air viscosity.

4.5 An airplane has a gun that fires bullets at the speed of 600 mph when tested on the ground with the plane stationary. The plane takes off and flies due east at 600 mph. Which of the following claims are correct, if any? In defending your answers, refer to the superposition principle and to Galilean relativity.

a) With respect to the ground, the bullet do begin moving east with a speed of 1200 mph. Superposition holds and the result is obtained by adding the two vectors.

b) When fired in the opposite direction, the bullets drop vertically downward.

c) If fired vertically downward, the bullets move eastward at 600 mph.

----

a) When fired directly ahead the bullets move eastward at a speed of 1200 mph.

b) Again vectors are added, this time giving the sum of zero, so that the bullets will have this speed relative to the ground and will fall straight down.

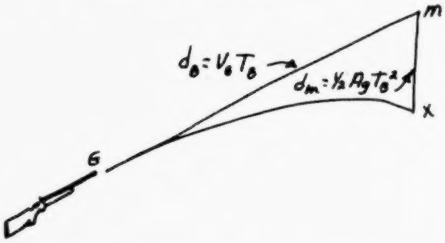
c) If fired vertically downward, the bullets have a component of their motion 600 mph eastward due to the plane's eastward motion. The initial velocity relative to the ground is obtained by superposition of the two vectors, which are at right angles to each other. This resultant velocity is 800 mph at 45° to the horizontal and in an eastward direction. (Notice that it takes more information to fix the direction of a vector in three dimensions.)

4.6 Two persons watch the same object move. One says it accelerates straight downward, but the other claims it falls along a curved path. Invent a situation in which they both could be right.

The condition described could take place if one person were in a train and let an object drop to the floor. To him, the path would be a straight line. An observer on the ground watching the train go by would see the object fall along a path which, in his frame of reference, was a parabola.

4.7 A hunter points his gun barrel directly at a monkey in a distant palm tree. Where will the bullet go? If the animal, startled by the flash, drops out of the branches at the very instant of firing, will it then be hit by the bullet? Explain.

Assume for a moment that gravity does not affect the problem. In that case, the bullet would travel along the straight line in the diagram and strike the monkey. If the distance were d_B, the the time required to travel the distance d_B is $T_B = d_B/v_B$.



Using the principle of superposition, we know that any vertical motion due to gravity can be considered separately and will merely add vectorially to the motion considered in the absence of gravity.

Therefore, we can calculate the position of the bullet after T_B seconds by adding to the displacement vector GM a vertical one due to gravity:

$$d_Y = \tfrac{1}{2} a_g T_B{}^2.$$

Since the monkey has also been falling vertically for T_B seconds, he will be at a position directly below his position at M. Thus,

$$d_M = \tfrac{1}{2} a_g T_B{}^2.$$

The two distances are the same which means that the monkey and the bullet will be arriving simultaneously at point x. Note that this event does not depend on the angle of fire or the speed of the bullet as long as the monkey is within range.

4.8 If a broad jumper takes off with a speed of 10 m/sec at an angle of 45° with respect to the earth's surface, how far would he leap? If he took off from the moon's surface with that same speed and angle, what would be the length of his leap? The gravitational acceleration of a body at the moon's surface is $\frac{1}{6}$th of that at the earth's surface.

----

a) 10.2 meters
b) 61.2 meters

$$v_x = v_y = 7.07 \text{ m/s}$$
$$x = v_x t$$
$$y = v_y t - \tfrac{1}{2} a t^2 = 0$$
$$t = \frac{2v_y}{a} = \frac{2 \times 7.07}{9.8} = 1.445$$

$$x = 7.07 \times 1.445 = \underline{10.2 \text{ m}}$$

on moon 6 times less or

$$x = 6 \times 10.2 = \underline{61.2 \text{ m}}.$$

4.9 Contrast rectilinear motion, projectile motion and uniform circular motion by:

a) defining each.
b) giving examples
c) comparing the velocity-acceleratior relationships.

----

(1) Rectilinear motion is motion in a straight line. Any acceleration is along the line of motion. Example: car moving along a straight road.

(2) Projectile motion is the motion of a body which is free to move in a force field but which does not supply its own propulsion. In this chapter a special case of projectile motion is considered in which the horizontal component of velocity is constant and the vertical component undergoes constant acceleration downward due to the force of gravity. Example: a rock thrown into the air.

(3) Uniform circular motion is motion at a constant speed along a circular path. In uniform circular motion the acceleration is at right angles to the velocity direction, that is directed toward the center of the circle and is constant in magnitude. Example: a point on a rotating turntable.

4.10 You are inside a uniformly accelerating moving van. If when the van is traveling at 10 mph (and still accelerating) you dropped a ball from the roof of the van onto the floor, what would be the ball's path relative to the van? What would be its path relative to a person driving past the van in the opposite direction of the van at a uniform speed? What would be its path relative to a person standing on road?

In every case the ball has an initial speed relative to the ground in the direction the van is moving of 10 mph. It also accelerates downward due to gravity. Relative to van a curved path toward the back of the van. Concave forward unless the van has terrific acceleration. To the man in passing car, a parabola, concave to back of van if speed less than 10 mph. Concave to front if greater than 10 mph. To man on road, parabola.

4.11 An object in uniform circular motion makes 20 revolutions in 4.0 sec.

a) What is its period T?

b) What is its frequency f?

c) If the radius of rotation is 2 meters, what is its speed?

----

a) $\frac{4}{20} = 0.2$ seconds

b) $f = \frac{1}{T} = \frac{1}{.2} = 5$ per sec

c) $d = 2\pi r$

$$v = \frac{d}{t}$$

$$v = \frac{2 \times 3.14 \times 2}{.2} = 62.8 \text{ m/sec}$$

4.12 Two blinkies were placed on a rotating turntable and photographed from directly overhead. The result is shown in the figure below. The outer blinky has a frequency of 9.4 flashes/sec and is located 15.0 cm from the center. For the inner blinky, the values are 9.1 flashes/sec and 10.6 cm.

a) What is the period of the turntable?

b) What is the frequency of rotation of the turntable? Is this a standard phonograph speed?

c) What is the linear speed of the turntable at the position of the outer blinky?

d) What is the linear speed of the turntable at the position of the inner blinky?

e) What is the linear speed of the turntable at the center?

f) What is the angular speed of each blinky in degrees/sec? Are they equal?

g) What is the centripetal acceleration experienced by the inner blinky?

h) What is the centripetal acceleration experienced by the outer blinky?

----

a) As measured on the photograph, the outer blinky flashes 17.6 times at a rate of 9.4 flashes/sec. The inner blinky flashes 17.0 times at 9.1 flashes/sec,

$$T = 17.6 \text{ flashes } \frac{1 \text{ sec}}{9.4 \text{ flashes}}$$
$$= 1.9 \text{ sec}$$

For the inner blinky,

$$T = 17.0 \text{ flashes } \frac{1 \text{ sec}}{9.1 \text{ flashes}}$$
$$= 1.9 \text{ sec}$$

$$\underline{T = 1.9 \text{ sec}}$$

b) $f = \frac{1}{T}$

$T = 1.9$ sec

$$f = \frac{1}{1.9 \text{ sec}}$$

$$f = \frac{1}{1.9 \text{ sec}} \times 60 \frac{\text{sec}}{\text{min}}$$

$$\underline{f = 32 \text{ min}^{-1}}$$

This is quite close to the standard rate of 33 1/3 min^{-1}.

c) $v = \frac{2\pi R}{T}$

$R = 15.0$ cm

$T = 1.9$ sec

$$v = \frac{2\pi \times 15.0 \text{ cm}}{1.9 \text{ cm}}$$

$$\underline{v = 50 \frac{\text{cm}}{\text{sec}}}$$

d) $v = \frac{2\pi R}{T}$

$R = 10.6$ cm

$T = 1.9$ sec

$$v = \frac{2\pi \times 10.6 \text{ cm}}{1.9 \text{ cm}}$$

$$\underline{v = 35 \frac{\text{cm}}{\text{sec}}}$$

e) $v = \frac{2\pi R}{T}$

$R = 0$ cm

$T = 1.9$ sec

$$v = \frac{2\pi \times 0 \text{ cm}}{1.9 \text{ sec}}$$

$$\underline{v = 0 \frac{\text{cm}}{\text{sec}}}$$

f) α = angular speed = $\frac{360°}{T}$

$T = 1.9$ sec

$$\alpha = \frac{360°}{1.9 \text{ sec}}$$

$$\underline{\alpha = 190° \text{ sec}^{-1}}$$

is true for both blinkies since their periods are equal.

g) $a_c = \frac{4\pi^2 R}{T^2}$

$R = 10.6$ cm

$T = 1.9$ sec

$$a_c = \frac{4\pi^2 \times 10.6 \text{ cm}}{(1.9 \text{ sec})^2}$$

$$\underline{a_c = 120 \frac{\text{cm}}{\text{sec}^2}}$$

h) $a_c = \frac{4\pi^2 R}{T^2}$

$R = 15.0$ cm

$T = 1.9$ sec

$$a_c = \frac{4\pi^2 \times 15.0 \text{ cm}}{(1.9 \text{ sec})^2}$$

$$\underline{a_c = 160 \ \frac{\text{cm}}{\text{sec}^2}}$$

This can be checked by using the relationship $a_c = V^2/R$.

$V = 50 \ \frac{\text{cm}}{\text{sec}}$

$R = 15.0$ cm

$$a_c = \frac{(50 \ \frac{\text{cm}}{\text{sec}})^2}{15.0 \text{ cm}}$$

$$a_c = 170 \ \frac{\text{cm}}{\text{sec}^2}$$

4.13 These questions are asked with reference to Table 4.2 on page 112.

a) Are the distances to apogee and perigee given as height above the surface of the earth or distance from the center of the earth?

b) Which satellite has the most nearly circular orbit?

c) Which are the most eccentric? How did you arrive at your answer?

d) Which satellite in the table has the longest period?

e) What is the period of Syncom 2 in hours?

f) How does the position of Syncom relative to a point on the earth change over one day?

g) Which satellite has the greater centripetal acceleration, Midas 3 or Syncom 2?

h) What is the magnitude of the centripetal acceleration of Vostok 6? Express answer in m/sec^2.

----

a) They are given in miles above the earth's surface.

b) Syncom 2 is the most circular with the distance from the surface (also from the earth's center varying by only 5 miles). If 4000 miles is taken as the radius of the earth, this is a difference of only about 0.02%. (5 mi/22,200 mi × 100% = 0.02%.)

c) Without actually calculating the eccentricity (which is defined in Chapter 7), it would be reasonable to estimate which satellite has the greatest percentage variation in its greatest and least distance from the center of the earth. This is Lunik 3.

d) Luna 4 has a period of 42,000 min (27 days).

e) Period in minutes given as T = 1,460.4. To express in hours

$$T = 1460.4 \text{ min} \times \frac{1}{60} \ \frac{\text{hours}}{\text{min}}$$

$$= \frac{1460.4}{60} \text{ hours} = \underline{24.34 \text{ hours}}$$

f) If headed westward at equator would remain nearly overhead.

g) $a_c = \frac{4\pi^2 R}{T^2}$

For Midas 3:

$R = 6140$ mi (using 4000 miles as the radius of the earth)

$T = 161$ min

$$a_c = \frac{4\pi^2 \times 6140 \text{ mi}}{(161 \text{ min})^2}$$

$$\underline{\text{Midas 3 } a_c = 9.3 \ \frac{\text{mi}}{\text{min}^3}}$$

For Syncom 2:

$R = 2.62 \times 10^4$ mi

$T = 1.46 \times 10^3$ min

$$a_c = \frac{4\pi^2 \times 2.62 \times 10^4 \text{ mi}}{(1.46 \times 10^3 \text{ mi})^2}$$

$$\underline{\text{Syncom 2 } a_c = 4.8 \times 10^{-1} \ \frac{\text{mi}}{\text{min}^2}}$$

Midas 3 has the greater centripetal acceleration.

h) For Vostok 6

R = 4120 miles assuming 4000 miles as the earth's radius

$$= 4120 \text{ miles} \times 1610 \ \frac{\text{meters}}{\text{mile}}$$

$$= 6,633,200 \text{ meters} \doteq 6.63 \times 10^6 \text{ m}$$

$T = 88.34$ min

$$= 88.34 \text{ min} \times 60 \ \frac{\text{sec}}{\text{min}}$$

$$= 5300.40 \text{ sec} \doteq 5.3 \times 10^3 \text{ sec}$$

$$a = \frac{4\pi^2 R}{T^2} = \frac{4 \times 9.4 \times 6.63 \times 10^6}{28.1 \times 10^6} =$$

$$\underline{9.36 \text{ m/sec}^2}$$

4.14 The following table shows the period and the mean distance from the sun for the three planets that most nearly go in a circular orbit.

Planet	Mean distance (r) from sun (in A.U.)	Period (T) in years
Venus	0.72	0.62
Earth	1.00	1.00
Neptune	30.06	164.8

(A.U. = astronomical unit = the mean distance of the earth from the sun; 1 A.U. = 92.9×10^6 miles.)

a) What is the average orbital speed for each planet (in A.U./year)?

b) Calculate the centripetal acceleration for each planet in A.U./yr^2.

c) Can you see any relationship between the mean distance and the centripetal acceleration a_c?

[Hint: Does it appear to be

(1) $a_c \propto r$; or (2) $a_c \propto 1/r$; or

(3) $a_c \propto r^2$; or (4) $a_c \propto 1/r^2$?

How can a graph help you to decide?]

----

a) $v = \frac{2\pi R}{T}$

For Venus:

$R = 0.72$ A.U.

$T = 0.62$ yr

$$v = \frac{2\pi \times 0.72 \text{ A.U.}}{0.62 \text{ yr}}$$

$$\underline{v \text{ (Venus)} = 7.3 \frac{\text{A.U.}}{\text{yr}}}$$

For Earth:

$R = 1.00$ A.U.

$T = 0.62$ yr

$$v = \frac{2\pi \times 1.00 \text{ A.U.}}{1.00 \text{ yr}}$$

$$\underline{v \text{ (Earth)} = 6.28 \frac{\text{A.U.}}{\text{yr}}}$$

For Neptune:

$R = 30.06$ A.U.

$T = 164.8$ yr

$$v = \frac{2\pi \times 30.1 \text{ A.U.}}{165 \text{ yr}}$$

$$v \text{ (Neptune)} = 1.14 \frac{\text{A.U.}}{\text{yr}}$$

b) $a_c = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$

We will use the latter relationship because it is generally better to use original data rather than derived quantities such as v which represents an intermediate calculation.

For Venus:

$R = 0.72$ A.U.

$T = 0.62$ yr

$$a_c = \frac{4\pi^2 \times 0.72 \text{ A.U.}}{(0.62 \text{ yr})^2}$$

$$\underline{a_c \text{(Venus)} = 74 \frac{\text{A.U.}}{\text{yr}^2}}$$

For Earth:

$R = 1.00$ A.U.

$T = 1.00$ yr

$$a_c = \frac{4\pi^2 \times 1.00 \text{ A.U.}}{(1.00 \text{ yr})^2}$$

$$\underline{a_c \text{(Earth)} = 39.5 \frac{\text{A.U.}}{\text{yr}^2}}$$

For Neptune:

$R = 30.06$ A.U.

$T = 164.8$ yr

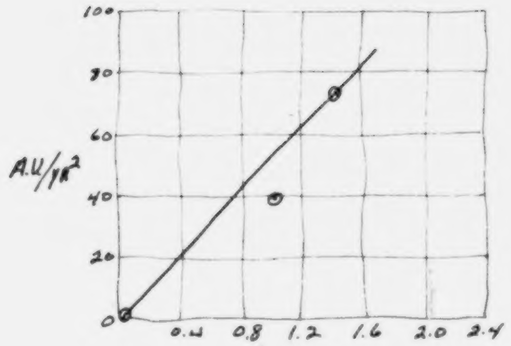
$$a_c = \frac{4\pi^2 \times 30.1 \text{ A.U.}}{(165 \text{ yr})^2}$$

$$\underline{a_c \text{(Neptune)} = 4.37 \times 10^{-2} \frac{\text{A.U.}}{\text{yr}^2}}$$

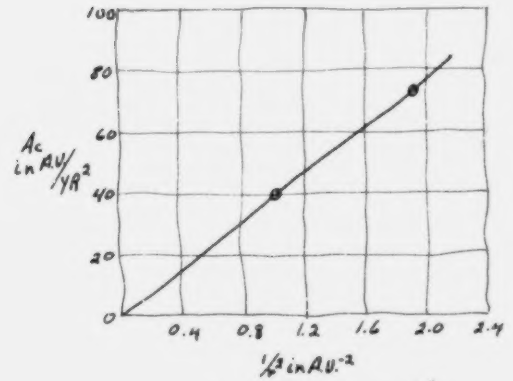
c)

Planet	a_c	r	1/r	$1/r^2$
Venus	74	0.72	1.4	1.9
Earth	39.4	1.00	1.00	1.00
Neptune	0.0437	30.06	0.0333	0.00111

The relationship is an inverse one. It may be possible to determine what the relationship is by graphing a_c vs. 1/r and $1/r^2$.



Graph of Ac vs 1/r



Graph of Ac vs. 1/r²

Since the points on the graph of a_c vs $1/r^2$ fall along a straight line, it appears that $a_c \propto 1/r^2$ is the correct relation.

4.15 Explain why it is impossible to have an earth satellite orbit the earth in 80 minutes. Does this mean that it is impossible for an object to circle the earth in less than 80 minutes?

A satellite is held in its orbit only by the pull of gravity. This speed calls for a centripetal acceleration greater than that gravity.

No. Put the rails of an elevated train underneath the trestle and run the train upside down.

4.16 The intention of the first four chapters has been to describe "simple" motions and to progress to the description of "complex" motions. Organize the following examples into a list from the simplest to the most complex, making whatever idealizing assumptions you wish. Be prepared to say why you placed any one example ahead of those below it, and to state any assumptions you made.

A "human cannon ball" in flight
A car going from 40 mph to a complete stop
A redwood tree
A child riding a ferris wheel
A rock dropped 3 m
A woman standing on an escalator
A climber ascending Mt. Everest

The organization of this list will certainly vary with the assumptions made. Here is one possible arrangement.

1. Woman on escalator. She moves with constant velocity.

2. Falling rock. It moves along a straight line with constant acceleration, if we neglect air resistance.

3. Car coming to a stop. This also moves along a straight line. It is not clear, however, that acceleration is constant. We will assume that it is.

4. Child on ferris wheel. We assume that the child travels in a circle with constant speed. The magnitude of the acceleration is constant, while its direction changes uniformly.

5. Human cannon ball. This is an example of projectile motion. Ideally, the path is a parabola and the velocity changes in magnitude and direction. Acceleration, however, is constant.

6. Climber on Mt. Everest. The displacement and velocity of the climber will undergo many complicated changes.

7. A redwood tree. Either the growth of the tree or its motion in a breeze constitutes a very complicated combination of velocities and accelerations.

4.17 Could you rank the above examples if you were not permitted to idealize? If yes, how would you then rank them? If no, why not?

It would be difficult to rank some of the examples. Irregularities in velocity and acceleration caused by air resistance, nonuniform circular motion, irregular braking are difficult to compare in their "complexity."

4.18 Using a full sheet of paper, make and complete a table like the one below.

Concept	Symbol	Definition	Example
		Length of a path between any two points as measured along the path	
			Straight line distance and direction from Detroit to Chicago
speed			
	$\bar{v}$		
			An airplane flying west at 400 mph at constant altitude
		Time rate of change of velocity	
	a_g		
centripetal acceleration			
			The drive shaft of some automobiles turns 600 rpm in low gear
		The time it takes to make one complete revolution	

Concept	Symbol	Definition	Example
total distance	d	Length of a path between any two points as measured along the path	The speedometer mileage recorded on a trip from Los Angeles to San Diego and back again
displacement	$\vec{s}$	The straight-line distance and direction	Straight line distance and direction from Detroit to Chicago
speed	v	Time rate of change of distance	A car travels at 40 mph
average speed	$\bar{v}$	Time rate of change of total distance	A car drives 5 miles through traffic in 20 minutes. $\bar{v} = 15$ mph
velocity	$\vec{v}$	Time rate of change of displacement	An airplane flying west at 400 mph at constant altitude
acceleration	$\vec{a}$	Time rate of change of velocity	A car accelerates at 3 $\frac{m}{sec^2}$ toward the north
acceleration of gravity	$\vec{a}_g$	The acceleration of a freely falling body	The acceleration of gravity in San Francisco is 9.800 $\frac{m}{sec^2}$ toward the center of the earth.
centripetal acceleration	$\vec{a}_c$	Time rate of change of velocity (toward the center of the circle)	A child on a merry-go-round
frequency	f	The number of complete cycles per unit of time	The drive shaft of some automobiles turns 600 rpm in low gear
period	T	The time it takes to make one complete revolution	The period of a drive shaft turning 600 rpm is 0.1 sec.

4.19 The diameter of the main wheel tires on a Boeing 727 fan jet is 1.26 m. The nose wheel tire has a diameter of 0.81 m. The speed of the plane just before it clears the runway is 86.1 m/sec. At this instant, find the centripetal acceleration of the tire tread, for each tire.

Assuming the speed of the tire tread to be the same as the speed of the plane:

$$a_c = \frac{v^2}{R} .$$

For the main wheels:

$v = 86.1$ m/sec

$R = 0.630$ m

$$a_c = \frac{(86.1 \frac{m}{sec})^2}{0.405}$$

nose tire $\underline{a_c = 1.8 \times 10^4 \frac{m}{sec^2}}$

main wheel $\underline{a_c = 1.2 \times 10^4 \frac{m}{sec^2}}$

4.20 Compare the centripetal acceleration of the tire tread of a motor scooter wheel (diameter 1 ft) with that of a motorcycle wheel (diameter 2 ft) if both vehicles are moving at the same speed.

----

a_1 = centripetal acceleration of motor scooter

a_2 = centripetal acceleration of motorcycle

$$a_1 = \frac{v^2}{R_1} \qquad a_2 = \frac{v^2}{R_2}$$

$$\frac{a_1}{a_2} = \frac{R_2}{R_1} = \frac{D_2}{D_1} = \underline{\frac{2}{1}}$$

4.21 Our sun is located at a point in our galaxy about 30,000 light years (1 light year = 9.46×10^{12} km) from the galactic center. It is thought to be revolving around the center at a linear speed of approximately 250 km/sec.

a) What is the sun's centripetal acceleration with respect to the center of our galaxy?

b) If the sun's mass is taken to be 1.98×10^{30} kg, what centripetal force is required to keep it moving in a circular orbit about the galactic center?

c) Compare the centripetal force in b) with that necessary to keep the earth in orbit about the sun. (The earth's mass is 5.98×10^{24} kg and its average distance from the sun is 1.495×10^8 km. What is its linear speed in orbit?)

----

a) $a_c = \frac{v^2}{R}$

$v = 2.5 \times 10^5 \frac{m\rightarrow}{sec}$

$R = 3 \times 10^4 ly \times 9.46 \times 10^{15} \frac{m}{1y} = 2.84 \times 10^{20}$ m

$$a_c = \frac{(2.5 \times 10^5 \frac{m}{sec})^2}{2.84 \times 10^{20} m}$$

$\underline{a_c = 2.2 \times 10^{-10} \frac{m\rightarrow}{sec^2}}$

b) $F_s = ma_c$

$m = 1.98 \times 10^{30}$ kg

$a_c = 2 \times 10^{-10} \frac{m\rightarrow}{sec^2}$

$F_s = 1.98 \times 10^{30} kg \times 2 \times 10^{-10} \frac{m}{sec}$

$\underline{F_s = 4 \times 10^{20} N}$

c) $F_e = \frac{mv^2}{R}$

$m = 5.98 \times 10^{24}$ kg

$R = 1.495 \times 10^{24}$ m

$v = \frac{2\pi R}{T}$

$$v = \frac{2\pi \times 1.495 \times 10^{11} m}{1 yr \times 365 \frac{day}{yr} \times 24 \frac{hr}{day} \times 3600 \frac{sec}{hr}}$$

$v = 2.98 \times 10^4 \frac{m}{sec}$

$$F_e = \frac{5.98 \times 10^{24} kg (2.98 \times 10^4 \frac{m\rightarrow}{sec^2}}{1.495 \times 10^{11} m}$$

$\underline{F_e = 3.55 \times 10^{22} N}$

F_e is about 100 times greater than F_s.

4.22 Here are a list of some possible investigations into simple harmonic motion.

1. How does the period of a pendulum depend upon

   a) the mass of the pendulum bob?
   b) the length of the pendulum?
   c) the amplitude of the swing (for a fixed length and fixed mass)?

2. How does the period of an object on the end of a spring depend upon

   a) the mass of the object?
   b) the spring constant, k, where the spring constant k is defined as the slope of the graph of force versus spring extension? Its units are newtons/meter.

----

1. a) independent of mass
   b) $T \propto \sqrt{l}$
   c) assuming small oscillation, independent of amplitude

2. a) $T \propto \sqrt{m}$
   b) $T \propto 1/\sqrt{k}$

4.23 The centripetal acceleration experienced by a satellite orbiting at the earth's surface (air resistance conveniently neglected) is the acceleration due to gravity of an object at the earth's surface (9.8 m/sec^2). Therefore, the speed required to maintain the satellite in a circular orbit must be such that the centripetal acceleration of the satellite is 9.8 m/sec^2. This condition can be expressed as follows.

$$a_c = \frac{V^2}{R} = a_g = 9.8 \text{ m/sec}^2$$

R, the radius of the earth, is 6.38×10^6 meters

$$a_g = 9.8 \text{ m/sec}^2$$

$$\begin{aligned} V^2 &= 9.8 \text{ m/sec}^2 \times 6.38 \times 10^6 \text{ m} \\ &= 62.5 \times 10^6 \text{ m}^2/\text{sec}^2 \\ V &= 7.85 \times 10^3 \text{ m/sec} \end{aligned}$$

What is the period T of this orbit?

What is the satellite's speed expressed in miles per hour? (Hint: 1,000 meters = .61 miles.)

$$V = \frac{2\pi R}{T}, \quad T = \frac{2\pi R}{V}$$

$$T = \frac{2 \times 3.14 \times 6.38 \times 10^6}{7.85 \times 10^3}$$

$$\begin{aligned} &= 5.11 \times 10^3 \\ &= 5110 \text{ sec} \\ &= \underline{85 \text{ min}} \end{aligned}$$

$$\begin{aligned} V &= 7.85 \times 10^3 \text{ m/sec} \\ &= 7.85 \times 10^3 \frac{\text{m}}{\text{sec}} \times \frac{1}{1.6 \times 10^3} \frac{\text{miles}}{\text{min}} \\ &\quad \times 3.6 \times 10^3 \frac{\text{sec}}{\text{hr}} \\ &= \frac{7.85 \times 10^3 \times 3.6 \times 10^3}{1.6 \times 10^3} \\ &= 17.65 \times 10^3 \\ &= \underline{17650 \text{ mi/hr}} \end{aligned}$$

4.24 The thrust of a Saturn Apollo launch vehicle is 7,370,000 newtons (approximately 1,650,000 lbs) and its mass is 540,000 kg. What would be the acceleration of the vehicle relative to the earth's surface at lift off? How long would it take for the vehicle to rise 50 meters? The acceleration of the vehicle increases greatly with time (it is 47 m/sec^2 at first stage burnout), even though the thrust force does not increase appreciably. Explain why the acceleration increases.

$$F = ma, \quad a = \frac{F}{m} = \frac{7.37 \times 10^6}{5.4 \times 10^5} = 1.36 \times 10^1$$

$$= 13.6 \text{ m/sec}^2$$

$$s = \tfrac{1}{2}at^2, \; t = \frac{2s}{a}$$

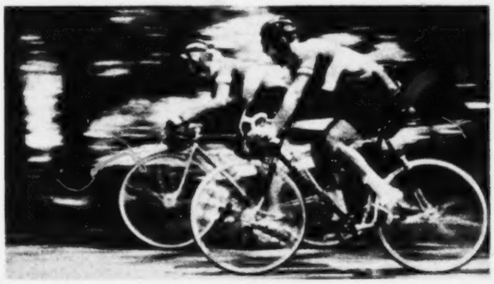
$$= \frac{2 \times 50}{13.6} = \sqrt{7.35} = 2.71 \text{ sec}$$

As fuel burns, mass of vehicle decreases.

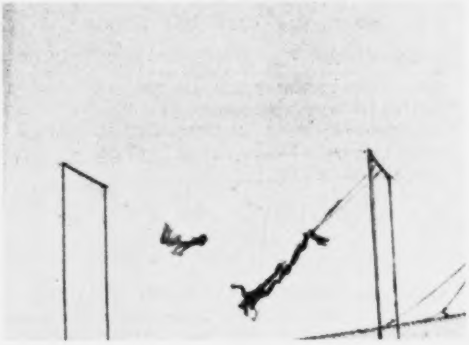
$a = \frac{F}{m}$, F(thrust force) is a constant while m becomes smaller. The quantity F/m (=a) increases.

4.25 Write a short essay on one of the following pictures.

Essay might touch on relative speeds, rotary motion or relation between linear and angular speeds.



Essay might touch on centripetal forces or projectile motion of acrobat.



Aid Summaries
Transparencies
Film Loops

Transparencies

T0 Using Stroboscopic Photographs

T1 Stroboscopic Measurements

Stroboscopic facsimiles of uniform speed and uniform acceleration are shown. Measurements may be taken directly, data recorded on tables and graphs plotted on grids.

T2 Graphs of Various Motion

Multiple examples of distance-time, speed-time and acceleration-time graphs. Useful for slope concept, area-under-the-curve concept, review and quizzes.

T3 Instantaneous Speed

Stroboscopic facsimile of body-on-spring oscillation, data table and grid. Find approximate instantaneous speed by approaching the limit and graphical estimation.

T4 Instantaneous Rate of Change

Determines v_{av} from enlarged portion of a distance-time curve as time intervals are decreased. Shows slope of chords must approach the slope of the tangent as a limit when Δt approaches zero.

T5 not available

T6 Derivation of $d = v_i t + \frac{1}{2}at^2$

Colored overlays illustrate graphical procedures using area-under-the-curve technique. Space is provided for teacher-directed derivation.

T7 not available

T8 Tractor-Log Paradox

Classic third law horse-and-wagon paradox is updated with this tractor-log version.

T9 Projectile Motion

Stroboscopic facsimile of objects projected horizontally and falling freely are analyzed graphically. Space is provided for derivation of equation of the trajectory.

T10 Path of a Projectile

"Demonstration Transparency" suggests that students approximate portion of circles, hyperbolas, parabolas and ellipses by throwing objects. Leads to determination of actual path of projectile. Use with T9.

T11 Centripetal Acceleration—Graphical Treatment

Stroboscopic facsimiles allow derivation of v^2/R and graphical measurement of a_c.

T12 not available

Film Loops

(L) = lab-loop; quantitative measurements can be made, but these loops can also be used qualitatively.

L1 Acceleration Due to Gravity—Method I

Slow-motion photography in one continuous sequence allows measurement of average speed of a falling bowling ball during two 50-cm intervals separated by 1.5 m. (Sec. 2.8)(L)

L2 Acceleration Due to Gravity—Method II

Slow-motion photography allows measurement of average speed of falling bowling ball as it passes through four 20-cm intervals spaced 1 m apart. (Sec. 2.8)(L)

L3 Vector Addition I—Velocity of a Boat

A motorboat is viewed from above as it moves upstream, downstream, as it heads across stream and as it heads at an angle upstream. Vector triangles can be drawn for the various velocities. (Sec. 3.2)(L)

L4 A Matter of Relative Motion

A collision between two equally massive carts is viewed from various stationary and moving frames of reference. (Sec. 4.3)

L5 Galilean Relativity I—Ball Dropped from Mast of Ship

A realization of the experiment gested in Galileo's Dialogue on the Great World Systems; the ball lands at the base of the mast of the moving ship. (Sec. 4.4)

L6 Galilean Relativity II—Object Dropped from Aircraft

A flare is dropped from an aircraft which is flying horizontally. The parabolic path of the flare is shown, and freeze frames are provided for measurement of the position at ten equally spaced intervals. (Sec. 4.4) (L)

L7 Galilean Relativity III—Projectile Fired Vertically

A flare is fired vertically from a Ski-doo which moves along a snow-covered path. Events are shown in which the Ski-doo's speed remains constant, and also when the speed changes after firing.

Due to a printing error, the directions are reversed, left to right, beginning at a point near the end of the "preliminary run." This does not affect the physics of the motions of the flare and the Ski-doo. In order to make the film available at a useful time early in the course, the error was not corrected in the preliminary version, although next year's final version will, of course, be corrected. (Sec. 4.4)

L8 Analysis of a Hurdle Race—Part I

Slow-motion photography allows measurement of speed variations during a hurdle race. (Sec. 1.5)(L)

L9 Analysis of a Hurdle Race—Part II

A continuation of the preceding loop. (Sec. 1.5)(L)

Reader

1. The Value of Science

by Richard P. Feynman

"The value of science remains unsung by singers, so you are reduced to hearing—not a song or a poem—but an evening lecture about it. This is not yet a scientific age." So Feynman begins in a conversational and readable style. In his view, knowing the great value of a satisfactory philosophy of ignorance is the vital aspect of science.

2. Close Reasoning

by Fred Hoyle

In this section of Hoyle's fine science fiction book The Black Cloud, Kingsley and his colleagues discuss the nature of the cloud and how to communicate with it. Kingsley alone has come to the conclusion that the cloud is a living entity and he tries to draw the others to his point of view. Their exchanges are a good approximation of an analytic conversation between scientists. This chapter will probably be popular with students. Guide students to the full book if they like a part of it! ($.50 paperback)

3. On "Scientific Method"

by P. W. Bridgman

Bridgman argues that scientific method is what working scientists do, not what other people or even the scientists themselves say about it. This will probably come as a great relief to your students who may have had to memorize tedious (and, for a working scientist, laughable) definitions of "the scientific method" at previous points in their school career.

4. How to Solve It

by G. Polya

This is Polya's one-page summary of his book in which he discusses strategies and techniques for solving problems. Polya's examples are from mathematics, but his ideas are useful in solving physics problems also.

5. Four Pieces of Advice to Young People

by Warren W. Weaver

In this short article Weaver urges students to concentrate on the fundamentals of science, not to worry early about fields of specialty; he advocates the balance of teaching/research and of scientist/citizen. His final advice is that science isn't the whole answer—or the only answer.

6. On Being the Right Size

by J. B. S. Haldane

A careful and exciting explanation of the importance of scaling in the structure of living organisms. Why can an insect fall without hurting itself? Why aren't very small animals usually found in cold climates? If a flea were as large as a man, how high would it jump? Few students will have difficulty reading this.

7. Motion in Words

by J. B. Gerhart and R. H. Nussbaum

Not only the scientist is interested in motion. This article comments briefly on references to motion in poetry.

8. The Representation of Motion

by Gyorgy Kepes

Kepes emphasizes that motion is of interest to the artist as well as to the physicist. He sees a similarity in Einstein's statement that relativity interprets events existing in space-time and Marinetti's manifesto:

> In painting a figure on a balcony, seen from within doors, we shall not confine the view to what can be seen through the frame of the windows; we shall give the sum total of the visual sensation of the street, the double row of houses extending right and left, the flowered balconies, etc....

9. Speed

by W. W. Sawyer

These two essays in an SMSG monograph. Students having difficulty understanding velocity and acceleration will probably find the essays helpful. The first section, "The Study of Speed," fits in nicely with Sec. 1.6 and Study Guide 1.2. If you want to give your classes additional material on velocity, the second section, "The Simplest Case of Varying Speed," can be correlated with Sec. 3.2, and is useful background for the difficult Feynman article later in the reader.

10. Motion

by William Kingdon Clifford

Most of Clifford's presentation is geometrical, and complements the Project Physics text. "Hence, any motion of a point, or any motion of translation, whatever, can be specified by a properly drawn curve of positions, and the problem of comparing and classifying different motions is therefore reduced to the problem of comparing and classifying curves."

11. Galileo's Discussion of Projectile Motion

by G. Holton and D. H. D. Roller

Galileo's discussion of the independence of the horizontal and vertical components of motion, presented in Sec. 3.4 in the text. Students will have read Galileo's dialogue with Sagredo, Salviati, and Simplicio in Sec. 2.3 and will be familiar with Galileo's style.

12. Newton's Laws of Dynamics

by R. P. Feynman, R. B. Leighton and M. Sands

Feynman treats the differential equations of motion as difference equations and solves them numerically. Students who have little difficulty with Chapter 4 will find this material exciting.

13. The Dynamics of a Golf Club

by C. L. Stong

This article tells the student how he can, with the aid of a slow motion movie camera and a cooperative friend, explore the dynamics of a golf club during the split second of the drive. Actual data are analyzed. The technical cinematic techniques are so carefully explained by Stong that some students may want to try for themselves.

14. Bad Physics in Athletic Measurements

by P. Kirkpatrick

Athletic events involve measurements of distance and time, and so bring in the same error considerations that one also meets in the laboratory.

15. The Scientific Revolution

by Herbert Butterfield

This is an article you should urge your students to read, if they respond to the historical orientation of the text. Butterfield's view is wide and his article brief, but he does manage to pull together many of the factors, political, economic, sociological, that contributed to an atmosphere conducive to drastic change. Students may be surprised that scientific developments are not created in a vacuum and are not isolated events, but are intrinsically rooted in and reflect their times.

16. How the Scientific Revolution of the Seventeenth Century Affected other Branches of Thought

17. Rigid Body (Sings)
Report on Tait's Lecture on Force:- B.A. 1876

by James Clerk Maxwell

Maxwell, the developer of electromagnetic theory (Unit 4), wrote light verse. The reference in the first line of the second poem is to the members of the British Association for the Advancement of Science.

18. The Vision of Our Age

by J. Bronowski

In tracing the relation of science to

other parts of modern life, Jacob Bronowski interviews an artist, Eduardo Paolozzi, an architect, Eero Saarinen, a physicist, Abdus Salam, and a writer, Lawrence Durrell.

19. Chart of the Future

by Arthur C. Clarke

An interesting attempt to try to project the future. Clarke starts at about 1800, goes up to the present time, and then tries to predict what is going to happen in various fields in the future, e.g., 2030, space mining and contact with extra-terrestrials. The article should not be taken too seriously or (in view of Clarke's past success in predicting) too lightly.

Programmed Instruction

Vectors - Part I. The Nature of Vectors. (A map of Washington, D. C. is provided with the booklet.)

Vectors - Part II. Adding Vectors. (A diagram page is provided with the booklet.)

Vectors - Part III. Components of Vectors.

Films

F1 PSSC film - Straight-line kinematics.

F2 PSSC film - Inertia

F3 PSSC film - Free-fall and projectile motion.

F4 PSSC film - Frames of reference.

F5 PSSC film - Vector kinematics.

Sources: PSSC films are available from

Modern Learning Aids
1212 Avenue of the Americas
New York, New York 10036

A list of District Offices of MLA, and addresses for N.A.S.A. and other film sources, is included in the Unit 2 Teacher Guide.

Notes for the new Project Physics film, "People and Particles," are on the following page. Extra copies of a leaflet, incorporating these notes with stills from the film, will be available for distribution to students.

Notes on **People and Particles**

In planning this course and preparing all the necessary texts, laboratory equipment, film loops, teacher guides and so forth, we felt that we should also make a film of what it is like to be working on a real physics problem at the research frontier. We did not want to film a set-up interview or a prepared lecture; we wanted to show people who are working in science.

Of course, we could choose only one out of the great variety of physics research problems of interest today. Our preference was to film a group of moderate size, to show the variety of people involved in experimental work. Also, we had to select a problem which was not too difficult to understand, since the first showing of the film will be near the beginning of the course.

We decided to focus the camera on a group of Harvard University physicists—students and professors—struggling with their work at the Cambridge Electron Accelerator (CEA). For over two years everything was filmed as it happened, using only a hand-held camera and portable tape recorder. The group being filmed soon forgot about our film crew. Over 50,000 feet of film was taken, from which was selected less than one-fiftieth (i.e., 900 feet) to fit into a twenty-eight minute movie.

The film traces the work of the participants in the experiment as they design, construct and assemble the equipment to a point where the physicists are prepared to take actual data. The experiment itself will take several more months. As this course is particularly concerned with the human element in the making of science, it is appropriate that the film should not emphasize advanced physical theory. Instead, it concentrates more on the style of work in a lab, on the men and women who are working together, on some of the joys and pains of doing original scientific work. It raises a number of themes—from the international character of science to the fact that work on this scale requires a great range of skilled people, including shop machinists, scientists, engineers, secretaries and so forth.

Most people have no way of being directly involved in any scientific work and cannot look over the shoulders of scientists. If they could, even through such a film, there might be fewer strange and false notions about work in a laboratory—false notions of exaggerated glamor just as much as of dark doings. We hope our film shows that work on a real research problem, whether in physics or in other sciences, or in any field, can be a truly human enterprise.

An extensive guide to the themes and physics of the film is being prepared, which can be studied in connection with a second showing of the film late in the course. Even before the first showing, however, the following brief notes will be helpful.

The term "pair production" refers to the production of a pair of elementary particles, an electron and an anti-electron (a positron). Under the right conditions such a pair of particles may be produced when a very energetic packet of light, a photon (which may also be considered an elementary particle) passes near a massive object, such as an atomic nucleus. The electron beam from the Cambridge Electron Accelerator is used to produce the particle pairs, by a several-step process. The motion of the particles is detected with spark chambers, in which electric sparks jump along the paths of the particles.

The Strauch-Walker group (named for the two physicists who head the group, Professors Karl Strauch and James Walker) is using the electron-positron pairs to look into a newly suspected flaw in one of the most solidly established theories of physics. They are trying to establish whether the present theory on the influence of electrical charges on one another is correct.

Some physicists think the present theory is bound to show its limitations when it becomes possible to experiment with charges that are extremely near each other.

This procedure has often occurred in science: by testing the limits of a theory, by looking into the contradictions between the experimental result and the theoretical predictions, scientists are led to new theoretical structures. Of the several themes that run through the film—the relation of research to education, the international character of science, communication among scientists, the relation of science to technology—the last is particularly evident on the first showing. Pure physics of the last 100 years or so has made possible the development of such practical devices as the oscilloscope, the electronic computer, the scintillation detector, the high-voltage generator, the Electron Accelerator itself. These devices are being put to use in this experiment to produce more advances in pure physics; without these technological devices the performance of this experiment would be impossible. Conversely, without the development of physics itself, these devices might never have been invented. It is a never-ending interplay.

Most of the equipment that is seen in the film are technological devices which are not based on the physical laws being tested. Such equipment is often referred to as "hardware." It is the construction of the hardware which is so costly, and yet it would be quite impossible to do experiments like this without it.

In a sense, the film does not and cannot show new physics "being done." It shows construction of new equipment, on the basis of known laws. From the operation of this equipment the "new physics" will be fashioned in the minds of the experimenters, where no film can enter.

Chapter 1 The Language of Motion

1.1 The motion of things.

The most significant case of motion in the development of science is not mentioned in this section, although it forms the main topic of Unit 2. The first, and perhaps philosophically most urgent, scientific problem facing man dealt with motion in the heavens. From earliest times, men questioned the nature and causes of the motions of the various astronomical bodies. As it finally turned out, Galileo's study of the motion of objects at the earth's surface (terrestrial motion) led to an understanding of the motions of the inaccessible heavenly bodies.

Thus, an understanding of the basic concepts of motion as formulated in seventeenth-century physics is taken up at the beginning of this course. These concepts are still useful in explaining and understanding much of the physical world that surrounds us. Moreover, the concepts have historical importance.

Do not devote much class time to the justification of starting the course with the study of motion. The students will not yet know enough physics to know what alternatives there are. It is more crucial to get the course going quickly; raise interesting questions and encourage student participation. (Some teachers, after having taught the course one year, have preferred to start with Chapters 5 and 6 and part of Chapter 7 to establish motivation for study of motion.)

Motion goes on about us all the time. Sometimes it is complex and confusing, or it may have regularities that make it simpler to understand and classify. To make students begin to think about the motions around them, ask them to classify several motions into those which are regular and those which are irregular. For example: a pendulum, a sewing machine, a leaf blowing on a tree, a bird in flight. Some motions may contain both regular and irregular features.

Just at an intuitive level, students will certainly recognize that while events may be commonplace—such as falling leaves or flying birds—they are not necessarily simple. As a first approximation, the motion of one object is more complicated than that of another if it is undergoing more erratic changes of direction and/or speed. In the long run, however, the distinction depends upon experience and our ability to find functional relationships with which to describe events or statements. Generally, when we undertake investigations in a relatively new field, we look around for what appear to be simple, straightforward examples of the phenomenon being investigated. The simplicity may later on turn out to be deceptive, but at least we have made a start. We can correct ourselves later.

The Greeks took uniform circular motion, rather than uniform straight-line motion, as the simplest. Both their physics and their metaphysics helped to direct attention to circular motion as primary. It is pointless to debate which is really simpler—but progress in physics was greatly helped by adoption of the latter view in the early and middle part of the seventeenth century.

1.2 A motion experiment that does not quite work.

Section 1.1 ends with a suggestion that we can learn from experiment. Section 1.3 suggests an experimental means for establishing regular time intervals, measuring distance as a function of time, leading to the definition of speed.

Section 1.2 is a bridge to help the student bring his intuitive feelings about motion and speed into an experimental environment.

1.3 A better motion experiment.

The main burden of teaching the student how to interpret and use strobe photographs rests on the laboratory and audio-visual aids under the direction of the teacher. The treatment in the book is not sufficient by itself.

Quickly get the students started on an analysis of motion and on laboratory work associated with it. Don't start the course with protracted philosophical discussions about the role of experiment or the nature of simplicity. After they understand more physics, there is time enough to come back to some of these questions.

Experiments are done so that events can be manipulated and made "simple." Furthermore, they can be reproduced and done over and over again while measurements and observations are made.

It is in the laboratory that students should learn about the role of laboratory in science. Laboratory experiments, which sometimes may seem very artificial and almost trivial, do lead to understandings which better explain the complex and interesting events seen in the world outside of a lab. If, however, we begin with the study of complex motions, such as falling leaves, we may never find the regularities for which we search.

If you use the student activity "Electronic stroboscope" (p. 19 in Student Handbook), develop the idea of "freezing" motion. However, this need not be done rigorously; the idea that regular motion viewed at regular intervals can cause the motion to seem to stop or move slowly will be enough.

1.4 Leslie's "50" and the meaning of average speed.

This section introduces the crucial definitions of average speed and interval and applies them quantitatively to a real motion of a kind that may have interested the student before.

The preceding section dropped rather casually the profound and essential notion that all measurements are approximations. This statement is incomplete, of course, until it specifies what measurements are approximations to. In this section, there is a clear example of one measurement (the overall average speed) which is an approximation to each of several other measurements (the average speeds over the 25-yard intervals). Likewise, each of these average speeds is presented as an approximation to "what (really) happened" at every moment of the motion. This may well be the student's first exposure to the idea of an experimentally unreachable concept that can nevertheless be approached, one step at a time, as nearly as one wishes, until one's measuring instruments are no longer good enough to improve the picture further.

1.5 Graphing motion.

This section presents no more than a bare outline of constructing and interpreting graphs related to speed. For a good many students this will be extremely elementary, and for them this section may be enough.

In addition to helping students to use graphs correctly, an effort should be made to get them into the habit of trying to interpret them physically. For example, you would like students to be able to look at a graph such as the one on page 23 and to describe in words the physical behavior of the balloon: its relative rate of ascent at various altitudes, the highest point reached, uniformity of rate of ascent, descent, etc.

PSSC Lab I-4 is an excellent exercise in graphing which can be done either under classroom supervision or as a homework assignment.

Below are some general rules which should be observed in plotting a graph. This format is far from complete and is used as a minimum standard. The following four ideas should be stressed with your students.

1. Proper graph format. Each graph prepared should include a title, experiment name or number, student name and date, presented in block form near the top of the graph.

Each axis should be labeled clearly with the quantity plotted and the unit of measure used. The scale values should also be clearly given. All of this information should be easily read from the lower right-hand corner of the graph sheet without rotating the page.

2. Size. All graph presentations should be large enough to show clearly the behavior of the quantities being plotted. The number of points included in the graph should also affect the choice of size. Poor choices include numerous points shown on too small a graph as well as very few points presented on too large a graph.

3. Scales. The choice of scale on any graph is arbitrary but should be made to maximize clarity. The range of values to be plotted should determine the placement of the origin and the maximum scale value for each axis. Whenever possible the scale should be chosen such that decimal multiples (such as 100, 10, 1 or 0.1) of the units being graphed can be located easily along the axis.

4. Plotting techniques. Experimental points should be plotted with small sharp dots. To avoid "losing" a data point a small circle should be drawn around each point.

The uncertainty in the values on the coordinate axes can be indicated by the size of the circle used, or perhaps by the length of horizontal and vertical bars drawn through the point as a cross.

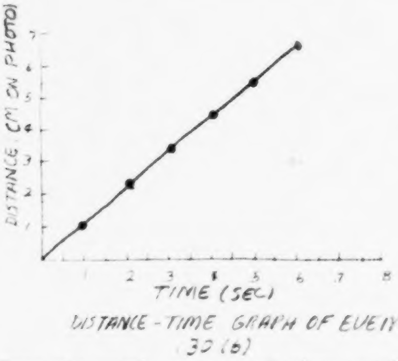
Seldom will data points fall on a really smooth curve. Whenever there is some reason to believe that whatever is being graphed actually does change smoothly a smooth curve should be drawn as close as possible to the data points plotted. Encourage students to consider what is implied in a broken line graph connecting data points in contrast to the implications of the smooth curve.

When two or more curves are plotted on the same graph students should use different colors for each curve, or dotted or broken lines may be used. In either of these cases a key or legend should appear on the graph defining the use of each line.

It may be very helpful to collect poorly prepared graphs during the year for display and file. Well-prepared and informative graphs might also be collected for these reasons but may be of greater value when returned to the student for his own reference. A poor graph is likely to be of little use to him.

1.6 Time out for a warning.

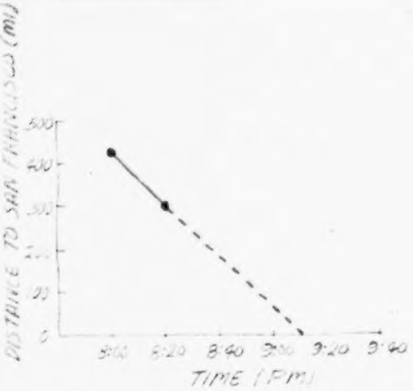
The process of estimating values between data points is called interpolation.



DISTANCE-TIME GRAPH OF EVENT 32 (b)

From the graph, we can see that at time .25 seconds the distance traversed is about 2.8 centimeters.

The process of predicting values that extend beyond the range of data points is called extrapolation.



From the graph we can see that the plane will be about 60 miles from San Francisco at 9:00 p.m. if it does not change speed and direction.

A discussion of the uncertainties involved in interpolation and extrapolation might be warranted. Stress the fact that interpolation is usually more reliable than extrapolation. Both are risky and should be undertaken with care and the values should not be ascribed greater certainty than is warranted.

The danger of extrapolation can be illustrated with a rubber band (about 3 inches) and a set of weights. Suspend the rubber band and load it successively with heavier and heavier weights, recording and graphing the amount of extension for each weight. After suspending 500 grams ask the students to predict by extrapolation the extension of the rubber band when the 1000-gram weight is suspended. The actual extension will turn out to be much less than the extrapolated value because the elastic characteristics of the rubber band change. This will appeal to your students, and it makes the point that not all graphs are linear. (You will find a graph like this in the Student Handbook on page 1.)

Note: the particular selection of weights will depend on the size of the rubber band available.

1.7 Instantaneous speed.

The concepts of instantaneous speed and limit are introduced here, although the latter concept is not given its customary name. The now-familiar notion of average speed is associated with its graphical representation as the slope of a chord joining the end-points of an interval on a distance-time curve. The student should already know about tangents from geometry, and this section should guide him to identify the slope of a tangent as a graphical representation of an instantaneous speed.

It may be puzzling to some students (and even disturbing to others) to learn that their common-sense notion of instantaneous speed corresponds to nothing that can be specified exactly in the "exact" science of physics. They may even resist the idea and thus miss the conceptual leap mentioned in lines 25-6 of page 26. Instantaneous speed is a conceptual "invention." It is justified in physics by its usefulness in describing and explaining motion and its consistency with other physics concepts. The point is not that there really is or isn't such a thing, but that the idea is fruitful.

Although the idea of speed is introduced with a car speedometer, a car speedometer does not give instantaneous speed any more accurately than our method. It also averages over a time interval. (Note the lag of a speedometer in registering as you begin with large acceleration.)

1.8 Acceleration—by comparison.

The scalar definitions of average and instantaneous acceleration are presented by analogy with the speed definitions. The common-sense basis of Galilean relativity also appears, but only casually.

Since Galilean relativity is going to play a major role in later chapters, it would be well to pause here and make a point of the fact that there is a real qualitative difference between speed and acceleration that the equations do not show (at least at this level). Every student's experience with carnival rides and automobiles is wide enough so that he can recall incidents that will make the distinction real to him, if he is stimulated to search his memory for them. If he comes out of this section convinced that "you can't tell when you're moving uniformly, but you can tell when you're accelerating," he will be better ready to tackle the physics of Galileo and Newton when the time comes.

Beginning physics students are sometimes confused by the units for accelerations, e.g., m/sec^2. It may be helpful to show that the units come from the definition of acceleration, $\Delta v/\Delta t$, and change in velocity per unit time, $\frac{cm/sec}{sec}$, is just written for convenience as cm/sec^2. If your students can't get comfortable with the "squared time," stick with the more obvious expression—e.g., m/sec/sec. Conventions of notation are the least important aspects of physics you can teach, and ought not to be purchased at the cost of understanding the ideas.

Chapter 2 Free Fall — Galileo Describes Motion

SEC. 2.1: THE ARISTOTELIAN THEORY OF MOTION

Background for Chapter 2

Many discussions of Galileo and his study of mechanics are quite critical of Aristotle. It is, perhaps, as unfair to condemn Aristotle for not accepting what the vacuum pump would prove as it would be unfair to criticize Galileo for not discovering radio astronomy or radioactivity.

It should be pointed out that the physics inherited by Galileo is really a very different and advanced kind of physics compared to the original work of Aristotle.

A frequently overlooked contribution to Aristotelian physics was made by the Arabs. After the decline of the Alexandrian period of Greek science (about 200 A.D.), the knowledge of the Greeks was not lost to the West. During the so-called Dark Ages in Europe, there was great activity in the Arab world. From the eighth century through the twelfth, there was considerable scientific and scholarly work done by the Muslims. Working in Damascus, in Baghdad, in Cairo and ultimately in several centers in Spain, the Muslims modified the work of Aristotle (and other Greeks) in many ways. Furthermore, the Muslims were influenced by the studies of the Persians, Hindus, Chinese and others in the East, and by certain Christians from the West.

Muslim science flourished in Toledo, Cordoba and other Spanish cities. As these cities were gradually reconquered by the Christians during the eleventh to fifteenth centuries, Muslim and ancient Greek knowledge filtered into Europe.

Before the time of Galileo, Aristotelian science had been blended with Christian philosophy, particularly by Thomas Aquinas. There were, however, various criticisms and interpretations made during the later Middle Ages at Oxford, Paris, Padua and other centers of intellectual activity. You should be aware of these points not in order to bring them up in class for discussion but rather so you will not overemphasize the conflict between Aristotle and Galileo and thus seem to imply that nothing happened during the 2000 years separating these two great men.

For a more complete (but still not extensive) treatment of Aristotle's physics the student may be referred to any of the following:

1. C. B. Boyer, "Aristotle's Physics," Scientific American, May 1950.

2. M. R. Cohen and I. E. Drabkin, A Sourcebook in Greek Science, McGraw-Hill, 1948. See pp. 200-203 on natural and unnatural motions, and especially pp. 207-212 on falling bodies.

3. Alexandre Koyre, "Galileo," pp. 147-175, in Philip P. Weiner and Aaron Noland (editors), Roots of Scientific Thought, (Basic Books, New York, 1957). Pp. 153-158 clearly describe the Aristotelian theory of motion.

There are other papers in this anthology of Weiner and Noland that will help you. We recommend the book for your library and probably the school library also.

4. O. L. O'Leary, How Greek Science Passed to the Arabs, London, 1948.

5. S. F. Mason, Main Currents of Scientific Thought, Henry Schuman, 1953.

The influence of China, India and the craft tradition in medieval Europe, as well as the influence of the Arabic world, is outlined in Chapters 7 through 11 (pp. 53-98).

6. A. C. Crombie, Medieval and Early Modern Science, (Doubleday-Anchor, 1959). Volume I deals with the fifth through the thirteenth centuries; volume II treats the thirteenth to the seventeenth centuries.

2.1 The Aristotelian theory of motion.

The Aristotelian scheme is a complex and highly successful one. For approximately 2000 years the ideas in this scheme dominated the thinking of intelligent men.

He was, perhaps, the first to realize that an explanation of the universe must be based on careful descriptions and classifications of what was in it. He was primarily an encyclopedist, and his writings were authoritative accounts of what was known at the time in such widely diverse fields as logic, mechanics, physics, astronomy, meteorology, botany, zoology, psychology, ethics, economics, geography, politics, metaphysics, music, literature and mathematics. He was among the first to understand and to discuss such things as the principle of the lever, the concept of the center of gravity and the concept of density.

Aristotle's notion that the motion of an object moving with constant speed requires a force proportional to the speed is not true for an object falling in a vacuum. It is true, however, for an object moving in a viscous medium, and most terrestrial motion is in the air, a viscous medium. Remember that vacuum pumps were invented nearly 2000 years later.

Students certainly should not be required to learn the details of the Aristotelian or medieval physics of motion. There are, however, some general points that might well be emphasized. These are:

1. The ideas on motion appear as logical parts of a larger theory about the nature and structure of the universe. In a sense, the "grand structure" existed first and the various aspects of it could be learned by deduction from this grand structure. This is in contrast to the modern approach in which individual topics and disciplines are studied and only gradually merge into a larger, more comprehensive structure.

2. The rules governing the motion of bodies on or near the earth were different from the rules governing the motion of non-terrestrial objects. Thus it was the nature of objects near the earth to be stationary once they reached their "natural place." There was no conflict in saying at the same time that for stars and planets to move continuously in circles is their natural behavior.

3. The Aristotelian scheme was essentially qualitative and non-experimental.

The main Aristotelian ideas about motion survived for a long time for many reasons. One of the reasons was that they did not seem to violate "common sense." Even today the instinctive physics of most people is probably Aristotelian. For example, recall how hard it is to convince students and laymen that a 10-lb weight and a 1-lb weight will fall at essentially the same speed. Recall also the difficulties of teaching Newton's first and third laws, not to mention special relativity theory or quantum mechanics.

2.2 Galileo and his time.

The roots of Galileo's thinking extend far back to the Greek tradition. He was able to apply the traditions of Pythagoras and Plato in a new context and to give them new vitality. Galileo contributed greatly to shaping the new science, but he did not do it alone and, indeed, he never entirely escaped from the past.

A thumbnail biography of Galileo cannot do justice to his colorful life and career. Students who would like to know more about Galileo should be referred to one of the following:

1. Laura Fermi and G. Bernadini, Galileo and the Scientific Revolution, Basic Books, Inc., New York, 1961. Short and readable.

2. I. B. Cohen, "Galileo," Scientific American, August 1949.

3. E. J. Greene, One Hundred Great Scientists, Washington Press, 1964.

The time-line chart on page 42 is the first of a series of similar charts which will appear throughout the text. These charts are to help students place the man and the events into the larger context of history. Most students know something about Shakespeare and Rembrandt and Galileo but frequently they are not aware that these men were contemporaries. We hope that the students will get more perspective by being able roughly to relate Galileo to his contemporaries: to Henry VIII, Luther, Calvin, and to the founding of Jamestown, the first American settlement.

Under no circumstances should students be required to memorize the names or dates appearing on these charts. The names and events here represent only a sampling. Students might wish to add additional names to the chart.

2.3 Galileo's "Two New Sciences."

The mention of the Inquisition and Galileo's confinement may stimulate students to raise questions about this whole affair. Most of the controversy had to do with the concept of the solar system, that is, with Galileo's astronomy. More about this will be encountered in Unit 2.

Since the focus in this chapter is on one aspect of Galileo's study of motion at the earth's surface, it may be well to put off the more dramatic aspects of Galileo's career until Unit 2. However, for the student who might like to prepare himself on the issue, you can recommend:

1. Georgio de Santillana, The Crime of Galileo, The University of Chicago Press, 1955.

2. F. Sherwood Taylor, Galileo and the Freedom of Thought, London, 1938.

The dialogue of Sagredo, Simplicio and Salviati is a discussion of a new book on mechanics by an unnamed author who is a friend of theirs. The "eminent academician" who wrote the book is, of course, Galileo, whose views are presented through Salviati.

Several copies of the Dover paperback edition of the Crew and de Salvio translation of Two New Sciences (see bibliography under Galileo) should be on hand for students who wish to locate these quotations and follow them in greater length.

A technique used in present-day experiments can be mentioned in discussing the argument between Simplicio and Salviati. It is easier to determine the difference between the outcome of two simultaneous events than it is to compare them individually. For example, it is easy to tell which runner has won a mile race, even when one leads the other by only a foot. It would be more difficult to make this determination by timing two runners in separate races.

2.4 Why study the motion of freely falling bodies?

This brief section emphasizes that our main interest is in studying the approach used by Galileo. The quotations from Two New Sciences show that Galileo himself realized that his work was of significance and that it would lead to a new science of physics.

2.5 Galileo chooses a definition of uniform acceleration.

This section and the three sections that follow it deal with Galileo's free fall experiment. There is some danger that the student will get lost before he has reached the end. For this reason, the opening paragraph of Sec. 2.5 summarizes the over-all plan of attack. Point out, especially to students who are not accustomed to involved derivations or proofs, that it is important to consider the plan of attack before beginning to study such a logical argument.

The summary and the marginal commentary accompanying it should prove useful, yet it is potentially misleading. This, like all summaries, makes the events sound more organized and systematic than they were. Galileo is, after all, giving an ex post facto description of work which he did over a period of years. Furthermore, he invented that description in a controversial document that has wider aims than merely presenting some research findings. Take care, therefore, to see that the student does not accept these activities as a model for all scientific endeavor.

2.6 Galileo cannot test his hypothesis directly.

Frequently a direct test of a particular hypothesis cannot be made. One or more of the quantities involved cannot be measured accurately because the means for making the measurement has not yet been established.

Suppose that, in an attempt to test directly the hypothesis that v/t is a constant, Galileo had permission to use a 20-story building 65 meters high. Suppose that he put marks on the building at 1, 4, 9, 16, 25, 36, 49 and 64 meters from the top. An object dropped from the top of the building should pass these marks at equal time intervals of a little less than 0.5 sec.

But observing that the marks are passed at equal time intervals is not really a direct test that v/t is constant. For this, he would have to determine the instantaneous speed as the object passed each mark. (The distance used for these speed measurements should be very small.) Suppose instead that he considered a rather large distance, one meter. The time to cover the first space interval, from 0.5 to 1.5 meter, would be about 0.23 sec. If we assume that Galileo could measure to within 0.1 sec, his probable error for the first distance would be about half the quantity he was trying to measure. The time elapsed while the object moved through the last interval, from 63.5 to 64.5 meters, would be less than 0.03 sec. To measure this time interval to 5 percent accuracy would require a clock which one could read to about 0.001 sec, at least 10 and probably 100 times better than anything Galileo had available. No wonder he resorted to an indirect test.

Now there are many methods available to us, such as stroboscopic pictures or electrically driven timers, which allow us to test directly whether or not v/t is constant for a freely falling body. But all of these methods depend on our ability to measure small time intervals (0.001 sec) with precision. Such methods were simply not available to Galileo.

2.7 Looking for logical consequences of Galileo's hypothesis.

Part of the reason for the scientific breakthrough begun in the 16th and 17th centuries was use of a mathematical approach to the study of motion.

The text derives $d = at^2/2$ but you should repeat the derivation carefully in class. The main point of this section is not to teach the derivation; it is to emphasize the value of mathematics in science. Simple algebra allows us to arrive at a relationship that is self-evident at the beginning. While the final equation contains no new information, it presents the information in a different and useful way. For instance, it allows us to make predictions concerning distances traveled by accelerating bodies that are not evident in the parent equations.

The word "constant" is used in several ways in physics. In the context of this section, constant means: in uniform acceleration, the numerical value of the ration d/t^2 is the same (i.e., is constant) for each and every interval for which distance and time measurements are made, provided other parameters are fixed. The numerical value of that constant ratio will depend on the value of acceleration in a particular case.

2.8 Galileo turns to an indirect test.

This section contains what is probably the largest conceptual leap in Galileo's argument. He assumed that the inclined plane was primarily a device for diluting free fall without changing its fundamental nature, and he was able to proceed to experimental tests of his hypothesis. If the students can be made to see that this is a reasonable assumption, although not necessarily true, the inclined-plane experiment should not be difficult.

One of the reasons for including the photographs of students doing this experiment is to encourage all students to carry it out. With reasonable effort the students will find that for any distance along the incline, the ratio d/t^2 will be constant for a given angle of incline. As a practical matter the experiment is limited to relatively small angles.

In summary, the main purpose of this section is to make the association between the inclined plane experiment and the overall problem of free fall. An understanding of the actual experiment itself should come from the laboratory.

2.9 How valid was Galileo's procedure?

Healthy scepticism is one of the characteristics of scientists. Students from the beginning should be encouraged to be critical of scientific claims and experiments. In a text book it is difficult not to sound authoritarian from time to time. A section such as this is intended to counteract that tendency.

2.10 The consequences of Galileo's work on motion.

Chapter 3 The Birth of Dynamics – Newton Explains Motion

3.1 The concepts of mass and force.

Dynamics is introduced by contrasting it to kinematics. The distinction between these was mentioned earlier in Chapter 2 (p. 48) when Salviati (i.e., Galileo) said that the time to talk about causes of motion was after accurate descriptions existed.

One of the preoccupations of science is to provide systematic explanation of observable phenomena. Newtonian mechanics (Newton's laws of motion, the law of universal gravitation, and various force functions) represent one such explanatory system. As the students progress through this chapter, studying the three laws individually, they should be reminded from time to time of this overall theme of explanation.

3.2 About vectors.

The concept of a vector is developed briefly in this section. The text states but does not show that acceleration can be treated as a vector. Actually the section will not stand alone as a way for teaching vectors. Previous experience has shown that the text is not a particularly good device for teaching vector concepts. For this reason, most students will need to study the programmed instruction booklet on vectors.

As a minimum it is necessary that all students understand what vector quantities are and why they are important. They should also be able to do vector addition and subtraction graphically.

3.3 Explanation and the laws of motion.

The purpose here is to place the kinds of motion we studied in kinematics into perspective. These are motions which Newton's laws will attempt to explain.

3.4 The Aristotelian explanation of motion.

Aristotelian ideas concerning motion should be presented so as to appreciate better the Newtonian development. Contrast between the two will and should be made. Aristotelian and so-called commonsense-everyday observations should be connected.

3.5 Forces in equilibrium.

Develop the ideas of unbalanced force and equilibrium for the condition of "rest." This will set the stage for the equilibrium condition of constant velocity with no unbalanced forces in Sec. 3.6.

3.6 Newton's first law of motion.

The significance of the first law of motion cannot be overstated. Most physics textbooks point out that the first law is really a special case of the second one, since by the second law the acceleration is zero if the force is zero. While this is true, it misses the point.

The law of inertia is fundamental to modern mechanics, for it states what is to be the starting place of the entire theory of motion. The first law makes it perfectly clear from the beginning what is to be basic, what requires further explanation and what does not. In so doing, the law of inertia dramatically exposes the difference between the Newtonian system and the Aristotelian system.

The main points to be emphasized about the first law of motion are these:

1. Fundamentally, the law is a definition. It states the convention to be followed in studying forces. Forces are not to be considered as the causes of motion but rather as whatever creates acceleration.

2. The law of inertia cannot be proven by observation or experiment. One reason mentioned later in Sec.   , is that the ordinary method for deciding whether or not there are unbalanced forces operating is to observe whether or not there is acceleration.

Many teachers demonstrate the plausibility of the first law by showing that as the retarding friction on a moving body is reduced, the object appears to behave more and more in accordance with the first law. The teacher should be extremely careful not to pass off a classroom demonstration as proof of the first law: definitions are not to be proven.

3.7 Newton's second law of motion.

Force and mass are very difficult concepts to master. This section postpones a definition of those terms and avoids consideration of the empirical content of the second law. The section does not explain the equation

$$\vec{F}_{net} = m\vec{a}.$$

The student should understand that for a single object, a is proportional to F. For different objects acted on by a constant force, a is inversely proportional to m. We want the student to realize that if the second law is true, then certain mathematical relationships must exist between force and acceleration and between mass and acceleration.

The newton is the only force unit mentioned in this chapter. Perhaps the student should know that there are other force units depending upon the system of units being employed. However, there is little to be gained from comparing them or being able to convert from one to the other.

Perhaps it is worth noting that implicit in the equation F = ma is a proportionality constant that does not appear because it was set equal to 1 (i.e., F = Kma and K = 1). An alternative approach would have been to define a standard unit of force as well as standard units of mass and acceleration, and then do experiments measuring force, mass and acceleration from which the value of the constant could be computed. Examples where this approach is used include G in the universal gravitation equation, and the spring constant in Hooke's law equations.

3.8 Mass, weight and gravitation.

Why do all objects in free fall at a given location fall with the same acceleration a_g? The answer is in the proportionality between weight F_g and mass m. It is imperative, therefore, that students understand the distinction between weight and mass. The only thing that really makes this difficult is that they are accustomed to using the terms interchangeably and usually think of weight as a measure of mass. (There is some dispute among physics educators about how weight ought to be defined. Here we are using weight as synonymous with gravitational force.) Perhaps the relationship between mass and weight is easier to understand if the second law equation is put in the form

$$a_g = \frac{F_g}{m} .$$

Then it is clear that the force is proportional to the mass; no matter what the value of the mass, the acceleration will remain constant.

Why in a given location do objects fall with uniform rather than nonuniform acceleration? That the acceleration is constant is an experimental fact and not dependent upon the second law.

3.9 Newton's third law of motion.

The purpose of this section is to enable the students to understand what the third law says.

Once students grasp the notion that forces always appear or disappear in pairs due to the interaction of real objects and that the two forces act on different objects, then the rest is not difficult. However, these ideas are contrary to everyday experience and are not easily accepted. The tremendous inertia of the earth, the ever-present forces of friction and the imperceptible distortion of rigid objects (like floors and walls) all help conceal action-reaction.

3.10 Using Newton's laws of motion.

The first law of motion is mathematically a necessary consequence of the second law, while the reverse is not true. One can conceive that the second law of motion might have been very much more difficult to formulate.

The first law does not really take on meaning and is not at all useful in the real world of physics until certain additional operational definitions have been given for such terms as rectilinear motion, equal time intervals and constant speed. Also, a frame of reference must be established for all the measurements.

The main point to be emphasized is that while the first law provides a general explanation of an event, the second law provides a quantitative and therefore more useful explanation. For example, when we can say that an object slows down because there is a retarding force of 4.0 newtons acting on it, we know a great deal more than when we merely say that it slows down because there is a retarding force.

It may be difficult to persuade some students that numerical answers are actually useful. The text illustrates the usefulness of quantitative results by pointing out that once numbers are obtained, a large variety of comparisons can be made between actual events. It may be helpful to give specific examples in class.

The third law allows one to examine a small part of a complex chain of events. The hammer example should cause no real trouble. Students find it hard to believe that the earth can exert a force on a runner. One way of demonstrating the value of the third law at this point is to ask students to invent an explanation for the acceleration of a runner that is not like the third law in form.

If forces are equal and opposite, how can an object accelerate? The point to be emphasized in going over this example is that the opposite forces act on different objects. In discussing the forces between two objects in a system, the third law is needed. It describes the location and magnitude of various force pairs. On the other hand, when one becomes interested in the motion of a particular object, then he must ask about the net unbalanced force acting on that object and apply the second law to determine its acceleration. Distinguishing which law to use is not easy and students should be furnished with other examples.

3.11 Nature's basic forces.

Treat as a reading assignment. The section generalizes and extends the laws of motion and helps introduce Chapter 4. The four basic interactions in nature are mentioned to reduce the complexity the world so far seems to present and to point to what lies ahead in our study of physics.

4.1 A trip to the moon.

What is simple and what is complex is not altogether easy to decide. Certainly one's purposes for making such distinctions will have something to do with the criteria. In general, there are three criteria being used in this unit: two are uniformity and symmetry. If the parameters that describe the motion are uniform or constant in value, or if the path of motion is symmetrical, the motion is regarded as "simple."

The third criterion has to do with dimensionality. Motion becomes more complex as you go from one or two to three dimensions. By this, projectile motion and uniform circular motion can be considered as being more complex than rectilinear motion and less complex than, say, helical motion.

4.2 Projectile motion.

Not much is gained by emphasizing the definition of projectile motion. Students should understand that a projectile is an object moving through space without the aid of any self-contained motive power.

The historical significance of the problem of projectile motion does not receive as much emphasis as it could in this section or in the chapter as a whole. Many historians of science feel that it was one of the key issues in the whole controversy over the nature of motion. Aristotle's theory was least able to explain projectile motion.

The concept of independent horizontal and vertical motion may be a difficult concept for students to accept because it conflicts with their commonsense notion that horizontal speed affects the rate of fall. The student should carry through the analysis argument. Demonstrate the apparatus that projects one sphere and drops another at the same instant (see Student Handbook). Also, students should make their own measurements, or at least see measurements made on photographs or transparencies similar to the one on page 97.

Two major and quite separate points need to be made.

1. It is an undeniable experimental fact that a short-range projectile launched horizontally will reach the ground at the same time as a similar object dropped at the same instant from the same height. This fact, that the gravitational acceleration of a projectile is exactly the same as the gravitational acceleration of an object falling freely from rest, comes from observation, not from deduction from first principles.

2. This experimental fact can be explained or rationalized by assuming that the observed motion of a projectile is the vector sum of two other motions which are completely independent of each other: uniform horizontal motion and accelerated vertical motion.

Students who are interested in projectile motion and who understand some trigonometry ought to be encouraged to analyze the general case of projectile motion, i.e., the case in which the projectile is launched at any angle. Once they have derived what they believe to be general equations, they should show that the equations used in this and the next section can be deduced from the general equations.

4.3 What is the path of a projectile?

The purpose of this section is to establish and demonstrate the power of mathematics in science and to justify the need for continued scholarly work in pure mathematics. In order to do this effectively, it is important that prior to the derivation of the equation of the parabola, two other points be made.

1. There is no a priori reason to favor one curve over another. In fact, there is no reason even to suppose that the trajectory of a projectile will always have the same mathematical shape.

2. The question cannot be settled simply by observing the path of projectiles with the unaided eye. For one thing, the angle of observation and problems of perspective make observation difficult. Secondly, many mathematical curves look very much alike and can be distinguished only by analysis. Finally, many objects that are thrown do not follow a parabolic path because of the large and changing air resistance they encounter along the flight path.

The difficulties in determining the shape of the projectile's trajectories can be easily demonstrated by throwing objects inside the classroom, or, even better, out on the playing field.

The support that experiment gives to the purely mathematical combination of motion can provide the student with evidence he may need more than we think: evidence that mathematical manipulation of symbols which express known principles can lead to new relations among the symbols—relations which are also true.

With a particularly able class, the teacher might wish to develop the relationship between range of a projectile and its velocity and angle of fire. This problem could also be assigned as a project for better students who are familiar with trigonometry. See "Projectile Motion" in Foundations of Modern Physical Science by Holton and Roller.

4.4 Galilean relativity.

For the first time in the course the frame of reference becomes important— and even here the term is not used. Make certain the student realizes that any motion he sees and analyzes *does* depend on the relative motion of the viewer. The *laws of motion*, however, are the same for *all reference* frames moving uniformly with respect to each other.

"The Perception of Motion," by H. Wallach, in the July 1959 *Scientific American* may be of interest. It concerns the fact that people view relative motion as if it were absolute. (For student or teacher.)

4.5 Circular motion.

This is an introduction to the study of circular motion and does not deserve great emphasis. However, it is probably worth demonstrating the difficulty of deciding whether or not an object is in circular motion when observing it from a frame of reference which is moving with respect to the object.

For some interesting results of being located on the earth in a non-inertial frame of reference, see the article by J. McDonald, "The Coriolis Effect," in the May 1952 *Scientific American*.

4.6 Centripetal acceleration.

The difficulty in this section is to show that the acceleration of an object moving uniformly in a circle is truly centripetal. The text gives only a plausibility argument and while it will convince some students, there may well be skeptics. In most classes it may be worthwhile for you to go through the derivation on the blackboard, or use transparency T11.

Material is included to provide the students with practice in thinking in terms of vectors. Also it provides an opportunity to review and compare the three kinds of motion dealt with: rectilinear, projectile and circular.

This might also be an appropriate time to suggest that circular motion is really a special case of projectile motion. This can be done in one of two ways. One way is to compare the vector relationship of an object moving with uniform circular motion to the relationship of a projectile at the top of its trajectory. The second way is to approach it through the use of a diagram such as Newton used (the figure on page 92).

The relationship $a = v^2/R$ is used to carry out an arithmetic solution to the same uniform circular motion problem solved graphically in the text. The derivation was omitted in order to provide some of the brighter students with an opportunity to work it out for themselves. One or more students can be assigned to do this and present the results to the class the next day.

If a sample problem is worked in class, an interesting example might be to find the acceleration of a point on the earth's equator due to the rotation of the earth ($r = 6400$ km, $t = 24$ hr $= 8.64 \times 10^4$ sec). The value of this calculation can be compared with the value of the acceleration of gravity. The question can then be asked: what would happen if the earth were rotating at a speed such that the centripetal acceleration were equal to the acceleration of gravity? This idea will be taken up again in Chapter 4.

This would also be a good place to review the facts concerning projectile motion.

1. The acceleration is always constant and always directed downward. This is true when the velocity of a thrown object is directed upwards, directed downwards or is at the point at the top of the trajectory when the vertical velocity is zero.
2. The horizontal component of the velocity is always constant.
3. The horizontal displacement is given by the relationship $d = vt$.
4. The vertical velocity is given by the relationship $v_i + at$.
5. The vertical displacement is given by the relationship $d = v_i + \frac{1}{2} at^2$.

4.7 The motion of earth satellites.

There is no new physics in this section. What the student has learned about circular motion up to this point is almost entirely theoretical or, at least, deals with cases (e.g., a blinky on a turntable) about which most students care very little.

The satellite Alouette is used because it has a nearly circular orbit and because it has some historical significance.

The trouble with a list such as the one in Table 4.2 is that it becomes obsolete almost as fast as it is printed. An effort was made to select satellites of continuing interest. However, the latest entry is over four years old. Perhaps some students should be assigned the task of finding the additional three or four entries needed to bring the list up to date. See Sky and Telescope magazine for frequent articles on satellites.

The important questions that will really show whether the student has learned what has been covered up to this point are: "Why does the satellite not fall back to earth?" and "Why does a satellite not fly off into outer Space?"

After most of the students in the class have been able to answer this successfully in terms of the kinematics of circular motion, they may be impressed by the progress they have made by asking them to respond to those questions as the Aristotelians probably would have.

The section ends by suggesting that the relationship between speed, distance above the earth and period of rotation are not independent variables. This is probably not the time to take up questions about how satellites get from one orbit to another, and what effect this has on their speed. However, it may not be possible to avoid the issue altogether, especially if some dramatic event has recently happened.

Although satellite orbits (i.e., planetary orbits) will be taken up in greater detail in Unit 2, the statement is made in this section that at a particular height a satellite must have a certain velocity in order to maintain a circular orbit. The question: "Why have all of the satellite launchings to date been in an easterly direction?" is useful. The answer involves having the students think about vector addition of velocities and at the same time think in terms of a frame of reference connected to the center of the earth rather than the more familiar one, that of the individual on the surface of the earth.

4.8 What about other motions?

This section should be treated merely as a reading assignment. Its only purposes are to remind the student that there are many interesting kinds of motion that we have not dealt with.

Demonstrations

D1 Recognizing Simple Motions

The following demonstration can aid in introducing the material in this section. Perform the events listed below. Ask the students to select the event which would be the best starting point for a study of motion. Ask them to give reasons for their selection.

a) roll a football

b) roll a marble

c) drop a sheet of paper

d) bounce a ball

e) swing an object around in a circle

D2 Uniform Motion, Using Accelerometer and Dynamics Cart

Tape a large liquid surface accelerometer to a dynamics cart and show the students that the water surface is horizontal when the cart moves with uniform motion (as when pushed by a toy tractor). Do the same with the cork-in-bottle accelerometer (see Student Handbook page 40), stressing the fact that the cork remains vertical when the motion is uniform. Contrast uniform motion with the rest condition.

D3 Instantaneous Speed, Using Strobe Photos of Body on Spring

In this demonstration-activity the class analyzes a complex motion—that of a body on a spring—which is definitely non-uniform. Simple equipment is used to develop step by step the quite sophisticated concept of instantaneous speed, introduced in Sec. 1.7 of the text. A stroboscopic record is made of one-half oscillation of the body-spring assembly and this record is used to estimate the instantaneous speed of the mass at one point of the observed oscillation.

Equipment

Body-and-spring assembly, hung so as to oscillate freely, with a light source taped to the oscillating body and a sliding pointer arranged so as to indicate the point of interest. (See Fig. 1.)

Polaroid camera
Motor strobe and disc
  or xenon strobe
Overhead projector, for projection
  of print. You may find transparency
  T3 useful for recording and analyzing the data.

Procedure

The body-spring assembly is shown to the class, extended, released and allowed to oscillate briefly. A problem is posed orally to the students: "How fast was the body moving?" Stated in the terminology of Sec. 1.7, this question is: "What is the instantaneous speed v of the body?"; the students will recognize that the body had different speeds at different instants and that they can't even begin to answer your question until you make it more specific. Now choose a point fairly near (but not at) the end of the oscillation and ask for the speed at that point. Attach the pointer to mark the point of interest. If we could tie a speedometer to the body, we could watch and record its reading as it passes the pointer. Since we cannot do this, we have to estimate the value of v from distance and time measurements. Two possible approaches are suggested in the text.

1. Measure the average speed $v_{av} = \Delta d/\Delta t$ over some interval that includes the point of interest. Begin with long time intervals and then progressively shorten the interval until there is no longer any trend in the values of v_{av} as the interval is reduced still further. This final value of v_{av} is, within the experimental uncertainty, equal to the value of the instantaneous speed v. (Note that as the distances and time intervals measured become smaller, the percentage uncertainty in v_{av} increases. Therefore, for small enough Δt, the calculated values of v_{av} will have random variations due to experimental uncertainties in Δd and Δt.)

2. Make a graph of displacement against time. Draw a tangent to the curve through the chosen point P and compute its slope, which is approximately the instantaneous speed at P.

Method (2) is a straightforward exercise in graphical analysis of a complex straight-line motion. Since the drawing of tangents to curves is not a very precise operation, many students will get more satisfying results from method (1). Method (1) also has the advantage of emphasizing the concept of approaching a limit.

1) Approaching the limit

Set up the body-spring assembly and camera as shown in Fig. 1. The best strobe rate to use will depend on the characteristics of your spring, and the mass of the body used; try a rate of 30 per second (6-slot disc, 300-rpm motor). You will want at least fifteen or twenty intervals to measure.

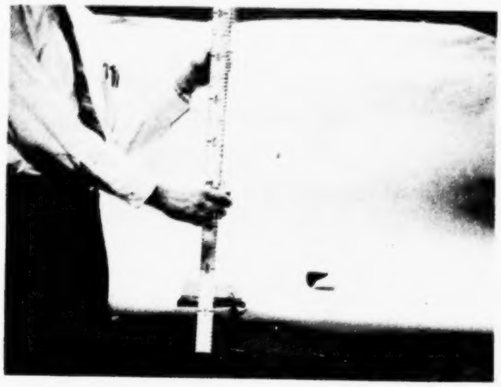


Fig. 1

Alternatively, if you have a polished steel ball that can be attached to the spring, a xenon strobe light gives good results.

With the apparatus aligned and the lights out, extend the spring by pulling straight down on the body. Open the camera shutter just before releasing the body, and then close the shutter again just as the body reaches its highest point and starts down again (to avoid the confusion of overlapping traces).

Hints on photography and techniques for making the information on a single photograph quickly available to the whole class are discussed in the notes on photography in the Equipment Notes part of the Teacher Guide following Unit 1.

Calculate $v_{av} = \Delta d/\Delta t$ for several intervals containing point P and having one end point in common (see Fig. 2, asymmetric intervals). Start with a Δt of 20 or so time intervals and work down to two intervals. Have some of your faster students repeat the process for a different interval; others might try symmetric intervals (see Fig. 2).

Sample results are shown in Table 1, as measured from a stroboscopic photo as shown in Fig. 2. Precision of this order is obtained by using the 0.1 mm scale and magnifier. (One millimeter in Fig. 2 represents one centimeter in real space.)

TABLE 1

t (in intervals)	d (mm on photo)
0	0.0
1	0.5
2	1.6
3	3.1
4	4.8
5	7.0
6	9.5
7	12.2
8	15.0
9	17.8
10	20.7
11	23.6
12	26.3
13	28.9
14	31.1
15	33.0
16	34.5
17	35.6
18	36.5

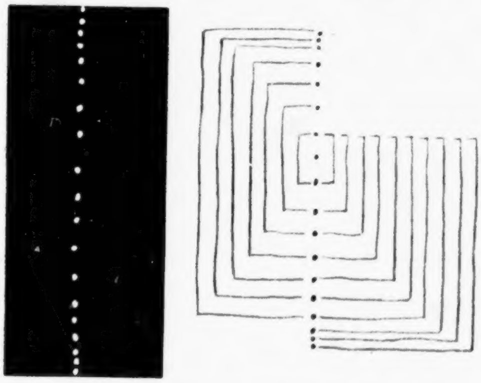


Fig. 2 A facsimile of a typical strobe photograph of the motion of a mass on a spring, showing two possible ways of choosing a set of decreasing time intervals.

TABLE 2

Δt one interval is 1/30 sec	Δd mm (on photo)	$v_{av} = \frac{\Delta d}{\Delta t}$ mm/$\frac{1}{60}$ sec	v_{av} cm/sec (real space)
18 intervals	36.5	2.03	60.9
16 intervals	35.1	2.19	65.7
14 intervals	32.9	2.35	70.5
12 intervals	29.9	2.49	74.7
10 intervals	26.3	2.63	78.9
8 intervals	21.9	2.74	82.2
6 intervals	16.8	2.80	84.0
4 intervals	11.4	2.85	85.5
2 intervals	5.7	2.85	85.5

We see in this example that the value of v_{av} does not change as Δt is decreased below six intervals. This value of v_{av} is equal, within the precision of this experiment, to the value of the instantaneous speed v at the point P at the center of each of the intervals tabulated above.

Have students graph the results tabulated in Table 2: average speed vs. size of time interval (Fig. 3). Ask students:

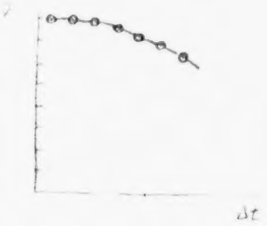


Fig. 3

"What would we find if we could make measurements over even smaller time intervals?" Is there a point on the curve whose value represents the instantaneous speed? You may be able to suggest that it is reasonable (since the body doesn't suddenly speed up or slow down at P) that v is the point where the curve would cut the v_{av} axis, and that in this case (the case of a limiting process) it is legitimate to extend the curve (extrapolate) to that axis. The important idea of extrapolation must be introduced with some care and a variety of examples.

2) Estimating v graphically

Graph d vs. t directly from Table 1. Draw chords centered on a chosen point P (corresponding to the intervals of Method (1) above) and compute their slopes, which are the various values of v_{av}. Construct a tangent to the curve at point P, and compute its slope as the value of v at P. The slope of the tangent at any point gives the value of v at that point.

Estimate the slope of the tangent at each of the data points, and plot a graph of v vs. t. Repeat the process, estimating the slopes of the v-t curve at the data points, and plotting acceleration vs. time. (See Fig. 4.)

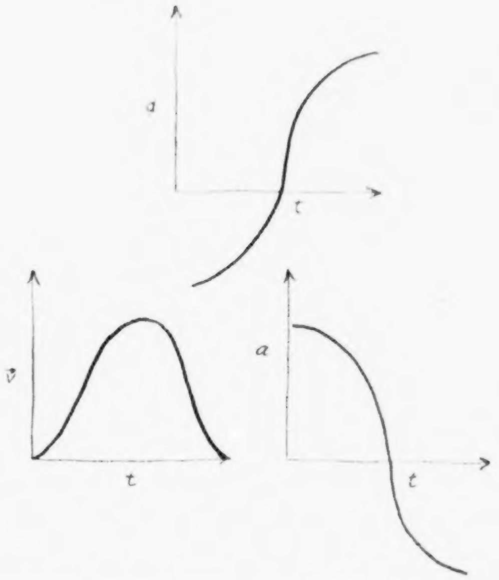


Fig. 4 Typical plots of d, v and a against t. The lowest point of the body's motion is taken as d = 0, t = 0.

Ask students, "At what point was the mass moving fastest? Was it at one of the data points? How accurately do you know where and when the maximum speed occurred?" Ask similar questions about the time (and position) of zero acceleration.

D3 Instantaneous Speed (Alternative)

As an alternative to the body-and-spring demonstration, a pendulum swing can be analyzed. In E4 (Uniform Motion) students measured the average speed of a bulldozer over long and short time intervals, and probably concluded that the bulldozer was moving with nearly constant speed. In this demonstration-experiment the class looks at and analyzes a more complex motion—the swing of a pendulum—which is definitely non-uniform. Simple equipment is used to develop step-by-step the quite sophisticated concept of instantaneous speed, introduced in Sec. 1.8 of the text. A value for the instantaneous speed of the pendulum bob at the bottom (center) of its swing is estimated experimentally.

Equipment

Pendulum, about 50 cm long, hung from rigid support
Polaroid camera
Motor strobe disc and light source taped to pendulum bob,
  or ac blinky,
  or xenon strobe
Overhead projector, for projection of print
Flexible scale, for measuring projection of print

Procedure

A pendulum is shown to the class, drawn back, released and allowed to describe a full arc. A problem is posed for the students: "How fast was the pendulum bob moving at the very bottom of its swing?" Stated in the terminology of Sec. 1.8, this question is: "What is the instantaneous speed v of the bob at the lowest point P?" This is the reading we might get from a speedometer at the moment of passing through the bottom position if we could possibly attach one to the pendulum. Since we cannot do this, we have to estimate the value of v from distance and time measurements. Two possible approaches are suggested in the text.

1. Measure the average speed, $v_{av} = \frac{\Delta d}{\Delta t}$, over some interval centered on the point P. The pendulum clearly moves more slowly the farther away it is from the bottom point. Therefore the longer the interval over which v_{av} is measured the lower v_{av} will be. All values of v_{av} will be less than the speed right at the bottom. To get an estimate of the instantaneous speed we must progressively shorten the time interval until there is no trend in the values of v_{av} as the time interval is further reduced. This value of v_{av} is, within the experimental uncertainty, equal to the value of the instantaneous speed v at P. (Note that as the distances and time intervals measured become smaller the percentage uncertainty in v_{av} increases. Therefore for small enough Δt, the calculated values of v_{av} will have random variations due to experimental uncertainties in Δd and Δt.)

2. Make a graph of displacement against time, and draw a tangent to the curve through the points for the highest observed velocities. The slope of the tangent is approximately the instantaneous speed at P.

The drawing of tangents to curves is not a very precise operation. For this reason, and because it emphasizes the idea of the approach towards a limit the first method is recommended.

Approaching the limit

There are several alternative experimental procedures here. The one described first is the simplest experimentally.

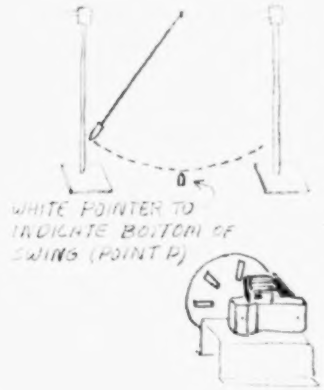


Fig. 1

Set up the pendulum, light source and camera as shown in Fig. 1. Use a strobe rate of 60 per second (12-slot disc, 300-rpm motor). Alternatively you may use the ac blinky, with some added mass, as the pendulum (in which case of course you do not need the strobe disc in front of the camera). It is important to set up a marker to indicate the bottom point of the swing, and to have a rigid stop so that the bob can be drawn back to the same position for each release. (Obviously the instantaneous speed at the bottom point depends on the amplitude of the swing.) Be careful not to pull down on the string prior to release—the stretch will disturb the motion of the bob. Photograph as much of the swing as possible. Be sure to close the camera shutter before the bob begins the return swing, and thus avoid the confusion of overlapping traces.

Hints on photography and techniques for making the information on a single photograph quickly available to the whole class are discussed in the notes on photography.

Calculate $v_{av} = \frac{\Delta d}{\Delta t}$ for several intervals centered on the bottom-most point or the bottom-most interval, depending on the particular photograph (Fig. 2). Start with a Δt of between 30 and 40 time intervals and work down to three or two intervals. The distance intervals Δd are measured along the arc, which requires using a flexible scale.

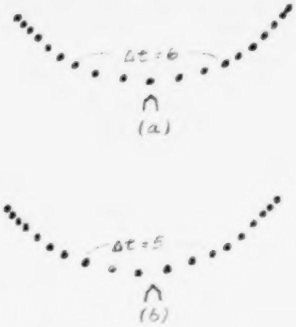


Fig. 2 (a) Trace is symmetrical about bottom-most point. (b) Trace is symmetrical about bottom-most interval.

Sample results are shown in the table. (One millimeter on the photograph represents one centimeter in real space.)

TABLE

Δt	Δd	$v_{av} = \frac{\Delta d}{\Delta t}$	v_{av}
one interval is 1/60 sec	mm	(mm/$\frac{1}{60}$sec)	cm/sec
	on photograph		real space
26 intervals	84.0	3.23	194
22 intervals	76.0	3.46	208
18 intervals	66.0	3.66	220
14 intervals	53.5	3.82	229
10 intervals	39.5	3.95	237
6 intervals	24.0	4.0	240
2 intervals	8.0	4.0	240

We see in this example that the value of v_{av} does not change as Δt is decreased below six intervals. This value of v_{av} is within the precision of this experiment equal to the value of the instantaneous speed v at the point P.

Alternative Procedure

A slight variation perhaps emphasizes more clearly that what we are doing here is measuring v_{av} over successively shorter time intervals. A series of strobe photographs are taken, each one at a higher strobe rate (smaller time interval between images) than the previous one. This is most conveniently done with the light source and disc strobe method by progressively opening up more slots. In this method one measures v_{av} over the lowest interval only on each trace. At the lowest strobe rate(s) it may be impossible to find an interval that is adequately centered on the bottom point. The change between the value of v_{av} over the longest Δt and its value over the shortest Δt will be less in this method, because the range of time intervals over which the measurement is made is less.

A calibrated xenon strobe and steel ball pendulum bob could be used for this method. Yet another possibility is to feed the ac blinky with various known frequencies from a (calibrated) audio oscillator via an amplifier and transformer. (Remember that the neon lamp does not glow below about 70 volts peak voltage.)

Possible extensions

1. Procedure (2) above: plot d against t, draw chords centered on P to find various values of v_{av}, and draw the tangent at P to find the value of v at P. The slope of the tangent at any point gives the value of v at that point (see Fig. 1.25 in text).

2. Plot a graph of the results obtained above—average speed against time interval (Fig. 3).

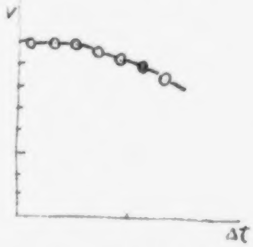


Fig. 3

Ask students: "What would we find if we could make measurements over even smaller time intervals?" Can they find a point on the curve whose value represents the instantaneous speed? You may be able to suggest that it is reasonable (since the bob doesn't suddenly speed up or slow down at P) that v is the point where the curve would cut the v_{av} axis, and that in this case it is legitimate to extend the curve (extrapolate) to that axis. The important idea of extrapolation must be introduced with some care and a variety of examples.

3. There are many other measurements that can be made with this simple experimental setup. It could be instructive for students to make graphs of d against t, v_{av} (measured over one interval) against t, and if possible of acceleration, a, against t. (Measure t and d from the point P, Fig. 4.)

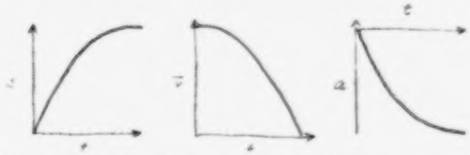


Fig. 4 Plots of s, v_{av}, a against t. The bottommost point of the pendulum swing is taken as $s = 0$, $t = 0$.

4. The concept of instantaneous speed will come up again in Unit 3. The kinetic energy of a body at a given instant depends upon its speed at that instant. The interchange of potential and kinetic energy in a pendulum will be referred to specifically. It is worthwhile to make photographs that encompass both the topmost point, the bottom point P, and some scale to give absolute measures of distance. The photographs should be kept for use later in Unit 3.

Questions for discussion

1. Could one ever measure v experimentally? How?

2. A car speedometer appears to measure instantaneous speed. Does any student know how it works? How is it calibrated? (This is done at constant speed, i.e. by measuring Δt for a known Δd while the speedometer reading is unchanging. So really all that we know is that the speedometer tells us instantaneous speed for the special case of uniform motion, that is when $v = v_{av}$ at every point.)

D4 Uniform Acceleration, Using Liquid Accelerometer

This demonstration allows you to show that when a cart moves with constant acceleration a, the surface of the liquid is a straight line tilted in the direction of the acceleration.

Give the cart a uniform acceleration by suspending an object over a pulley as in Fig. 1.

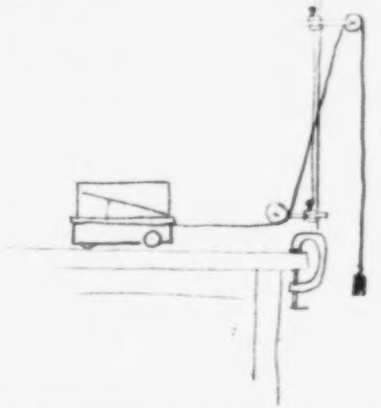


Fig. 1 Arrangement to demonstrate uniform acceleration.

It is best to use objects whose masses range from 100 to 400 grams. It is important to keep the string as long as possible, so that you use the entire length of the table. By changing the mass of the suspended object, you can vary the acceleration of the cart. Notice that the slope of the liquid increases with greater acceleration. The slope is thus a measure of the acceleration. It can be shown that $\tan \theta = a/g$ so that $a = g \tan \theta$. You will find detailed comments on Quantitative Work with the Liquid Surface Accelerometer in the Equipment Notes for Unit I.

D5 Comparative Fall Rates of Light and Heavy Objects

Drop several pairs of objects, such as a marble and a lead shot, simultaneously from the same height. Decide whether the theory of Aristotle or that of Galileo agrees best with the observations. Account for any discrepancies.

On a large book place several objects such as a small piece of paper, a marble and a paper clip. Drop the book. Do the objects fall at the same rate and stay on the book?

D6 Coin and Feather

If the equipment is available, do the coin and feather experiment.

D7 Two Ways to Demonstrate the Addition of Vectors

Method I

Apparatus:

20" × 20" board
Two dynamics carts
Two "baby bulldozers" (noisemaker springs removed)
Two sheets of clear plastic (Kodak Safety 3, for overhead transparencies, 9 3/4" by 11 3/4"
Three Pentel marking pens of different colors
Clamps, stands, etc., to support Pentels
Stopwatch
Three people to operate bulldozers and stopwatch

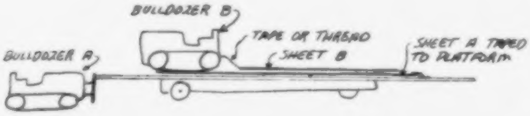


Fig. 1 The rolling platform and the arrangement of the plastic sheets.

Fasten the two carts underneath the board to form a rolling platform, as shown in Fig. 1. Hook up one bulldozer to pull the platform along the table; this is Sheet A.

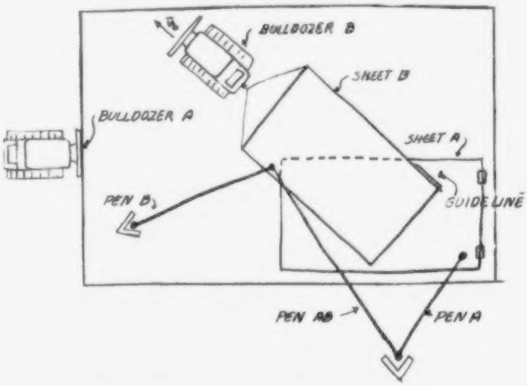
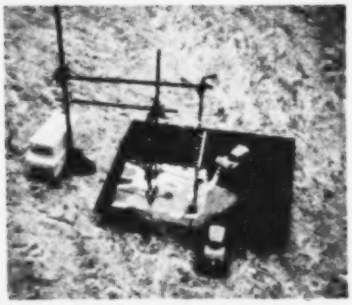


Fig. 2 The apparatus as seen from above.

Attach the other plastic sheet (Sheet B) to the other bulldozer (Bulldozer B) as shown, so that the bulldozer tows the sheet along smoothly behind it. Adjust the tow-rope so that $\vec{v}_b$ (the velocity Bulldozer B) is parallel to the long edges of Sheet B.



Choose a direction for $\vec{v}_b$, and aim Bulldozer B in that direction, laying Sheet B across Sheet A as shown. Draw a guide line on Sheet A, using the edge of Sheet B as a ruler (this is your only record of the direction of $\vec{v}_b$).

Attach one Pentel marker (Pen A) by means of a ringstand and stiff wires so that it makes a line on Sheet A as the platform rolls along. From the length and direction of this line, you will be able to figure out the magnitude and direction of $\vec{v}_a$, assuming that $\vec{v}_a$ is constant (how?).

Pen B is attached to the rolling platform. It makes a line on Sheet B that indicates the motion of Bulldozer B relative to the platform.

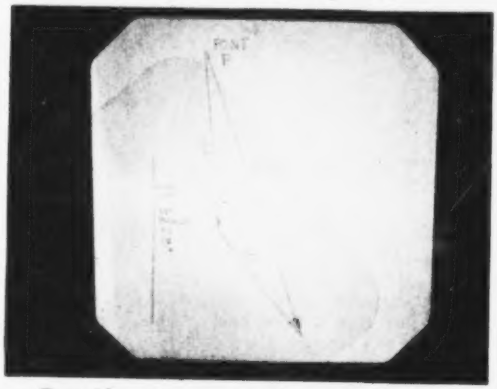
Pen AB also marks on Sheet B, but it is fastened to the stationary ringstand on the table. The motion of Sheet B with respect to the table is made up of the two simple motions added together (vectorially, of course). From the line that Pen AB makes, you can deduce the vector $(\vec{v}_a + \vec{v}_b)$.

Adjust Pen B and Pen AB so that they begin at the same point on Sheet B (call this point P).

With the pens in place, set the bulldozers in motion at the same time (this will take a little practice). Shut them off, again simultaneously, when the longest line that has been drawn is four to six inches long. Use the stopwatch to time the motion.

You now have three lines of different lengths, colors and directions. If you make certain assumptions (what assumptions?), you can treat these lines as direct representations of $\vec{v}_a$, of $\vec{v}_b$ and of $(\vec{v}_a + \vec{v}_b)$. Add an arrowhead to each line to indicate the actual direction of the velocity that it represents (careful!). Remove both plastic sheets from the apparatus and slide Sheet B over Sheet A until the head of $\vec{v}_a$ is at Point P. Be sure to keep the edge of Sheet B parallel to the guide line.

Does $(\vec{v}_a + \vec{v}_b)$ seem to be the vector sum of $\vec{v}_a$ and $\vec{v}_b$, using the parallelogram rule? Convince yourself that if these velocities have added as vectors, the three vectors should form a triangle. Is this the case (see Fig. 3)?



Try the same procedure for a few other directions of $\vec{v}_b$ ($\vec{v}_a$ and $\vec{v}_b$ parallel, opposite, at right angles, etc.).

Method II

Apparatus:

DC blinky, set to about 1 flash per second
The same rolling platform as in Method I, painted black
Two "baby bulldozers"
Polaroid camera mounted on tripod (for 3000 speed film, the lens setting is about EV 16)
Bench stand and pointer to indicate the starting point of the blinky
Three people, to operate bulldozers and camera

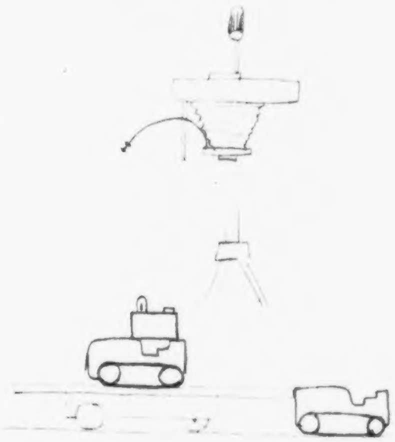


Fig. 1 Arrangement of apparatus when v_1 and $\vec{v}_2$ parallel.

Set up the rolling platform, towed by Bulldozer A as in Method I; the velocity of the platform as it moves past the camera is again $\vec{v}_a$. Place Bulldozer B on the platform; its velocity with respect to the platform is $\vec{v}_b$. Point the camera downward, so that it takes a picture of the apparatus from directly overhead. Mount the blinky on Bulldozer B and position the pointer so that you will be able to put the blinky back in its original position after taking the first picture.

Turn out the lights, open the camera shutter and set Bulldozer A in motion. Let it tow the platform across a good part of the camera's field of view. Which velocity can you obtain from the strobe record of this motion?

Replace the platform so that everything is back the way it was before the last step. Repeat the process (without advancing the film—when you are through, you will have a triple exposure), but this time have only Bulldozer B in motion. Which velocity can you figure out from this strobe record?

Return to the starting point again, and take a third picture (on the same film) of the motion of the blinky, this time with both bulldozers moving.

Develop the print and calculate the three velocities (speeds and directions) $\vec{v}_a$, $\vec{v}_b$ and $(\vec{v}_a + \vec{v}_b)$. Skip the first interval in each strobe record: it takes the bulldozers a little time to get up to speed. Draw arrows representing the three velocities, and check as in Method I to see if the parallelogram law of vector addition holds for the motion you have observed. Do the three vectors form a triangle? Should they?

Again, try the cases where $\vec{v}_a$ and $\vec{v}_b$ are parallel, anti-parallel, at right angles and at several other angles of your choosing.

D8 Direction of Acceleration and Velocity

Using the same arrangement shown in Fig. 1, demonstrate that acceleration and velocity can have different directions. Hang an object of 100 or 200 gm mass over the pulley; give the cart a push to the left so that it goes nearly to the end of the table before it stops and reverses direction. You should try to give a short smooth push so that the liquid reaches its steady state quickly.

Once the water has reached its steady state, the surface is a straight line whose slope does not change, even when the velocity reverses direction. The explanation, of course, is that the acceleration is constant and independent of the velocity. Only the weight of the object over the pulley determines the acceleration of the cart.

D9 Direction of Velocity and Acceleration —An Air Track Demonstration

Mount the small accelerometer on an air track cart. When the track is horizontal and the cart is at rest or moving with uniform speed, the surface of the liquid is also horizontal. Only when a horizontal force causes the cart to accelerate—for example, when the cart starts or stops or collides with something else—is the slope of the surface not horizontal.

Next, place the track at a slight incline. When the cart slides freely on the air track, however, the surface is parallel to the track. These interesting facts are explained in the theory section.

Again, you can show that velocity and acceleration can have different directions. Give the cart a push up the incline. If friction is negligible, the slope of the liquid remains the same while the cart slows down, reverses direction and moves down the incline. If frictional forces are increased by adding mass to the cart, the slope will decrease when the cart begins to move downhill.

D10 Non-Commutativity of Rotations

One of the points frequently made about vector addition is that it is commutative, i.e., the order of addition does not affect the sum. Students are frequently convinced from their experience with arithmetic that this is true of all operations. It is useful to be able to show them an example of an operation that is not commutative.

If a closed book is placed on the desk in front of a student and he rotates the book up 90° about an axis along the spine of the book, and then rotates it up another 90° about an axis parallel to the near edge of his desk, the final orientation of the book is different than if the opposite order of the two operations is followed.

D11 Newton's First Law

There is an aesthetic appreciation in science for simple statements that describe very complex phenomena. $E = mc^2$ is a recent example of such a statement. Newton's first two laws and the equation $F = ma$, which follows from them, are an early example. For the teacher, these simple statements often create difficulties because the students fail to realize their importance. There is a tendency to feel that what is not complex and filled with mathematical symbols cannot be very important. Nothing could be farther from the truth. Many men contributed to the eventual development of the three laws, and Newton's own work was a perplexing amalgam of intuition, definition and experiment. While one cannot say precisely how Newton came to his conclusions, he was deeply familiar with the related phenomena. Therefore we suspect that the students' introduction to the laws of motion should avoid the didactic and favor direct experience and an intuitive approach.

The demonstration described below may seem trivial, but first-hand experience with very low-friction motion is valuable for understanding Newtonian physics. While this is listed as a demonstration, it should be conducted as an informal experiment. This has always been a very enjoyable experiment for the students, who frequently mention it as their favorite.

Equipment

Several pucks with balloons or plastic beads
Puck table
Large rubber band
Air track (optional)

Procedure

Student lab groups are given single pucks without the balloons or plastic beads, with the instructions that they are to play with them for several minutes so as to be able to describe how the pucks move. Then a brief discussion is held to establish what happens to the pucks' motion under various circumstances—e.g., just sitting on the table, being pushed briefly, being pushed steadily, tilting the table, blowing, etc. Friction may be mentioned, and hopefully someone will suggest what the motion would be like without friction.

Immediately demonstrate the low-friction capability of the pucks, and supply students with balloons and/or plastic beads for another short period of investigation. (Half the class could use balloons, half use plastic beads, to make the results more general.) The instructions are, as before, to be able to describe the motion of the pucks. (Fences made from the large rubber bands are invaluable as reflectors because they allow long runs.)

The leveling of the surface may be a problem, especially with the balloon pucks. The concluding discussion might become heated on the how-do-you-know-when-there's-no-force paradox, but that's fine—if students argue about these things, they are aware of the issue.

The disc magnets or air track can be used for a further extension of frictionless motion.

D12 Newton's Law Experiment (Air Track)

With the calibrated accelerometer we could perform experiments on Newton's laws which enable us to define forces in terms of the accelerations of objects whose masses are known. The accelerometer would enable us to determine the accelerations directly.

D13 Effect of Friction on Acceleration

The above demonstration works only if friction is negligible. Since the direction of the frictional force F_{frict} is always opposite to the velocity, you can show the effect of friction on acceleration by attaching tape with adhesive on both sides to the wheels of the cart.

When the cart moves to the right, the horizontal forces acting on it are illustrated as in Fig. 2. The acceleration is then

$$a = \frac{T - F_{frict}}{M},$$

where M is the mass of the cart plus the accelerometer. When the cart moves to the left, however, the forces act as in Fig. 3. The acceleration is now

$$a = \frac{T + F_{frict}}{M}.$$

The tension T is simply the weight of the object hanging over the pulley and is independent of the velocity.

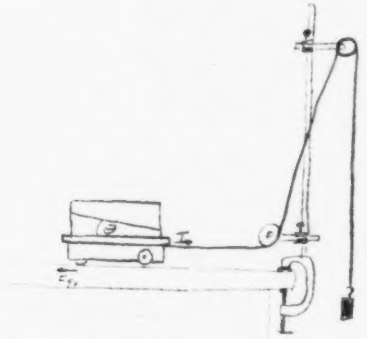


Fig. 2 Force diagram when cart is moving to right.

Since the acceleration is less when the cart moves to the right than when it moves to the left, the slope of the water when the cart moves to the right will also be less. This difference in slopes is slight, but noticeable.

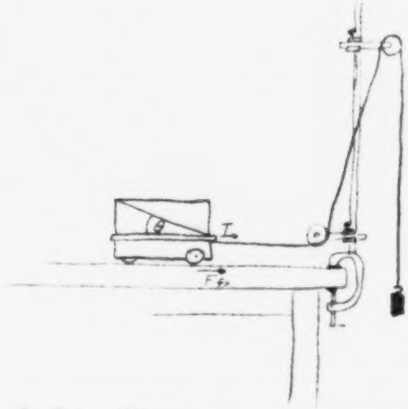


Fig. 3 Force diagram when cart is moving to left. The change in theta is exaggerated.

D14 Demonstrations with Rockets

Introduction

The demonstration experiments suggested in this note can accomplish two things. They are exciting, which makes them ideal as motivating experiments at the beginning of the course. Rockets and space flight are matters of great public interest today, and experiments like these could do much to arouse interest (and perhaps increase enrollment) in a physics course. The experiments can also be used to teach quite a lot of physics: free fall, force, impulse, conservation of energy, application of trigonometry, etc.

We cannot stress too highly the need for strict supervision by the teacher at all times. Get permission and support from local officials and school administrators before starting model rocketry.

Small solid-fuel rocket engines, lightweight rockets and a considerable body of supplementary information can be purchased from Estes Industries, Inc., Box 227, Penrose, Colorado 81240. Their catalog is available on request from the address given. Prices for rocket engines range from \$0.21 to \$0.33 each, in lots of 12; rockets come in kit form and range from \$0.50 to \$4.00. We have tested the "Scout" which costs \$0.70, the "Corporal" which costs \$1.50, and the "V-2" which costs \$1.75. Assembly for these models ranges from one to two hours and could be done by students.

When used with some care under strict supervision of the teacher, these rockets are probably considerably safer than a good number of other experiments that are performed in the classroom. However, students should not be permitted to take home rockets from the school's supply or to use the school's rockets during school hours without careful supervision. Although quantitative experiments of real precision are probably mathematically too involved, students can learn much from a series of demonstrations that permit some student participation.

Rocket engines come in a variety of sizes with maximum thrusts of either 21 ounces or 9 pounds and thrust durations from 1.7 sec to 2 sec. In addition, a special purpose engine [B.8-0(P)] for use in static tests is available.

Experiments with rockets in free flight

If a large, open space is accessible to the class, a number of experiments can be performed with free flight rockets. For example, one may use successively more powerful engines in several otherwise identical rockets. Another set of experiments would make use of rockets of identical exterior design but of different mass; in fact, one might make one of the rockets so heavy that it will not lift off. We all get a thrill from firing the small rocket and seeing it rise rapidly several hundred feet. Students should stand at known distances, at least 100 feet from the launching pad, each with a simple altimeter, consisting of a protractor with a small plumbline and a viewing tube, made, for example, from a large soda straw.

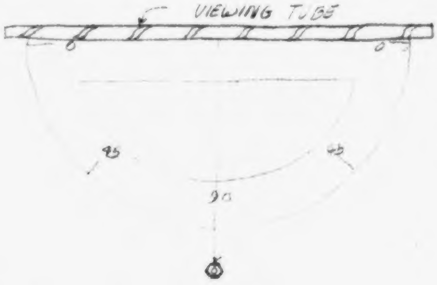


Fig. 1

Each student should try to measure the angle of elevation of the rocket at the same moment, preferably when the rocket has reached its maximum height. The teacher can call out the time for this measurement. Using simple trigonometry, each student can calculate the height of the rocket. If there is little wind and the rocket rises vertically, he can calculate the height knowing the distance and elevation angle. Each student will find a value for H. A comparison of the results will provide an opportunity to discuss errors of measurement.

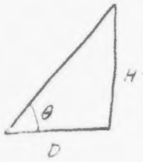


Fig. 2

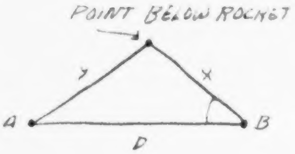


Fig. 3

In most cases, the rocket will not rise vertically. The computations will become fairly involved unless students can measure both the angle of elevation at points of maximum H as well as the angle through which they must turn from a fixed line when measuring H. For example, if two students, A and B, stand at a fixed distance D, and each has to turn from the line connecting their position by angles of θ and ϕ, respectively, we can then at once find the point above which the rocket was at its highest point and determine the distances x and y. Knowing x and y, each student can calculate H (see Fig. 3). Write to Estes Industries for copies of their Technical Report TR3 "Altitude Tracking" which gives detailed instructions. This exercise and excursion into trigonometry, although not directly part of a physics course, is useful in showing the need for mathematics as a tool.

When firing rockets, all possible safety precautions should be followed. Estes Industries will supply an outline (Attachment #3) of how to handle the rockets and what methods to employ to prevent accidents. In fact, the safety code as supplied by Estes has an educational value in showing students how to handle potentially dangerous situations.

Experiments with test stands

The design of a simple test stand for rocket engines requires knowledge of fundamental physics principles. Basically, one wants to measure as accurately as possible the force (thrust) a rocket exerts as a function of time. Since the burning times of these rockets are short (from a minimum of 0.17 sec to a maximum of 1.4 sec), one needs to use a recording device. In order to measure thrust correctly, the apparatus should be truly static, i.e., there should be as little motion as possible while the engine fires. If a spring is used to provide the balancing force, precautions must be taken to avoid oscillations; in fact the damping should be critical and furthermore should be velocity-dependent so that the recording pen will always return to the same zero position.

Test stands can be designed in a variety of ways. Two designs have been tested:

A. The first test stand consists of an engine holder (Figs. 4 and 5), made from a rocket body tube (Estes Cat. #651-BT-40, 0.765" I.D., 0.028" wall thickness) connected to an aluminum rod R free to move in two bearing blocks B. Attached to the far end of the rod R is a tube P into which a Pentel pen or some similar easy and light writing device can be inserted. The rod R carries a pin which serves two purposes: it compresses a spring as the engine is fired and it prevents the rod from turning about its axis by riding in a key slot attached to one of the bearing blocks. The spring constant should be chosen so that a steady (static) force of 20 newtons will give a compression of approximately 3 inches. Friction in the bearings may just provide the necessary damping force; otherwise, one can add some damping by pressing a cloth strip against the rod. The test stand is set up radially near a turntable so that when no force is applied to the spring, the pen will leave a circular trace near the edge of a circular sheet of paper attached to the rotating turntable. When the rocket engine is fired, the pen is pushed toward the center of the turntable and plots a graph that can be analyzed for a measure of the force applied to the spring. With the turntable rotating at 33 rpm, a "firing" of an Estes A8-0 (P) rocket engine will leave a polar coordinate record that covers almost a complete revolution, indicating that the force was applied for approximately 1/33 min, i.e., just under 2 sec (Fig. 6). If a linear chart drive is available which will move the paper at a high enough speed so that the graph is spread out over a reasonable distance (i.e., at least 10 inches per second), you can substitute this for the turntable. However, there is merit in using a polar graph, if only to show students a different method of recording and analyzing data.

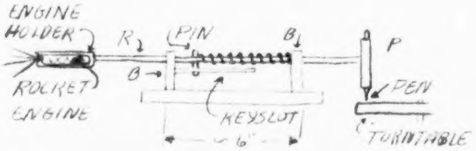


Fig. 4

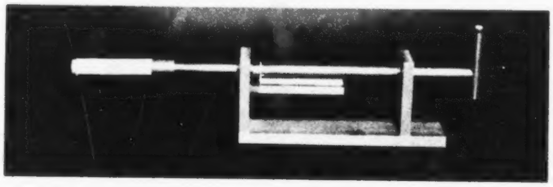


Fig. 5

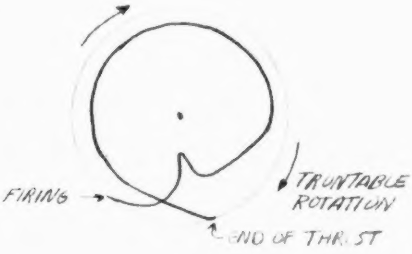


Fig. 6

To translate the curve drawn by the recording device during the firing into a force vs. time plot one needs to calibrate the test stand. This can be done by applying known forces (e.g., weights applied via a pulley) to the spring and turning the turntable by hand through sections for each applied force. A calibration curve, relating displacement to (static) force can then be drawn.

Note that the spring used was non-linear. The reason for this is that the initial large force acts for a short time only and thus the impulse due to this force is fairly small. To measure accurately the much smaller sustained force a spring is needed which will give reasonably large deflections for the small force acting over the longest part of the firing. Again, there is additional educational benefit to be derived from the fact that another illusion is shattered for most students (and many teachers), namely that springs by nature are linear and that Hooke's law can be applied without thought.

It might be worthwhile to point out that there is another problem involved in this analysis, namely that the force applied by the rocket engine is an "impulsive" force, acting for a short time only, whereas the calibration of the test stand is done statically.

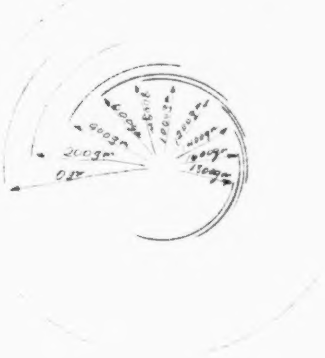


Fig. 7(a)

Students would benefit from transferring the polar-coordinate graph to a cartesian-coordinate graph. They can then compute the total impulse of the engine (∫Fdt) by finding the area under the curve. If this impulse is assumed to occur in a short time compared with the total flight, a first approximation gives ∫Fdt = (mv) final - (mv) initial, when m is the mass of the rocket plus engine and v is the speed of the rocket after the impulse has been applied. If we neglect all external forces except gravity, we can find the maximum height to which it would rise from simple kinematic considerations ($v_f^2 - v_0 = 2gh$). The actual height to which the rocket will rise is much less than the computed one.

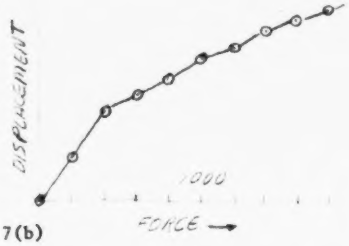


Fig. 7(b)

B. A second type of test stand (Figs. 8 and 9) can be assembled easily in most schools from odds and ends. It involves a 15" wooden ruler in which a vertical shaft has been placed at the 3" mark; the ruler can turn freely about this shaft in a horizontal plane. (Inexpensive steel shafts with bearings are available from radio supply houses, e.g., Allied #44Z094, panel bearing assembly with 3" shaft, costs 33¢ complete.) At the 1" mark a rocket motor holder is fastened securely by gluing it with a good contact cement, then tying it with string (Fig. 10). Finally, paint the string and motor holder with glue, coil dope, shellac or some other material which will bind to the ruler, string and rocket motor holder.

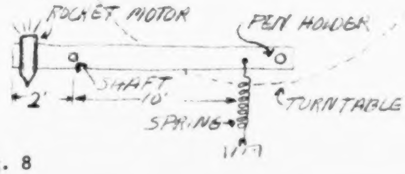


Fig. 8

Fig. 9

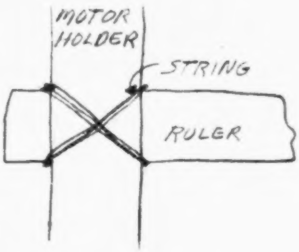


Fig. 10

At the 13" mark a spring is fastened which will extend not more than 3" when a force of 4 newtons is applied to it. Again a non-linear spring would have the same advantages explained earlier.

NOTE: There are various ways to make non-linear springs. In this particular case one could, for example, have two springs attached (Fig. 11) such that for small forces spring 1 will stretch, but spring 2 will not be under any tension. As spring 1 stretches, eventually the string that connects spring 2 to the ruler will become tight and the force constant of the combination will become the sum of the force constants of both springs.

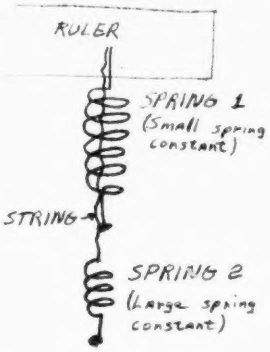


Fig. 11

A second method to obtain a non-linear spring uses one single spring and a thin string loosely tied between some of the coils of the spring (Fig. 12). As the spring is stretched, all coils will open up at first, until the string becomes taut. From then on, only those coils can extend which are outside the tied-down section of the spring. It is easy to adjust the relative spring constants simply by shifting the position of the string, holding back more or fewer of the coils.

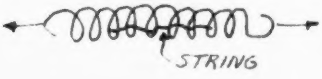


Fig. 12

A light tube is attached to the far end of the ruler, near the 14" mark, which can hold a Pentel marker. We again use a turntable so that the pen can trace a graph of its excursion as a function of time on a paper disc fastened to the turntable.

The reason for using the unequal lever arms in this design is to have the rocket engine move through as small a distance as feasible, thus approaching a true static test, and also to have the moving parts of the device be as light as feasible while still giving a reasonably large trace on the graph paper.

Figure 13 shows the result of a firing using a linear spring and no damping force. A number (at least four) of oscillations following the initial excursion of the pen can be seen.

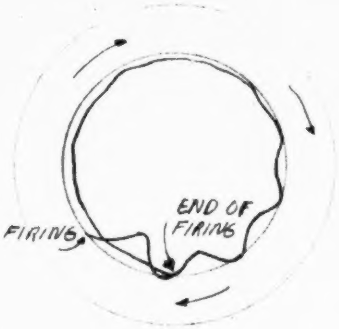


Fig. 13

Damping can be applied in a variety of ways and will provide a very interesting exercise in applied physics. The most obvious way to decrease oscillations is to apply a frictional force. A bottle brush held perpendicular to the ruler near the 12" mark and pushed against the flat side of the ruler (Fig. 14) will help dampen out the vibrations (Fig. 15), but the damping is not critical, and in

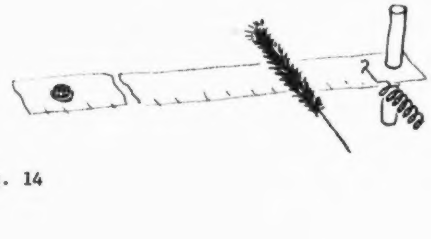


Fig. 14

addition the friction will introduce sizeable shifts in the zero position. Ideally the damping force should be velocity-dependent. We have tested a viscous device, consisting of a metal vane being pushed through oil and find that it is also non-critical (Fig. 16) but does not have a zero correction. Another method would be to use a metal plate moving in a strong magnetic field (eddy-brake).

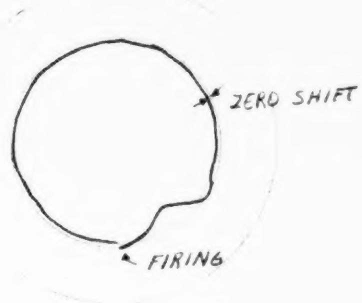


Fig. 15

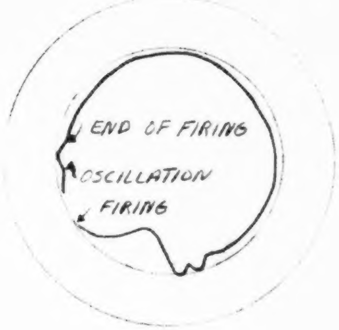


Fig. 16

This part of the project is completely open-ended. Students can undertake a systematic experimental study of damping forces and begin to appreciate the problems of the scientist or the engineer. They will also begin to realize that through systematic study of a problem one will slowly be able to approach better and better solutions.

Postscript

This demonstration can teach a good deal about free fall, propelled flight, the operational meaning of force, momentum, conservation of energy, the use of trigonometry, experimental uncertainty, the scattering of data, etc., but it can also be justified as a motivating experiment which is interesting and exciting. Rockets and space flight hold today a unique position in the public eye, and the interest with which millions follow each firing of a major rocket from Cape Kennedy is without precedent. It seems reasonable to make use of this interest in attempting to attract students to the physics course. There is no question that the news of such firings in a course will spread rapidly through a school. As a consequence, students who otherwise might not have found out about the excitement and challenges of physics may become interested.

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D15 Making an Inertial Balance

An inertial balance may be an aid to help the students distinguish between mass and weight. One end of a hacksaw blade is clamped to a bench so that it can vibrate in a horizontal plane. Various masses are attached to it, but their weight is supported by suspending the masses from a string. The hacksaw blade is pulled to one side and then released so that it swings.

D16 Action-Reaction Forces in Pulling a Rope

Attach a heavy spring balance to a wall and find two students whose maximum pull is about the same. Then place the spring balance between the two students and have them pull against each other with their maximum force. The balance will read the same in each case. This should help bring home the point that a "pushed or pulled" object such as a wall will exert an opposing force whenever a force is applied to it.

D17 Action-Reaction Forces in Pulling a Rope

Place a student on each of two carts and pass a rope between them. First have one student pull alone, then the other, and finally both. Start the carts from the same position each time and note the place where they meet. Ask the class whether an observer, watching the carts alone, could tell which student was actively pulling in each case.

D18 Reaction Force of a Wall

When you lean on a wall does it exert a force on you? Stand on a cart or roller skates and lean against the wall.

D19 Newton's Third Law

The following simple demonstrations dramatically illustrate Newton's third law. Their simplicity, moreover, gives some indication of the elegance and profundity of this remarkable law.

To show that forces exist in pairs on different objects, and that the paired forces act in opposite directions, set up a linear equal-mass explosion between two dynamics carts. Propel the carts apart with a steel hoop, magnets, streams of water, or any other forces you can think of. See Fig. 1 for some suggestions. Stress that this concept of force-opposite-force is valid for all types of forces.

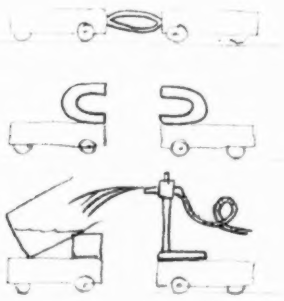


Fig. 1

The experiment on conservation of momentum (E22) gives detailed instructions about the explosion using the steel hoop. You can take a strobe photograph of the explosion, and show that if the carts have equal masses, they move apart at equal speeds. If the carts have equal speeds, the accelerations they received during the explosion were equal in magnitude. Since the carts have equal masses and since the duration of the interaction is the same for each cart, Newton's second law implies that they experienced equal forces during the explosion.

A more direct method to show that the forces are equal in magnitude is to modify the demonstration by propelling the two dynamics carts with large magnetron magnets. Move the magnets back about an inch on the carts. Place a pencil or dowel in the hole at the front of each cart and loop a three-inch rubber band around the pencils. When you release the carts, they will separate, stretch the rubber band, oscillate, and finally come to rest.

When the carts are at rest, the forces acting on each cart are those shown in Fig. 2. The tension in a rubber band is uniform, so $T = T'$. Since each cart is at rest, then $T = F$ and $T' = F'$. Thus $F = F'$: the magnetic forces on the carts are equal. Note that in this demonstration the carts can have different masses.

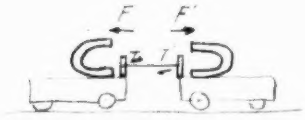


Fig. 2

Another exciting way to illustrate Newton's third law is to mount a sail on the fan cart that was used to illustrate uniform acceleration (D4), and let the propeller blow against the sail. Since the sail bends forward, clearly there is a force on it. But the cart does not move because when the propeller pushes against the air, the air exerts

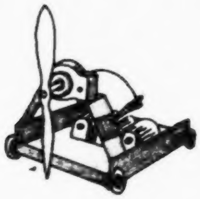
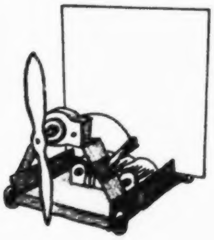


Fig. 3

A reaction force against the propeller. Thus the net force on the glider is zero. (If the sail does not catch all the air from the propeller, the cart may move slightly.) If you remove the sail, the only force on the glider is the reaction force exerted by the air on the propeller. This force causes the glider to move backwards.

Fig. 4



The fan cart rigged for uniform acceleration is sketched in Fig. 3. The placement of the sail to show action and reaction is sketched in Fig. 4.

D20 Action-Reaction Forces between Car and Road

Demonstrate the coupling of forces between a car and the road. Obtain a motorized toy car. Place a piece of cardboard on top of some plastic beads or an upside-down skate-wheel cart. Then place the wound-up car on the cardboard roadway; the opposing forces will cause the roadway to move backward.

D21 Action-Reaction Forces in Hammering a Nail

Hammer a nail into a plank. First place the plank on a bench, then place the plank on a soft pillow. The force exerted on the nail depends not only on the hammer but also on the opposing force of the plank.

D22 Action-Reaction Forces in Jumping Upward

When you jump off the floor does the floor push harder on you in order to cause the upward acceleration? Jump up from a bathroom scale and watch the scale.

D23 Frames of Reference

The following demonstration illustrates the idea that different motions can appear the same when observed from different reference frames.

One familiar example is the situation of two trains in a station on parallel tracks. An observer in one train cannot tell which train is moving, or whether both trains are moving, unless he watches the station.

In the following demonstration, a camera photographs a blinky, with either the camera or the blinky moving at constant velocity. From the photograph, one cannot tell which object was moving. The photos in the two cases are identical, unless part of the laboratory also appears in them.

This idea that an observer's view of a motion will depend on his frame of reference will be a major theme in Unit 2. To an observer on the earth, the sun seems to move daily around the earth. But he would see the same apparent motions if the sun were stationary and the earth rotated on an axis. The impossibility of distinguishing between the two motions caused much intellectual controversy in the sixteenth and seventeenth centuries.

Equipment

Polaroid camera, cable release and tripod. With 3000-speed film, use the EV 15 setting.
Two dynamics carts
Two baby buldozers
DC blinky
Black screen
Turntable

Procedure

Mount the blinky on one cart and the camera on the other. Use the toy bulldozer to push the carts. It may be necessary to increase the mass of the cart with the blinky, so that both carts are driven at the same speed. Arrange the apparatus as shown in Fig. 1.

Take two photographs, one with the blinky moving and the camera stationary, and the other with the cart moving and the blinky stationary. Use the cable release and be careful not to jar the camera when you open the shutter.

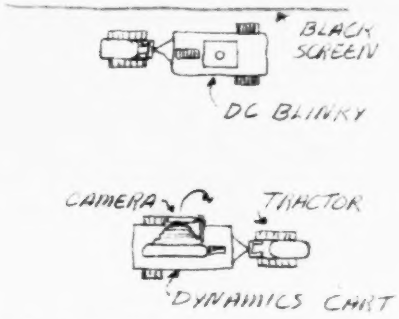


Fig. 1 Apparatus for linear motion.

Circular motion

Mount the camera on the tripod and attach the blinky to a turntable. Aim the camera straight down. Figure 2 shows this arrangement.

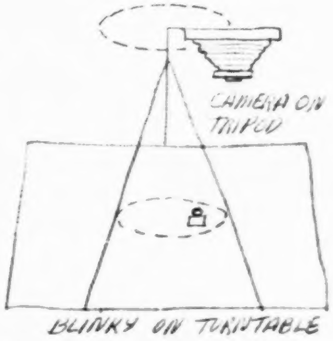


Fig. 2 Apparatus for circular motion.

Take a time exposure with the camera at rest and the blinky moving one revolution in a circle. If you do not use the turntable, move the blinky by hand around a circle drawn faintly on the background. Then take a second print, with the blinky at rest and the camera moved steadily by hand about the axes of the tripod. Try to have the camera at the same rotational speed as the blinky moved in the first photo.

Extension

The observer in the train cannot tell which train is moving if there is a relative velocity between the trains. If there were a relative acceleration, however, he could tell which train was accelerating. He could detect the acceleration of his train, for example, with a liquid-surface accelerometer. If the acceleration were great enough, he would also feel himself being pushed back or thrown forward. An object cannot accelerate unless a force acts on it.

Strictly speaking, our observer could not be sure he is accelerating. According to Einstein's principle of equivalence, the effects of a uniform acceleration $\vec{a}$ are indistinguishable from those of a uniform gravitational field $-\vec{a}$. In the train, however, the observer can be reasonably confident that his accelerometer detects an acceleration, not some bizarre gravitational field.

D24 Inertial vs. Non-Inertial Reference Frames

Have a student toss a ball straight upwards and catch it again while walking at a constant speed. Ask for descriptions of the path of the ball as seen by the ball tosser and by a seated student. How do the accelerations compare as measured by the walker and by the seated students? (They are the same.) How would the path appear if the ball tosser had stood still and the students had moved sideways with the original speed of the walker? (It would appear to be the same as before to both viewers.)

Now toss the ball as you accelerate, walking faster and faster, and again as you slow down. Also toss the ball as you walk in a circle. Show that, in these cases, the two frames of reference give two different accelerations.

You might want to discuss this idea again in Chapter 4 where the idea is developed that acceleration is caused by an unbalanced force. An accelerated frame of reference requires apparent (or fictitious) forces to explain accelerations which are not present when viewed from a fixed frame of reference.

D25 Uniform Circular Motion

To demonstrate the acceleration in uniform circular motion, place the accelerometer along the diameter of a phonograph turntable. When the turntable rotates, the liquid surface is parabolic. Figure 3 shows this situation. The acceleration increases with the distance from the center and is always directed inward. By changing the speed of the turntable, you can show that the acceleration is greater for higher speeds of rotation. This is also discussed in the Theory section.

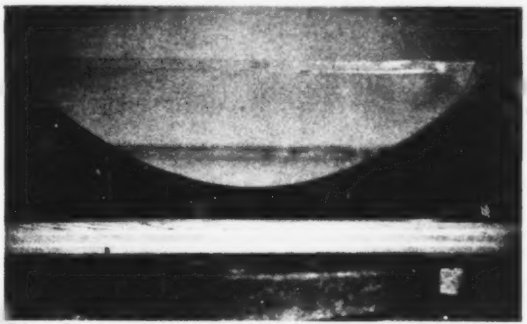


Fig. 3 Accelerometer on rotating turntable. The surface of the liquid is parabolic.

For a further discussion of this profound idea that an observer rotating uniformly with respect to an inertial frame can tell that he is rotating, see the articles in A Physics Reader on Newton's bucket experiment.

D26 Simple Harmonic Motion

Harmonic motion can be demonstrated as an example of a more complex motion. To show that harmonic motion can be discussed in terms of circular motion, set an object moving in uniform circular motion, such as a peg on a phonograph turntable. Then illuminate this motion from the side and project its shadow onto a screen so that all that can be seen is a back-and-forth motion. Harmonic motion can be developed further, but it is probably enough just to give several examples of objects that have this motion, such as a vibrating tuning fork, a pendulum, an object suspended on a spring, etc. When Hooke's law is discussed in Chapter 4, students can be reminded that forces which obey that law give rise to harmonic motion.

D27 Simple Harmonic Motion—Air Track (Optional)

By attaching a long rubber band or string to each end of the cart and pulling back and forth, you can make the cart move in approximately simple harmonic motion. The class can see qualitatively that the acceleration is directed opposite to the velocity and is at maximum when the cart is farthest away from the equilibrium position.

Equipment Notes

Notes: Polaroid Photography

Cameras

Almost any Polaroid Land camera can be used in classroom demonstrations and experiments in physics. The notes refer in detail to (a) the experimental model 002 cameras supplied by Project Physics, and (b) the older models 95, 150, 800 that can be bought relatively inexpensively and are already being used in many classrooms. A third section of these notes on photographic techniques refers to all models.

A. Experimental model 002 Polaroid Land Camera

This camera is a modified version of the model 210 camera, in which exposure time is controlled automatically by the electric eye. The manufacturer's instruction booklet describes the normal use of the model 210—loading the film pack, processing, etc.

The modifications consist of:

1. a cover for the electric eye that makes it possible to take bulb exposures. When the eye is covered the camera shutter remains open as long as the shutter release (or cable) is held depressed. There are very few, if any, experiments for which you will use the eye to control exposure time automatically. Always keep the eye covered when the camera is not in use to prevent rapid rundown of the internal battery.

2. a cable release clamped semi-permanently on the shutter release button

3. a base plate with locking thumb screw. For most classroom work the camera is used as a fixed-focus camera. It is convenient to use the camera at distance that gives a 10:1 photographic reduction. The locking screw is used to fix the camera bellows at the correct extension. The base plate also has a screw hole which takes a standard 1/4"-20 screw for mounting it on a camera tripod or motor strobe disc unit.

4. a close-up accessory lens, which clips onto the camera lens to give an approximately 1:1 reduction for photographing traces on oscilloscope screen, etc.

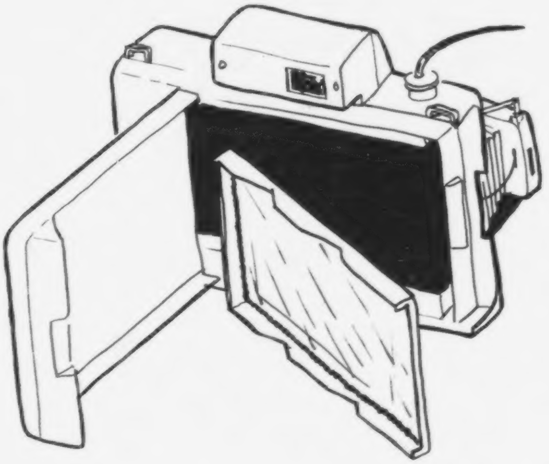
5. a clip-on slit, to be used in conjunction with the motor strobe unit (see notes on strobe photography).

Focussing

The camera has a range finder (non-automatic). Look through the finder window: the position of the arrow on the scale at the left of the window indicates the focal distance, in feet. Focus is adjusted by pushing the buttons marked "1" back and forth.

For most classroom use, it is convenient to work at a standard distance from the event being recorded. A distance of about 1.2 meters (47 inches) gives a 10:1 photographic reduction. We recommend that one of the first things students do with the camera is to establish precisely what the 10:1 distance is.

A focussing screen can be made very simply as follows. Take a discarded film pack apart into its three component pieces. One of these pieces is a frame which encloses an area the size of the picture. Fix a piece of ground glass (about 4½" × 3¼") in the frame, so that the ground surface faces toward the lens when the grame is put into the camera. If ground glass is not available, a satisfactory screen can be improvised by sticking scotch tape (not the clear variety) on a piece of flat glass, or using tracing paper.



Insert the frame in the camera, just as if it were a film pack, ground glass surface towards lens. Leave the camera back open. Set up a meter stick about 1.2 meters in front of the camera; the stick should be well illuminated. Cover the electric eye, set the speed selector to 75. Open the shutter and keep it open, by keeping the no. 2 button or the cable release depressed. (The cable release can be locked by tightening the set screw.) Look at the image on the focussing screen and adjust the range finder until the image is sharply focussed. Measure the image of the meter stick on the screen. Adjust the camera-stick distance and focus until the sharply focussed image of the meter stick is 10 cm long.*

Once the 10:1 distance has been found and the camera focussed, use the thumb screw provided to lock the camera bellows in this position. Measure the lens-to-object distance. It will now be easy to set up and photograph an object or event at 10:1 reduction. Do not refocus the camera or loosen the locking thumb screw unnecessarily.

This preliminary exercise can be extended to establish two important points about using the camera to record events at the 10:1 distance:

a) what is the field of view at this distance? (It should be just under 1 meter.)

b) is the photographic reduction uniform over the print; i.e., is it the same near the edge as at the center, or is there some distortion? (There is in fact very little distortion—the 10:1 factor can be used on all parts of the print.)

Exposure

a) Aperture. Students' attention may also be directed at this time to the effect of the "Film Selector" (manufacturer's instruction booklet, pages 16, 18). Remove the screen, open the camera shutter and look through the lens with camera back open. At the 3000 setting the lens aperture is small; at 75 the aperture is 3000 times larger (in area).

For most strobe photography, use the 75 setting, even though the camera is loaded with 3000-speed film. A slight decrease in the 75 aperture can be effected by placing the clip-on slot over the camera lens. (The numbers refer to the ASA "speeds" of the two types of film. For normal outdoor use the selector is set to 3000 for 3000-speed black-and-white film and to 75 for 75-speed color film. But this does not apply to our special classroom use of the camera. Although 3000-speed film will be used in our experiments, in many instances the 75 setting is needed. The lighten/darken control—see page 16 of the manufacturer's instruction booklet—is effective only when the electric eye is open.)

b) Time. If the electric eye is open, the exposure time is controlled automatically. If the electric eye is closed, the shutter will remain open as long as the cable release (or shutter release) is held depressed. For strobe work, cover the electric eye and control the exposure time manually. Try to keep the shutter open for the minimum time necessary to record the event. The longer the camera is open the poorer will be the contrast in the picture.

Lighting

The strobe photography experiments and demonstrations that are described in detail in the Teacher Guide and Student Handbook do not require a darkroom. In many cases it is not even necessary to turn off the room lights, unless there is a light directly over the lab table.

But a dark background is essential in strobe photography—use the black cloth screen provided by Project Physics (see Fig. 1).

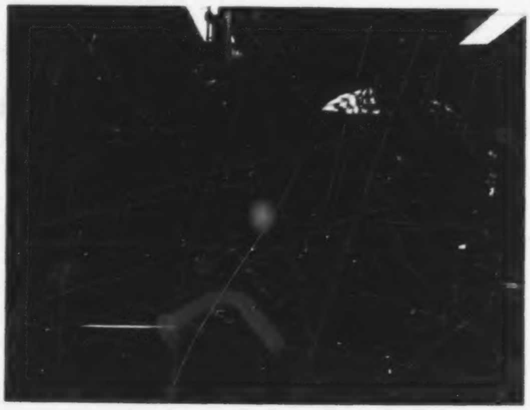


Fig. 1 Blinky photograph taken with model 002 Polaroid Camera. Note 1, room lights were on; 2, use of black cloth screen.

It is often useful to record both the strobe event and a scale (meter stick) in the same picture. Table 1 summarizes conditions for the various strobe techniques.

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*Unfortunately, the screen is just less than 10 cm long. Therefore the meter stick must be set up obliquely. Or alternatively, adjust until a 90 cm long part of the meter stick gives an image 9 cm long.

Table 1. Suggested exposure conditions using experimental camera model 002.

Strobe Technique	Lighting	Film Selector (aperture)	Procedure
light source and disc strobe	normal—but not directly overhead	75	single bulb exposure records both event and scale
xenon strobe	normal—but not directly overhead	3000	single bulb exposure records both event and scale
blinky	darkened room—but not dark room	75	single bulb exposure records both event and scale

When working at 75 aperture, a small decrease in exposure can be effected by adding the clip-on slit over the camera lens.

Close-up accessory lens

With the accessory lens clipped in place over the regular camera lens the camera can be used for close-up work. Focussing with this lens is quite critical and must be done with a focussing screen. The object should be between 12 and 14 cm from the front surface of the accessory lens, depending on the magnification you want. With the camera bellows fully collapsed—i.e., camera focus set to infinity—the ratio of image size to object size is about 0.85, and with the bellows fully extended the ratio is about 1.2.

For most classroom work the camera is used at a bellows extension that gives a 10:1 reduction (without the clip-on lens), and it is convenient to keep the bellows fixed in this position. You can use the camera for close-up work without changing the bellows extension: add the accessory lens, insert a focussing screen in the camera back and focus on the object by moving the whole camera towards or away from it. The magnification will be approximately unity.

Photography of traces on the oscilloscope screen

Remove any colored plastic window that may be in front of the screen. Clip on the close-up lens and focus the camera as described above. For stationary patterns set the film selector to 3000 and give a bulb exposure of about 1 second duration. It is not necessary to darken the room. For single-trace work it may be necessary to set to 75 and darken the room or add a light shield around the oscilloscope face, long enough to reach to the camera. Keep the shutter open for the minimum time necessary to record the trace.

B. Models 95, 150, 160, 800 Polaroid Land Camera

These cameras all use roll film and give a picture size just under 3 × 4 inches. Models 150, 160 and 800 have a range finder; model 95 does not. On these cameras one adjustment determines both the lens opening and the time the shutter stays open. Speed and aperture combinations corresponding to the EV numbers of the various cameras are given in the table.

Notice that to convert EV numbers given for a model 95B, 150 or 800 to values for a model 95 or 95A, one must subtract 9; and vice versa. A setting of 15 on one series gives the same exposure as a setting of 6 on the other series. In this note and others in this Teacher Guide, we will give both settings—for example, EV 15(6).

A decrease of one unit in EV number means that twice as much light reaches the film. This is true for "instantaneous" photographs, but not necessarily so for time exposures. Note from the table that at all settings below EV 13(4) the camera lens is wide open. For time exposures any further decrease in EV will not affect the amount of light reaching the film.

All these cameras have a little knob on the camera face close to the lens. This can be set to either I—for "instantaneous"—exposures (exposure times as given in Table 2), or to B—for "bulb"—exposures (shutter remains open as long as the shutter release or cable release is held depressed). This knob returns to the I position automatically after every bulb exposure, and must be reset to B for each time exposure. Failure to reset it is the most common cause of unsuccessful exposures. (Possibly the second commonest cause is forgetting to check that there is film in the camera.)

Table 2

Models 95A, 95B, The 700, 150, 160, and 800 Cameras				Model 95 Camera		
Shutter No. Models 95A, The 700	Shutter No. (EV Scale) Models 958, 150, 800	Lens Opening	Shutter Speed	Shutter No.	Lens Opening	Shutter Speed
1	10	f/ 8.8	1/12 sec	1	f/11	1/8 sec
2	11	f/ 8.8	1/25 sec	2	f/11	1/15 sec
3	12	f/ 8.8	1/50 sec	3	f/11	1/30 sec
4	13	f/ 8.8	1/100 sec	4	f/11	1/60 sec
5	14	f/12.5	1/100 sec	5	f/16	1/60 sec
6	15	f/17.5	1/100 sec	6	f/22	1/60 sec
7	16	f/25	1/100 sec	7	f/32	1/60 sec
8	17	f/35	1/100 sec	8	f/45	1/60 sec

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Film

The most useful type of film for classroom use is the 3000-speed, type 47. It is the most sensitive and has the shortest development time—10 seconds. The two transparency films are useful occasionally but are less sensitive. One of them (46-L) also needs longer development time.

Table 3

Film	ASA Speed Value	Development Time	
47	3000	10 seconds	prints
46-L	800	2 minutes	half-tone transparency, for slides
Polaline 146-L	120	10 seconds	black-and-white transparency (high contrast for line drawings)

If prints are to be kept more than a few days, they should be coated soon after exposure with the squeegee supplied with each roll of film. Prints are normally somewhat curled; flatten prints by pulling over a straight edge, picture side up, before coating them. Transparencies are preserved by immersion in "Dippit" liquid for at least 20 seconds, and can then be mounted in easily assembled frames for projection. Read the instructions supplied with film, with "Dippit" and with slide frames for more details.

Exposure

It is impossible to give hard-and-fast rules about exposures, as these will vary according to local conditions. Exposure values given in the notes on particular experiments and demonstrations must be regarded as suggestions only. In all kinds of multiple-exposure photography (blinky, strobe), it is important to increase contrast as much as possible. It is not necessary to have a completely blacked-out room. Regular opaque shades are quite adequate; some venetian blinds are satisfactory. A black background (such as the cloth screen provided by Project Physics) will improve the contrast enormously. In the particular conditions of our laboratory at Harvard, we have found the following values useful starting points. (One must always be prepared to move up or down one or two EV units after taking the first picture.)

Photography of moving blinky: EV 15(6).

Photography of moving light source (pen-light cell and bulb) with 300 rpm disc strobe: EV 14(5).

Xenon strobe photography—falling steel ball: EV 16(7).

Xenon strobe photography—white mast on dynamic cart: EV 15(6).

C. Photographic techniques (refers to all models)

All blinky or strobe photographs could be called multiple-exposure. By multiple-trace we mean the recording of more than one motion (of the same or different bodies) on one photograph. An example is the Unit 1 demonstration, "Vector Addition of Velocities." Usually it is necessary to move either the object or the camera slightly between each trace, or to tilt the camera a bit, to prevent successive traces overlapping. The shutter must be recocked (model 002) or the knob returned to B (model 95, etc.) each time. The background light level is more important in this sort of work, but up to 20 blinky traces have been recorded on a single print.

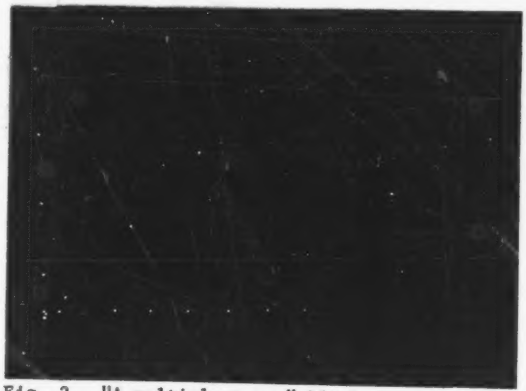


Fig. 2. "A multiple trace" blinky photograph. Shutter setting for 3000-pound film was EV 16 (7). On model 002 experimental camera set selector to 75.

How to use the pictures

Ideally, each student team will be able to make and analyze its own photographs.

Students can probably best make measurements using a magnifier, made of a 10X set of magnifying lenses and a transparent scale. Even quite dark prints can be measured with the magnifier in good light. (Hold the print against a window pane or put it on the stage of an overhead projector, or use a reading lamp close to the print.) The scale is made much more visible by backing it with 1/16" wide white sticky tape (ACS tape).

Also satisfactory is a technique using dividers and millimeter scale.

Because the protective coating takes several minutes to dry, it will save time to measure before coating; however, the uncoated emulsion is soft and easily scratched.

Students can also use the "negative" to take measurements; this halves the number of exposures needed. To preserve the "negative" wash it with a damp sponge and coat it in the usual way.

If it is impossible for each team to produce its own print, the information on one print can quickly be passed on to the class by projection. Carefully make a pin prick at each dot on the print and use the overhead projector to project onto a sheet or pad of paper pinned to the wall. You may need a sheet of glass or corner weights to keep the print flat. A trial may be needed to find the best hole size for your projector. Each student makes his own copy of the print by making a mark at each point of light on the projected image; he then takes down or tears off his sheet, and the next student in turn makes his enlarged copy of the print. These enlarged copies can be measured with rulers.

It is possible to make projection transparencies from black and white prints on a thermofax copying machine. Do not coat the print before making a copy. A high contrast print and careful adjustment of the lighter/darker knob on the copier are important.

For demonstrations it may be useful to project at high projection magnification directly onto a meter stick and simply read off the positions of the dots.

When using projection techniques, make sure that the projected image is not distorted—i.e., that the projector is set perpendicular to the wall so that no "keystoning" exists. A quick way to check your projector for keystoning is to place a transparent ruler in the position later to be occupied by the photograph and to see if the scale in the projected image remains similar to the original one. Measure distances between equivalent points, e.g., cm marks. Most projectors introduce some distortion near the edge of the picture area.

Opaque projection of prints is only marginally successful. Most opaque projectors do not have a lamp that is bright enough.

Polaroid transparency film (types 46-L and 146-L) can be projected, using either an overhead or 3" × 4" slide projector. This is more successful than opaque projection of prints, but in general has been found less useful than the projection of pricked-through prints. Transparency film is not available in pack form and so you cannot use the technique with experimental model 002 camera.

Scale

For many experiments and demonstrations, distance measurements can be made in arbitrary units: millimeters on the film is most convenient. Similarly, time intervals can often be expressed in multiples of an arbitrary unit—flashes of the blinky or the strobe. But there are instances in which it is necessary to know actual distance and time values in conventional units. In the determination of the acceleration of a freely falling body, for example, you must convert distances and times into some familiar units to compare your result with known values.

Fig. 3 Single exposure, 3000 film, model 002 camera. Falling light source, disc strobe. Selector at 75, room lights on, electric eye covered.

It is quite easy to take a picture that shows both a moving object and a scale (e.g., a meter stick). For the scale image to be useful, the scale must be in the same plane as the motion being photographed.

With the experimental camera (model 002), both the moving object and the scale can be photographed in a single exposure (Fig. 3). See Table 1 for recommended exposures and lighting conditions. If you are using one of the older cameras (95, 150, etc.) a double exposure may be necessary (Fig. 4).

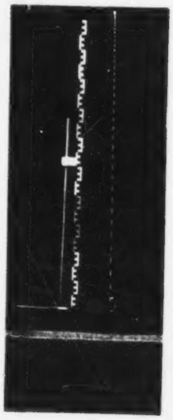


Fig. 4 Double exposure photograph, 3000 speed film.
(a) To photograph scale: EV 13(4), instantaneous exposure, with room lights on. This type of scale, with one centimeter wide bars is easier to photograph than a scale with millimeter divisions.
(b) To photograph falling light: EV 13(4), bulb exposure, darkened room. Disc strobe with 18 slots, 300 rpm. For explanation of the elongation of the images see article on stroboscopic photography.

It may be worth bearing in mind that if the 10X magnifier is used to measure photographs taken at 10:1 reduction, each millimeter on the print is one centimeter in real space.

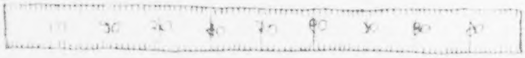
If two points in real space are known to be a certain distance apart when photographed, it is possible to reestablish the real scale by projection. Move the projector to or from the screen until the images of the two points are the same distance apart as the objects were.

Checklists of the actual operations involved in using the two types of camera appear in the Student Handbook.

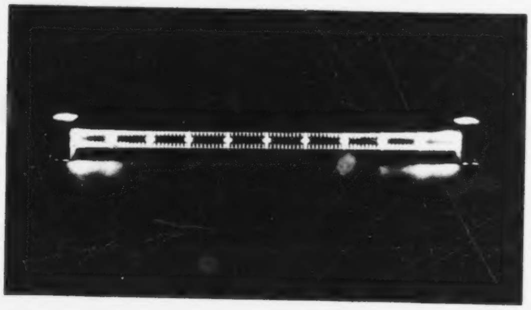
Illuminated scale for Polaroid photography

Often you want to include a scale (meter stick) in a strobe photograph so that you can convert measurements taken on the photo into real units. Although you can do this by double exposing a real meter stick, the special illuminated scale described here makes this sort of photography easier and produces very impressive pictures.

Take a piece of 1/4-inch thick lucite, about 1 1/2 inches wide and one meter long. Use an engraving tool to inscribe lines at exactly one centimeter intervals. The lines should be about 1 mm deep. Make every tenth line the full width of the rule. Number the 10, 20, 30...cm marks IN REVERSE, scribing the numbers carefully and being careful not to scratch adjoining surfaces of the lucite. The scale (engraved side up) should now look like this:



To use the scale, shine light into the stick from the two ENDS. One way of doing this is shown.



Light is scattered in all directions wherever there is a scribed line on the stick, causing the numbers and lines to show up bright against a dark background. Set up the stick so that you view it from the UNSCRATCHED face. You will then see the numbers in proper position. (If the numbers were on the front of the stick, some light would be scattered from them back into the ruler, and be reflected from the back, causing a double image.)

Notes: Stroboscopic Photography

Introduction

Many of the experiments and demonstrations described in the Teacher Guide require stroboscopic photographs. There are several reasons why we use this technique so often.

a) The strobe photograph can sometimes give at a glance a qualitative idea of the time-displacement relationship in a particular motion (for instance, uniform circular motion, free fall and trajectories).

b) The strobe photograph is a permanent record. Measurements made on a permanent record can be more precise and unambiguous than those made during the fleeting moment while the event is occurring. The measurements can be checked several times if necessary. (Strobe photographs are by no means the only permanent records that will be used in this course. See, for example, the experiment on uniform motion, the photography of spectra and the use of a strip chart recorder. This corresponds to a very modern tendency in the research lab—namely, to let the event "record itself"—on an xy plotter, or on-line computer, etc.)

c) Measurements can be made over rather short time intervals, so that rapidly moving objects and short-duration events can be analyzed.

Someone familiar with strobe techniques can often very quickly take a photograph to illustrate a point discussed in class. The more familiar one is with the camera and stroboscope equipment and their use in the particular local conditions of background illumination, etc., the more easily these demonstrations can be performed and the more effective they become. Of course this familiarity comes from experience. These notes may be useful to those who are unfamiliar with the various strobe techniques.

Techniques

It is convenient to classify three kinds of stroboscopic photography. Most of the experiments and demonstrations described in this Teacher Guide can be done by any of the three methods.

a) The moving object is illuminated intermittently by an intense light—e.g., by a xenon strobe.

b) The moving object carries a flashing light source—e.g., a blinky (relaxation oscillator with neon bulb).

c) The moving object carries a steady light source, and light from this source to the camera is interrupted by a chopper in front of the camera lens, e.g., by a motor-driven disc strobe.

Xenon strobe

Xenon strobe photography has the advantage that often nothing needs to be added to the moving object. But xenon stroboscopes are expensive: prices range from $58 (Stansi model 1812W, Stansi Scientific Co., 1231 N. Honore Street, Chicago 22, Illinois) to about $275 (Strobotac by General Radio Co., West Concord, Massachusetts), or more. The Strobotac is calibrated and can be set to flash rates between 110 and 25,000 per minute. The Stansi strobe gives much more light, but is uncalibrated (see notes on "Calibration of Stroboscopes"). Of course, once a xenon stroboscope is available, much more can be done with it than simple strobe photography: for example, the measurements of rates of rotation and some very effective visual demonstrations which depend upon the "freezing" of various motions.

As in all strobe photography, a suitable background is very important: black cloth or a surface painted flat black are good. A clean blackboard or cheap paper used in roofing and flooring can be used. But even these surfaces will give a surprisingly bright and troublesome reflection if the stroboscope is not carefully placed. It should light the background at a glancing angle, if at all.

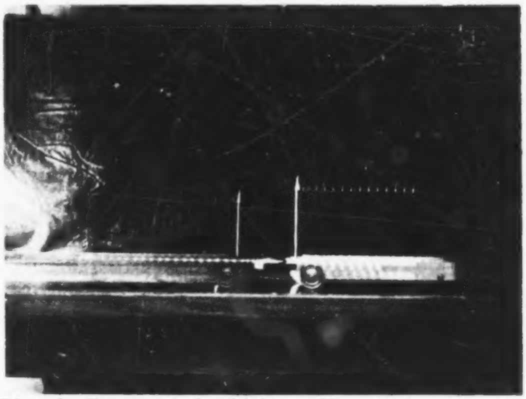


Fig. 1 Xenon strobe light reflected from black cloth background.

The moving object is illuminated from the side (or occasionally from above or below). The background should, if possible, be some distance behind the event. Small screens can be used to make sure that the background is in shadow while the object remains well lighted. The stroboscope must not be in front of the object near to the camera. Make sure that the object is illuminated by strobe light throughout the motion that you want to photograph. Sometimes it is helpful to have a student hold the stroboscope and follow the moving object. Figures 2 and 3 show typical arrangements.

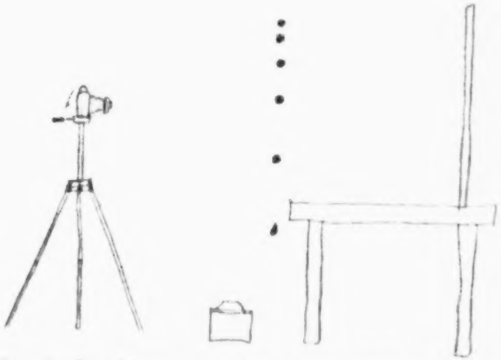


Fig. 2 In this set-up for a free fall demonstration, the xenon strobe on the floor illuminates the falling steel ball, but not the background.

Objects to be photographed using xenon stroboscope:

a) A golf ball will look more like a ball than any other object due to its surface texture.

b) A ping pong ball, if clean, will give a white disc.

c) A steel ball provides a sharp, bright point of light due to the convex mirror effect of the spherical surface. These points are ideal for taking measurements from a photograph, but the focussing effect may introduce a small error. (The camera "sees" the virtual image of the light source reflected in the polished surface of the sphere. As the relative positions of light source and ball change, the virtual image will shift. The maximum possible error that can be introduced in this way is one ball radius; for most setups it is less, and for any ball less than 1-inch diameter the error can usually be ignored. The size of the virtual image also depends on the radius of the ball. For very small balls the image may be so small that it is hard to photograph.)

d) Dynamics carts can be strobed. It is important, however, to have some bright object to serve as a reflector. A pencil painted black, except for the sharpened end which is painted white, can be fixed to the cart in a vertical position. Reflective tape (Scotchlite silver), knitting needles and metallized drinking straws are good also.

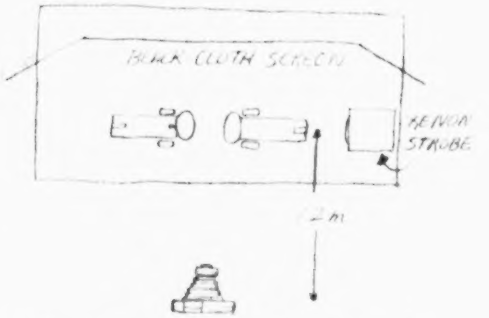


Fig. 3 Xenon strobe photography of dynamics carts. Note position of strobe and cloth screen, which is not immediately behind event to be photographed.

As always, optimum camera setting will depend upon local conditions. The photographs shown here (Figs. 4 and 5) were taken using a Stansi strobe, with a black cloth background behind the moving object.

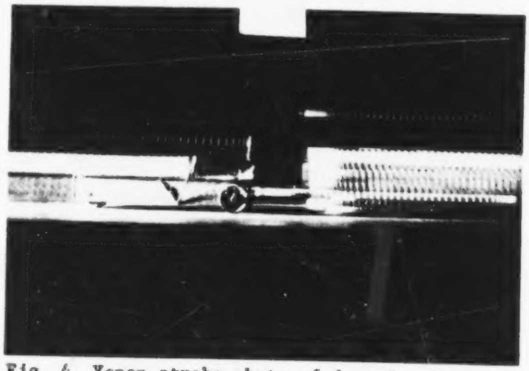


Fig. 4 Xenon strobe photo of dynamics carts. In the particular conditions of our laboratory a setting of EV/5 on model 800 camera was used. For model 002, set film selector to 3000. Strobe rate about 60 per second.

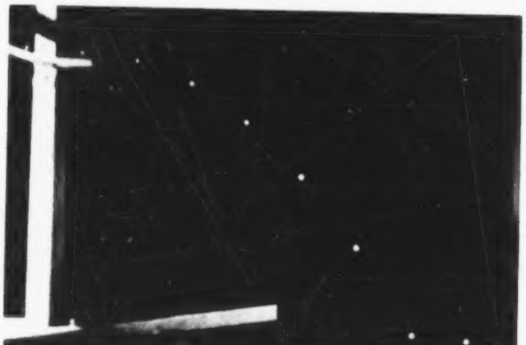


Fig. 5 Xenon strobe photograph of trajectory of steel ball. Strobe rate about 20 per second. Aperture setting of EV/16 on models 800 and 150. EV/7 on model 95. Film selector to 3000 on model 002.

Blinky

The blinky supplied by Project Physics can be made to flash at rates between about 20 and 200 per minute. It is rather expensive (largely because of the three thirty-volt batteries it contains), and it is fairly massive (300 g). But the principle of strobe photography is probably most easily explained using the blinky. The so-called "ac blinky" is certainly lightweight; it flashes at a known frequency (line frequency), but is not self-contained: it must always be attached to an ac outlet of at least 90 volts. Because of its higher flash rate the ac blinky is suitable for faster moving objects such as pendulums. It is possible to make a simple variable frequency blinky (20-2000 per second) using an audio-oscillator, amplifier, transformer and neon bulb.

Although the blinky is not always the most convenient of the three strobe methods discussed here, students will probably find a photograph taken with the blinky technique the easiest to understand. The blinky is the first choice for demonstrations early in the course (uniform motion, vector addition of velocities, etc.). Because the light output of the blinky is rather low, it is important to keep the background illumination low so that fairly wide apertures (low EV numbers) can be used without losing contrast. The data given in Fig. 6 should be regarded as only a starting point from which to establish optimum conditions for your own local situation.

Fig. 6 Blinky photograph: three traces. Model 150 camera was set to EV/17 (model 95 setting would be 8). With model 002 set film selector to 75.

Strobe-disc photography

The small light sources supplied by Project Physics have a mass of about 25 g, and so their mass can often be ignored if they are used on the 1 kg dynamics carts. But their mass can very definitely not be ignored if they are added to air track gliders, the smallest of which has a mass of about 30 g.

The heart of the motor strobe kit is a 300 rpm synchronous motor. The disc supplied by Harvard Project Physics has 12 equally spaced slots. This gives a maximum strobe rate of 3,600 per minute or 60 per second. By taping over some of the slots (so that the open slots are equally spaced), the rate can be reduced—to 300 per minute (5 per second) when only one slot is open. (The requirement that the open slots be equally spaced limits the possible rates to submultiples of the maximum frequency.)

Of course, by changing the motor or the disc, the range can be extended. Synchronous motors of various speeds are available from most radio-supply houses (Lafayette, Allied, Radio Shack, etc.). Extra discs can easily be made of paper.

Table 1. Strobe rates for 300 rpm motor

Number of Slots Open	Rate	Time between Successive Images
12	3,600/min	$\frac{1}{60}$ sec
6	1,800/min	$\frac{1}{30}$ sec
4	1,200/min	$\frac{1}{20}$ sec
3	900/min	$\frac{1}{15}$ sec
2	600/min	$\frac{1}{10}$ sec
1	300/min	$\frac{1}{5}$ sec

Fig. 7 Disc strobe photograph of dynamics carts: 1.5 v light source on each cart. Six slots, 300 rpm (30 per second). Shutter setting EV/14(5), on old cameras; film selector to 75 on experimental model 002.

Fig. 8 Disc strobe photograph of D 2.5—uniform acceleration. One-slot disc, 300 rpm (5 per second). 1.5 v light source; EV/14(5), or film selector to 75.

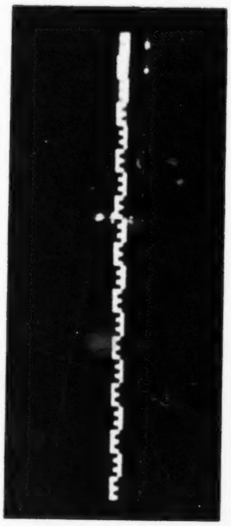


Fig. 9 Free fall, disc strobe technique, showing elongation of images.

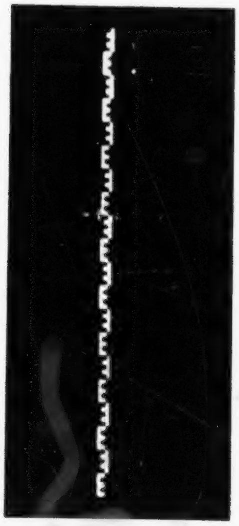


Fig. 10 Free fall, disc strobe technique. Slit on camera lens reduces elongation of images.

A variation on the disc-strobe can be used if it is impossible or undesirable to attach a light source to the moving object. Illuminate the event strongly (two 100-watt bulbs with reflectors) and photograph the object by the light it reflects. Of course a highly reflecting object, like a steel ball, is needed.

An important point to remember when using the disc strobe technique is illustrated by the pair of prints shown in Figs. 9 and 10. A slot 0.5 cm wide in a disc of 10 cm radius rotating at 300 rpm takes about 0.005 seconds to pass in front of the camera lens (diameter about 1.5 cm). The camera lens will be "open' for this time. In 0.005 sec a body moving at 4 m/sec (the speed of a freely falling body 80 cm below release) will move about 2 cm. This explains the elongation of the images—which increases with the speed of the object—in Fig. 9. This elongation is reduced, but not completely eliminated, by taping a fixed slit (supplied with the kit) to the camera lens (Fig. 10). This slit should be parallel to the slot in the rotating disc as it passes in front of the lens. The length of the streak could be further reduced by using a narrower slit on the lens, but image brightness will be reduced by lens slots narrower than the disc slot.

The duration of a blinky flash is about 0.010 seconds, but since the blinky is unlikely to be used for fast-moving objects, the problem of image elongation is unlikely to occur. The duration of a xenon stroboscope flash can be several orders of magnitude less, ranging from about 5 microseconds in more expensive strobes to around 10 or more microseconds in others. In all experiments that one is likely to do in the classroom, the object will be effectively "frozen" by stroboscopic illumination.

Calibration of Stroboscopes

Inexpensive stroboscopes are usually uncalibrated; that is, the numbers on the frequency-control dials don't correspond to actual frequencies. Below are several methods for finding the dial readings that correspond to a set of known frequencies. A calibration graph is constructed by plotting the dial values against the known frequencies, and drawing a smooth curve through the plotted points. Dial readings can then be converted to frequencies by interpolating the calibration curve.

1. Oscilloscope method

a) "Linear trace" on oscilloscope.

Connect a phototube (such as the IP39 tube which is part of the phototube module supplied by Project Physics) to the vertical input terminals of the oscilloscope. Notice that no voltage source is needed in this circuit.

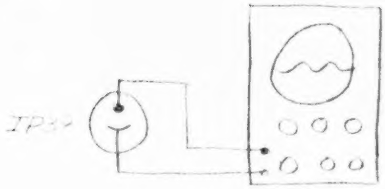


Fig. 1

Set the horizontal sweep rate to about 10 per second. Adjust the vertical gain until you see a 60-cycle trace on the oscilloscope (the phototube has a very high impedance, and the wires to it act as an antenna picking up 60-cycle noise). Adjust "sync" control of oscilloscope until this 60-cycle pattern is stable.

Position stroboscope so that light from it falls on the phototube. Each flash will produce a sharp vertical line on the trace. Adjust the flash rate until there is one flash per cycle of the 60-cycle pattern (about 80 on the high range of Stansi stroboscope 1812W).

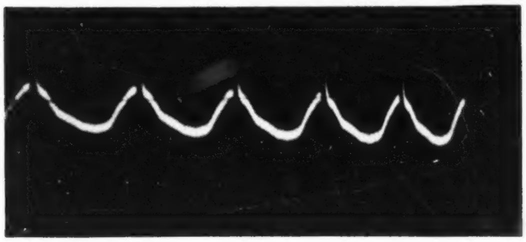


Fig. 2

With the flash rate f slightly above 60 per second the lines will be slightly less than one "wavelength" apart, and will move to the left. And vice-versa: if f < 60 per second the lines will be more than one wavelength apart and will move to the right. Only when the strobe rate is exactly 60 per second will the vertical lines be stationary on the 60-cycle trace.

Now reduce the strobe rate. The next simple frequencies to recognize are 30 per second (about 35 on the Stansi strobe, high range), and 20 per second (0 on Stansi, high range).

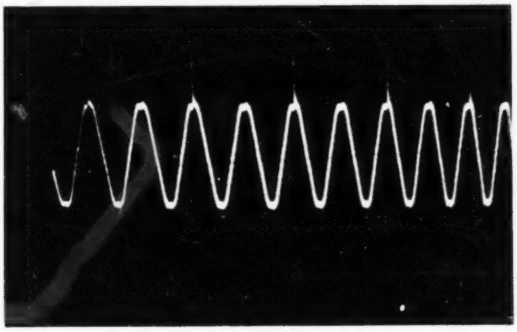


Fig. 3

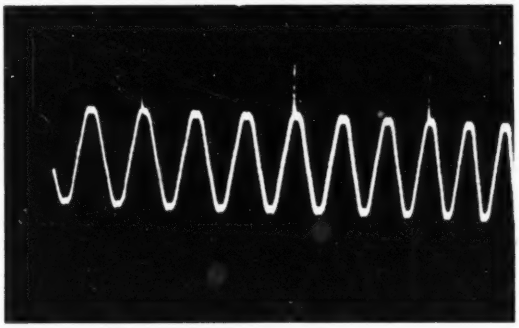


Fig. 4

Patterns for other fractional frequencies of 60 per second can also be recognized and interpreted.

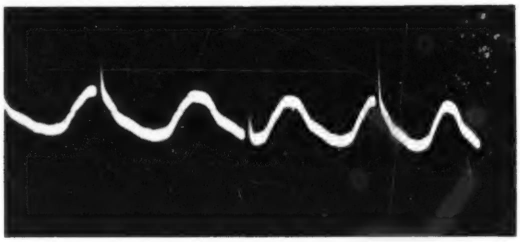
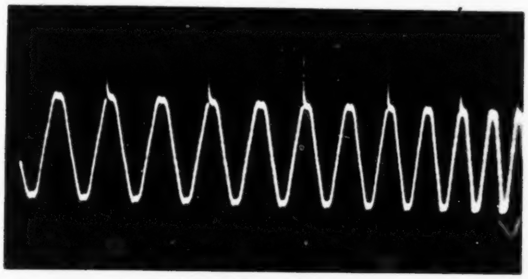


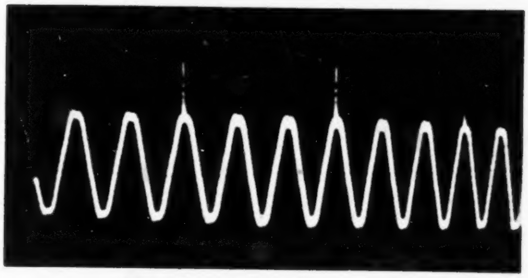
Fig. 5

In Fig. 5 there are two flashes for every three 60-cycle periods. The time between flashes is therefore $\frac{1}{2} \times 3 \times \frac{1}{60} = \frac{1}{40}$ sec. So the frequency is 40 per second.

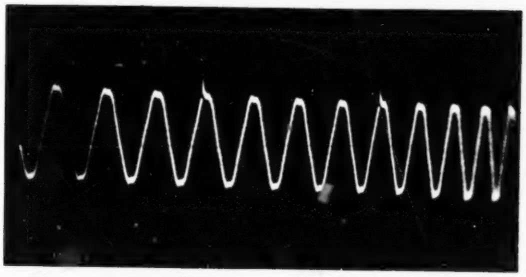
On the low range of the Stansi strobe the following stationary patterns were observed:



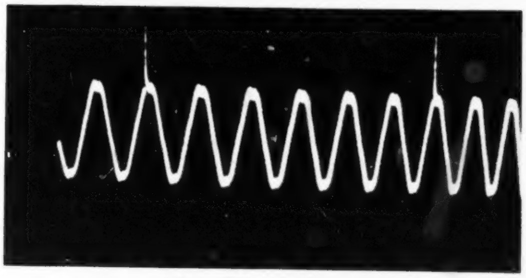
f = 1/2 × 60 = 30 per second



f = 1/3 × 60 = 20 per second



f = 1/4 × 60 = 15 per second



f = 1/6 × 60 = 10 per second

Fig. 6

b) "Circular" trace on strobe.

Connect the phototube to the vertical input as above.

Establish a circular or elliptical trace on the oscilloscope face either by (a) setting the horizontal frequency selector to line sweep; or by (b) setting to external input and connecting the horizontal input terminal to the 60 vibrations per second calibration signal available on the scope (or simply attaching a short wire to the horizontal input which will act as an antenna to pick up 60-cycle noise). Adjust horizontal and vertical gain as necessary to obtain an open figure.

The electron beam is now tracing out one revolution of this figure in 1/60 of a second. Turn on the stroboscope. Every flash will cause a sharp vertical peak. Adjust the flash rate until this peak is stationary. The simplest figure to interpret is one flash per cycle:

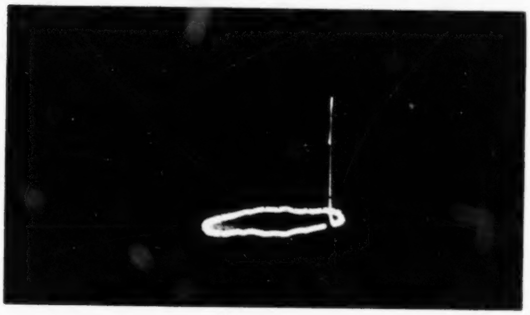


Fig. 7

As the flash rate is reduced other stationary patterns will be produced and can be interpreted. For instance, 30 flashes per second will produce a peak every second revolution of the spot.

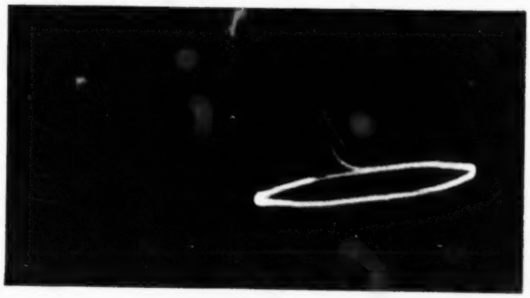


Fig. 8

Notice the subtle difference between this pattern and the previous one. Here the vertical spike is superimposed on a closed ellipse.

As the flash rate is reduced further this pattern will recur at $f = \frac{60}{n}$ per second where n is an integer, i.e.,
$f = \frac{60}{2} = 30;\ \frac{60}{3} = 20;\ \frac{60}{4} = 15;\ \frac{60}{5} = 12;$
$\frac{60}{6} = 10;\ \frac{60}{7} = 8.6;\ \frac{60}{8} = 7.5;\ \frac{60}{9} = 6.7;\ldots$

Other series of stationary patterns can be produced.

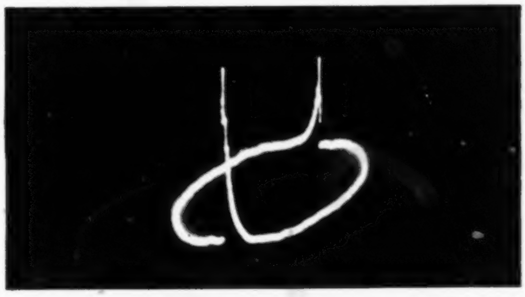


Fig. 9 Two spikes in 1/60th sec indicate a frequency of 120 per sec (but few strobes can flash at this rate—the Stansi strobe cannot).

The pattern below will occur if the strobe flashes twice in every 3, 5, 7, etc. cycles, corresponding to flash rates of $\frac{60}{3} = 20;\ \frac{60}{5} = 12;\ \frac{60}{7} = 8.6$, etc. per second.

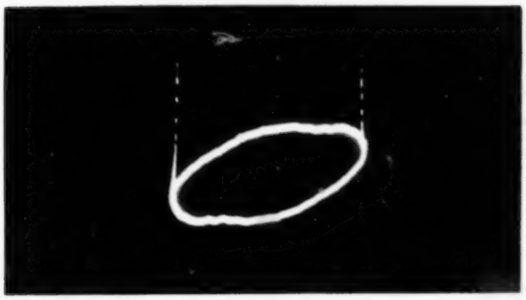


Fig. 10

Patterns containing 3 and more spikes per cycle can also be obtained.

Linear versus circular trace method

Clearly the circular technique needs more careful interpretation than the "linear trace" method described above. However, it is particularly useful at low flash rates. It may not be possible to get more than 6 cycles of 60 cycle signal on the oscilloscope face and this puts a lower limit on the frequency at which the "linear trace" method can be used: one spike in 6 cycles means $f = \frac{60}{6} = 10$ per second. The circular trace method can be used down to the lowest frequencies.

The circular trace method has the advantage that it is easier to obtain a stationary pattern on the circular trace than on the linear trace. On the other hand the linear trace is much easier to interpret. A combination of the two methods is useful. Use the circular trace to establish a stationary pattern. Then at the same flash rate switch to linear trace for interpretation.

2. Rotating disc method

Any rotating object with a known rate of rotation can be used. A synchronous motor with a suitable disc is the most reliable; some electric fans and other rotating machines have speed ratings given and could be used. Rotation rates of less than about 300 per minute are not very satisfactory—see below.

The method will be described here in terms of a specific example. Be quite careful about generalizing to other situations (i.e., other discs rotating at different rates).

Mount a disc with 12 equally spaced marks on the shaft of a 300-rpm synchronous motor (e.g., the motor strobe kit supplied by Project Physics). Add another single mark, such as a white star or a piece of masking tape, between two of the slots (Fig. 11). Start the motor, darken the room and turn on the strobe. As the strobe rate is changed, different stationary patterns will appear.

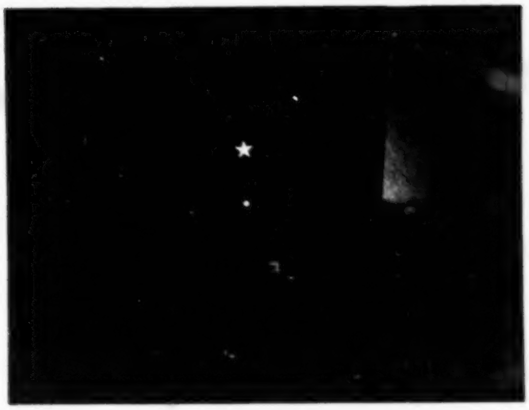


Fig. 11

The simplest pattern to interpret is one that shows twelve slots and twelve stars; the strobe is flashing twelve times for each revolution of the disc and the strobe rate is twelve times the rotation rate of the disc: 12 × 300 = 3,600 rpm.

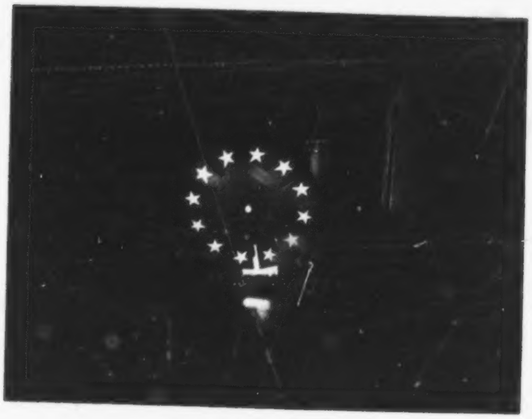


Fig. 12

Reduce the strobe rate slowly until a stationary pattern showing 12 slots and 6 stars is observed. The strobe rate is now six flashes per revolution:

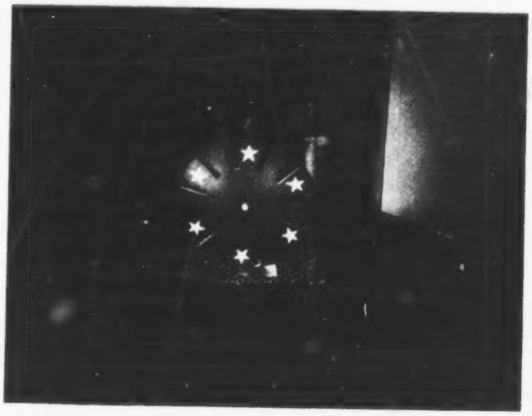


Fig. 13

Other patterns that are easy to interpret are:

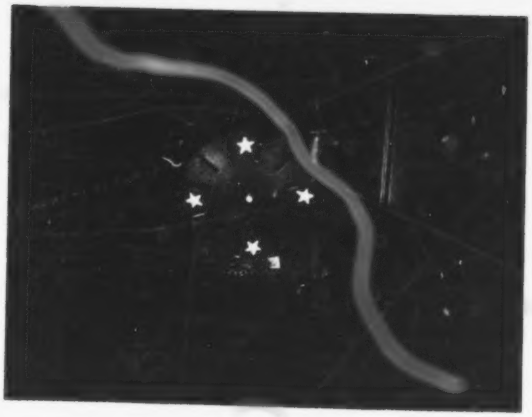


Figure 14

Flash rate = 4 per revolution
(= 1,200 per minute)

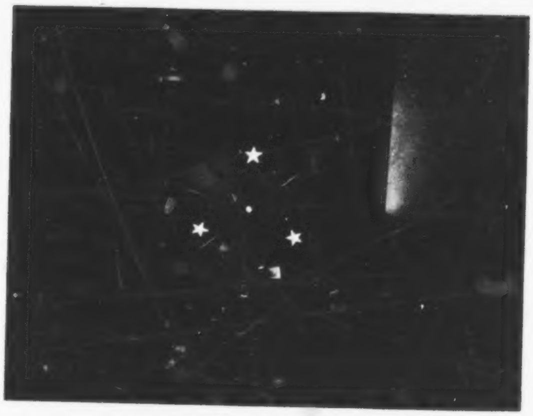


Figure 15

Flash rate = 3 per revolution
(= 900 per minute)

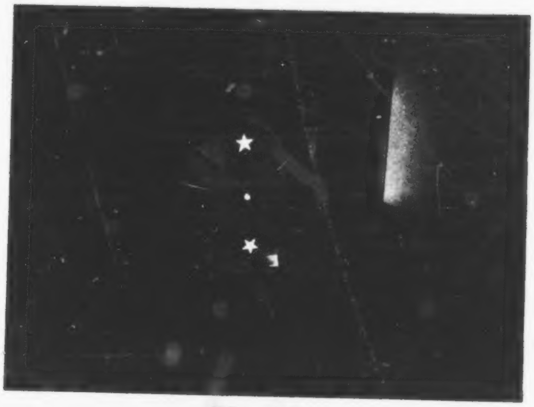


Figure 16

Flash rate = 2 per revolution
(= 600 per minute)

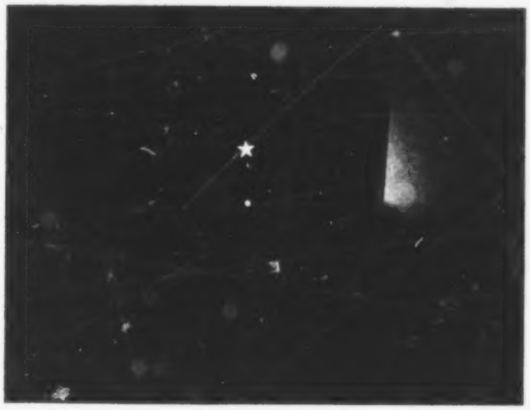


Figure 17

Flash rate = 1 per revolution
(= 300 per minute)

The last two figures, those obtained at flash rates of 10 per second and 5 per second respectively, bring us down to rates slow enough to be counted directly.

This really completes the simple calibration of a stroboscope by this method. However, it is probably worthwhile mentioning some of the other stationary patterns that can be observed, and their interpretation.

Figure 17 (above) is the pattern observed if the lamp flashes once per revolution. The same pattern would also be seen if the lamp flashed once for every two revolutions of the disc, and once for every three revolutions and so on. But in fact there need be no confusion, for two reasons: first, if one works from high to low flash rates in this calibration the first time that Fig. 18 is observed it will correspond to one flash per revolution, the next time to one flash per two revolutions, and so on; and second, in the particular case of a 300-rpm motor the flash rates concerned are low enough (5, 2 1/2, 1 1/4...flashes per second) to be identified by direct counting.

The same sort of thing will happen at other flash rates too. Consider for instance Fig. 15. The strobe flashes 3 times for each revolution of the disc. Suppose it flashed once for every 1 1/3 (=4/3) revolutions of the disc: the same pattern would be obtained. Similarly Fig. 14 would be obtained with one flash for 1 1/2 (= 5/4) revolutions of the disc as well as at four flashes per revolution. And in general a figure with n stars (which is obtained at a flash rate of n flashes/revolution) is also obtained when the rate is one flash for every $\frac{n+1}{n}$ revolutions.

Other stationary patterns can be observed in which more than 12 slots are seen. For instance, a flash rate of 8 per revolution (2,400 per minute with 300-rpm motor) will give a pattern showing 24 slots and 8 stars (Fig. 18).

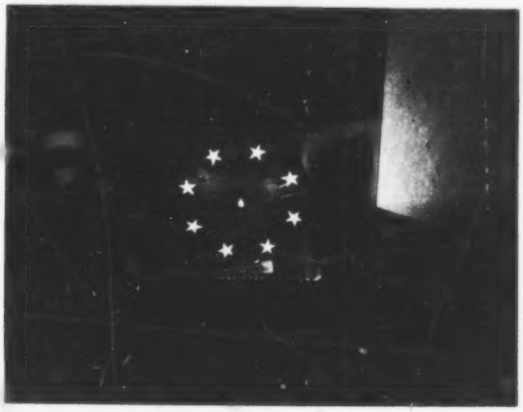


Fig. 18

Flash rates of more than 12 per revolution will give more than 12 slots, of course. At a flash rate of 16 per revolution, 16 stars, 48 slots are seen (Fig. 19).

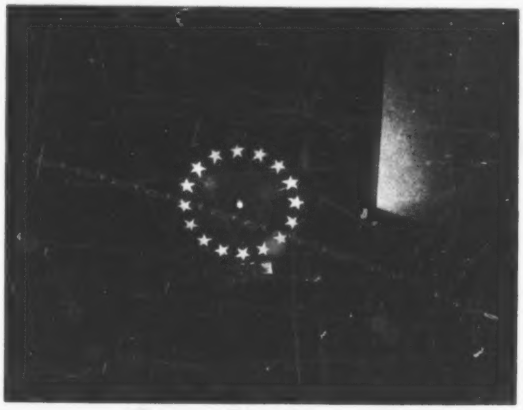


Fig. 19

Discs that rotate at 78 rpm (called phonograph turntables) are easy to obtain, but their usefulness for strobe calibration is limited. They can only be used for slow flash rates.

With a turntable rotating at 78 rpm carrying a disc with 6 symmetrical radii, a stationary image of the disc is observed for flash rates of 156 (= 2 × 78); 234 (= 3 × 78); 468 (= 6 × 78) per minute (Fig. 20). At 936 flashes per minute a disc with 12 radii is seen (Fig. 21). At higher flash rates the number of radii grows and counting them gets difficult.

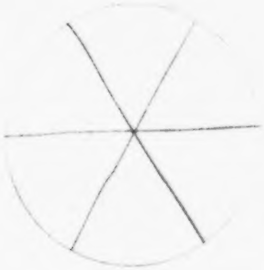


Fig. 20

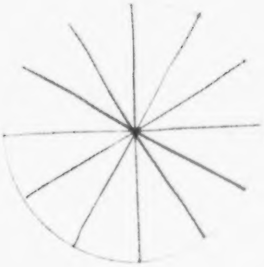


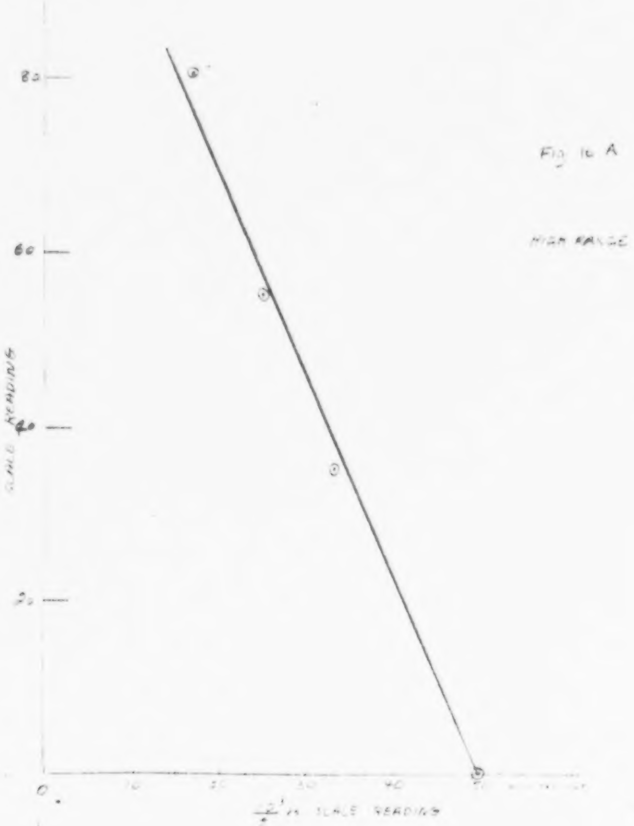
Fig. 21

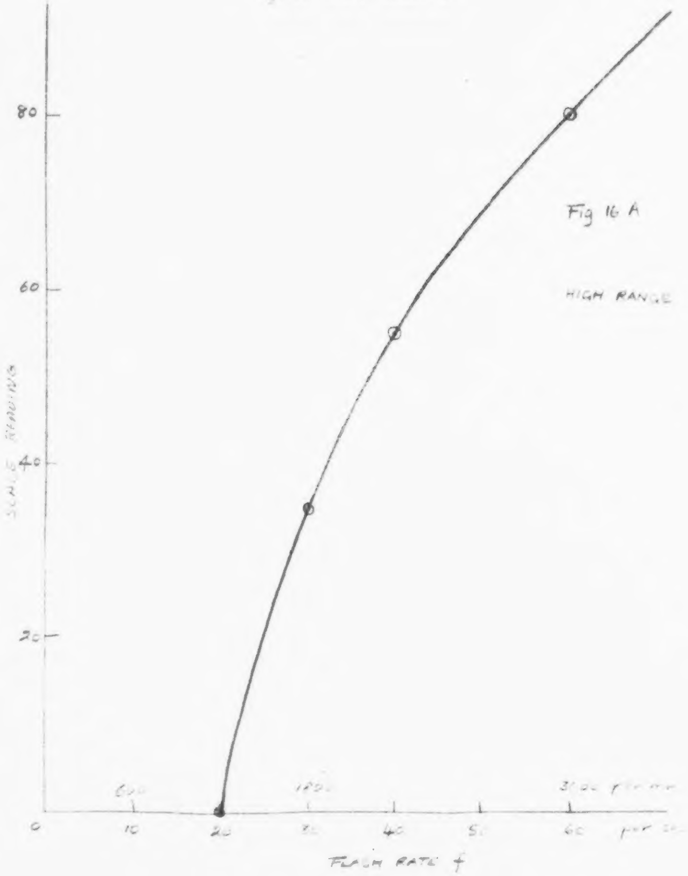
Sample results

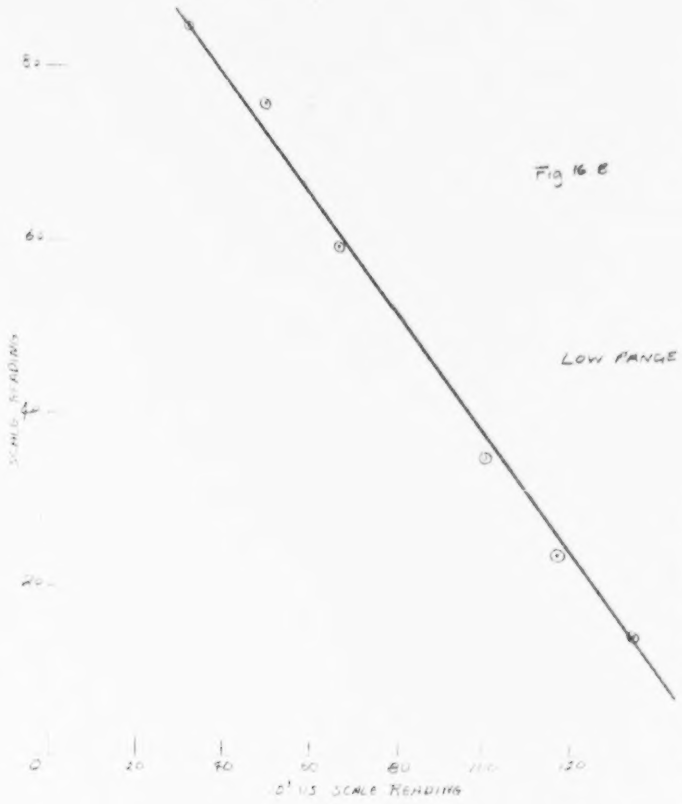
Figures A and B are calibration curves obtained for a Stansi strobe obtained by method 1 (oscilloscope). The vernier adjustment was kept at its upper limit. The plot of flash rate against scale reading is quite non-linear.

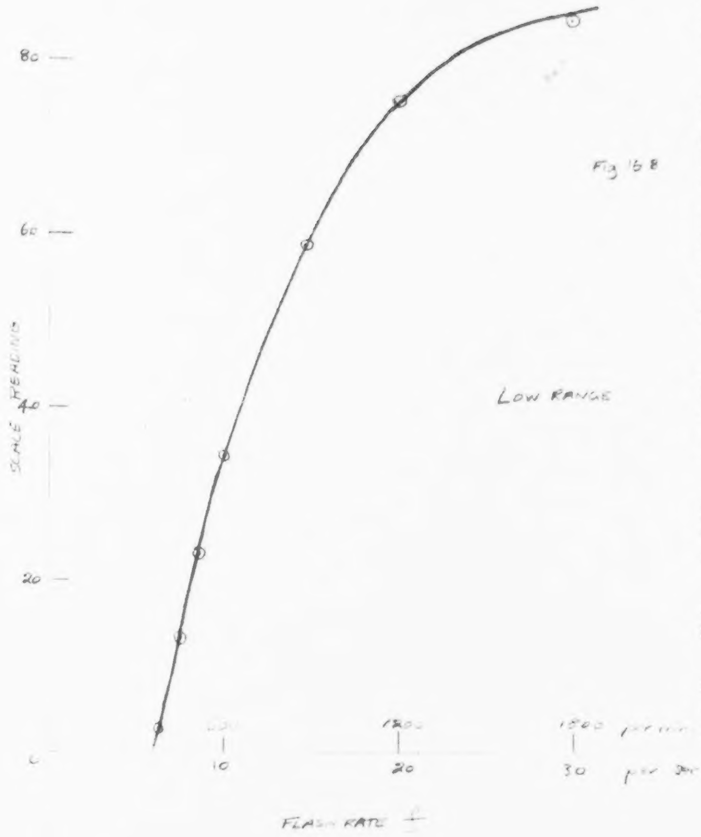
It turns out that a plot of $\frac{1}{f}$ (the period, or time interval between flashes) against scale reading is linear. (This is because the period is determined by the time constant of an RC circuit in the stroboscope, and turning the knob adjusts the resistance, evidently in a linear manner.) The plot of $\frac{1}{f}$ against scale reading makes interpolation and particularly extrapolation easier, and fewer points are needed to complete the graph.

(Note that a plot of f against $\frac{1}{\text{scale reading}}$ is not linear: this is because there is a fixed resistor in the circuit as well as the variable one controlled by the knob.
$f \propto \frac{1}{R_{\text{variable}} + r_{\text{fixed}}}$, so f is not proportional to $\frac{1}{R_{\text{variable}}}$.)









The Blinky

A simplified circuit diagram of the blinky is shown in Fig. 1.

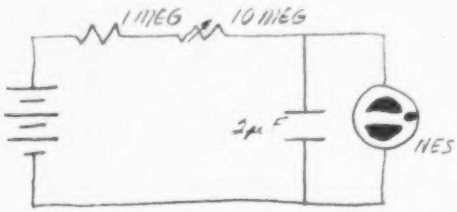


Fig. 1 The blinky

Since it goes through a certain sequence of actions periodically, on its own initiative, the blinky is an oscillator. It is one of a class known as relaxation oscillators.

The three 30-volt batteries E charge the capacitor C through the resistance R.

The neon lamp remains non-conducting as long as the potential difference across it remains below the "breakdown" voltage which is about 70 volts. The voltage across the neon lamp is of course, equal to the voltage across the capacitor. The capacitor continues to charge up until the neon lamp becomes conducting at 70 volts.

Once the neon lamp becomes conducting, the capacitor begins to discharge through it. The neon lamp continues to discharge even when the potential difference across it has fallen below the breakdown voltage. In fact, it continues to conduct and the capacitor continues to discharge through it until the potential difference across them reaches about 53 volts. This all happens very quickly; the whole process just described takes on the order of 10 milliseconds (0.01 seconds).

The capacitor now begins to charge up again from the batteries, and the neon remains non-conducting until the "breakdown" voltage is reached again. Then the neon bulb glows briefly as the voltage drops down to about 53 volts.

The knob on top of the blinky box adjusts the variable resistance which controls the rate at which the capacitor charges between discharges.

There is no switch. The only way to stop the blinky from blinking is to remove the bulb.

Do not worry about the batteries running down. The current drawn from them is very small. It is the "shelf life" of the batteries that determines how long they will last. This can be extended by keeping the blinky cool (as in the refrigerator) during the summer.

The most likely reason for a blinky not to blink is poor contact between one of the 30-volt batteries and its holder.

ac blinky

(This is not supplied by Project Physics, but is easy to make and is a useful piece of equipment for motion studies and photographs.)

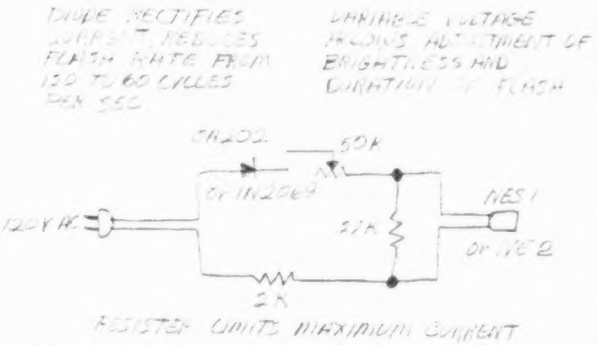
An ac blinky is a neon glow lamp circuit that operates directly from the 110-120 volt ac line. The intensity and duration of the flashes can be varied, but the flash rate (frequency) cannot: it is fixed at 60 per second.

Two factors make the ac blinky especially useful:

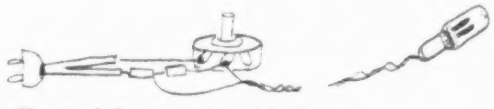
a) The flash rate is accurately known (line frequency is usually maintained very precisely at 60 per second).

b) The flash rate is high, making it useful for rapidly moving objects.

But, unlike the regular (dc) blinky the ac blinky is not a self-contained unit: it must always be plugged into the line.


DIODE RECTIFIES CURRENT, REDUCES FLASH RATE FROM 120 TO 60 CYCLES PER SEC
VARIABLE VOLTAGE ALLOWS ADJUSTMENT OF BRIGHTNESS AND DURATION OF FLASH
CR202
50K
or 1N2069
120V AC
22K
NE51
or NE2
2K
RESISTER LIMITS MAXIMUM CURRENT

(a) Schematic of ac blinky circuit



(b) Physical layout of ac blinky

ac blinky

Air Tracks

The linear air track supplied by Project Physics is an inexpensive model. Although it is quite adequate for many demonstrations and experiments, it is not a high precision device.

Little needs to be said about the operation of the track. Any medium-to-large household vacuum cleaner which can be used as a blower should be adequate. The air flow will be increased if you remove the dust-bag from the cleaner. If you use a large industrial-type cleaner (e.g., one borrowed from the school shop or from the janitor), you may find that it helps to plug it into a variac; too strong an air flow will cause the gliders to float too high and to "hunt" along the track. We have found that the compressed air supply sometimes available in laboratories is generally not enough to operate the air track.

To test the track, raise one end a few inches and release a glider from the top. The glider is running satisfactorily if it rebounds from the rubber band at the lower end of the track to within ten inches of its starting point.

Use the leveling screws to adjust the track so that a glider, released from rest, has no tendency to move towards one end or the other. Because of the slight drop in air pressure along the track, this balance will not necessarily be achieved when the track is perfectly horizontal.

The two small gliders supplied by the manufacturer have equal masses. The one large glider has twice the mass of either small one (± 2%). The three gliders allow you to perform equal-mass elastic collisions, unequal-mass elastic collisions and unequal-mass inelastic collisions. Note that if the gliders are carrying light sources for strobe photography, the mass ratios will not be 1:1 or 2:1.

The range of mass ratios can be extended by taping extra mass to the gliders. Be sure that the added mass is distributed symmetrically. It is important to keep the center of mass low and therefore it is better to add mass (equally) to the two sides of the cart than to the top. Check the glider for free running after you have added extra mass, by doing the rebound test described above. The large cart should support an extra load of at least 40 grams.

The set up sketched in Fig. 1 can be used to impart the same initial velocity to a glider on consecutive trials. Attach a small block to the glider. Draw the pendulum bob back and let it strike the block. If the pendulum is always

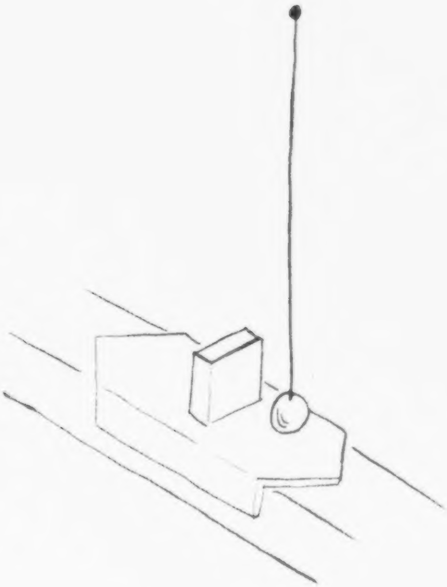


Fig. 1 Air track

released from the same point and the glider is in the same position(so that the bob hits it at the bottom of its swing), the glider will always acquire the same initial velocity.

Quantitative Work with Liquid Surface Accelerometer

Theory predicts that the slope of the liquid surface is given by

$$\tan\theta = a/g$$

Figure 1 shows an accelerometer moving horizontally with constant acceleration a.

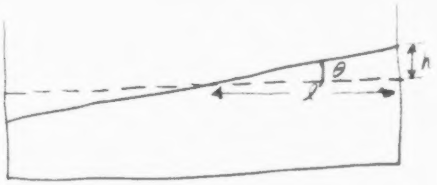


Fig. 1

If the cell has length 2ℓ and the liquid rises to a height h above its rest position at the end of the cell, then the angle θ that the surface makes with the horizontal is given by

$$\tan\theta = \frac{h}{\ell}.$$

So

$$\frac{h}{\ell} = \frac{a}{g}$$

and

$$a = \frac{h}{\ell}\,g.$$

I.e., the ratio of the two lengths h to ℓ gives the acceleration in gs.

The matter can be simplified further. Since g is almost 10 m/sec^2, if we make the length ℓ = 10 cm, then

$$a = \frac{h\ \text{cm}}{10\ \text{cm}} \times 10\ \text{m/sec}^2 = h\ \text{m/sec}^2;$$

the height h, in centimeters, is equal to the acceleration in meters/sec^2.

To read h it is convenient to stick some centimeter tape to the front surface of the cell, with the scale vertical, and exactly 10 cm from the center of the cell. The zero-mark of the scale should be at the height of the undisturbed horizontal level of the liquid, usually about half way up the cell. It also helps to stick a slightly wider piece of white paper or tape on the back of the cell, opposite the scale. This gives a definite background against which to observe the liquid level (Fig. 2).

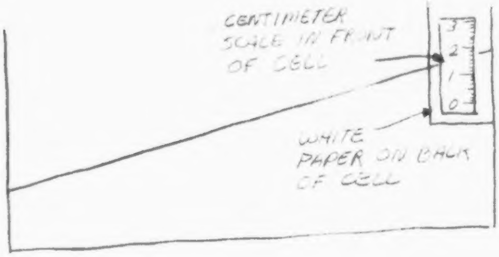

CENTIMETER SCALE IN FRONT OF CELL
3
2
1
0
WHITE PAPER ON BACK OF CELL

Fig. 2

Calibration of the accelerometer

The theoretical derivation described above can be confirmed experimentally by the following procedure. Use a conventional string, pulley and mass set up to produce uniform acceleration of a dynamics cart carrying the accelerometer. The actual acceleration can be measured from a strobe photograph. (The strobe rate and photographic reduction must be known, of course. The calculation is much simplified if the strobe rate is 10 per second, and the reduction is 10:1.)

From the same photograph the height h can be measured on successive images and the average value of $\frac{h}{\ell}$ calculated. (A variation of less than 10% was found.)

This is repeated with several different falling weights (or masses on the carts) to produce a range of values of a. The average value of $\frac{h}{\ell}$ is plotted against the average value of a for each photograph. A typical result is shown in Fig. 3.

As an alternative (which is less precise, but involves more students) have several students stationed along the cart's path and let each one observe the value of h as the cart passes him.

For further details and theoretical derivation of the formula mentioned above, see J. Harris and A. Ahlgren, Physics Teacher, vol. 4, p. 314-315 (October 1966).

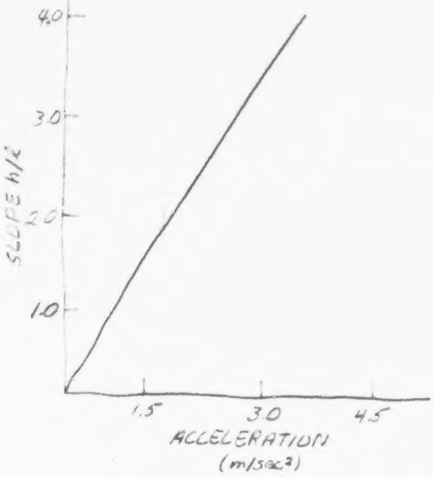


Fig. 3

A Versatile "Cannon"

A very versatile and inexpensive rubber band-powered "cannon" can be built, either as an individual activity, or as a mass-production class activity. Four of the immediate uses we have tried are:

1. a launcher for range of projectile demonstrations;

2. a launcher for the "Monkey in the Tree" demonstration;

3. a device for reproducible forces for accelerating carts, air track gliders, etc;

4. a sighting tube for astronomy (made more accurate by taping a plastic soda straw along the top of the barrel—paper gets soggy and bends in damp night air).

Our model (see Fig. 1) consisted of a 20 cm length of 8 mm bore brass tubing, with a piece of solid brass brazed to the middle. An alternative to avoid brazing would be a length of aluminum tubing with a wooden dowel fastened to it with epoxy cement and a small metal strap around the aluminum tube (see Fig. 2). The plunger consists of a wooden dowel with a larger piece of wood screwed to the end. A slot cut across the end of the wooden piece keeps the rubber band from slipping off the end of the plunger. A plastic protractor is glued to the side of the tube. A short pin is glued in the reference hole in the protractor, and a thread and washer is attached to it for determining a plumb line. For use as a sighting instrument, the handle can be put through a hole in a piece of wood which is pivoted on a flat board marked off in degrees.

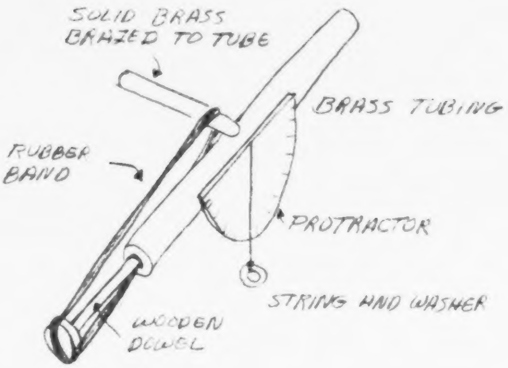


Fig. 1

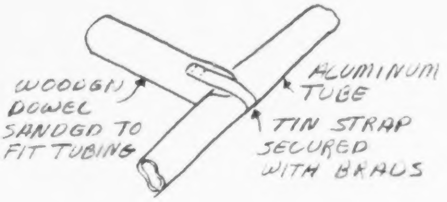


Fig. 2

Range prediction experiment

1. Determine the muzzle velocity by firing the cannon vertically, measuring h and substituting in $v = \sqrt{2gh}$.

2. Estimate the horizontal range, knowing v from above, and h, height above the floor, from the relations:

$h = \frac{1}{2} gt^2$, $t = \frac{2h}{g}$, and range = vt.

Mark the expected range on the floor and try to hit the mark.

3. For the more advanced student, develop (or have him derive) the general range formula, $R = \frac{v^2 \sin 2\theta}{g}$, and then try the experiment.

4. In Unit 3 the energy concept can be used for the same situation:

a) Make a graph of force vs. length for the rubber band.

b) From the graph, find maximum F and minimum F when the rubber band is used for a particular shot.

c) Find the mass of the plunger and cannon ball.

d) Find the estimated velocity, using

$F_{avg} \times d$ = kinetic energy ($\frac{1}{2}$ mv^2).

Sample results

Using method 3, a measured value of 4.34 meters was obtained for an estimate of 4.20 meters.

The discrepancy was slightly larger for method 4. The predicted muzzle velocity (from force-extension curve) was 6.9 m/sec; the measured velocity (from the height to which the ball rises when fired vertically) was 6.4 m/sec. A direct check with a strobe photo or two photocells and an oscilloscope would be excellent.

Note that the force vs. extension curve for method 4 (Fig. 3) does not pass through the origin because the rubber band is already stretched before the plunger is pulled back at all. There is a finite force for a zero extension (on our scale). The energy given to ball and plunger is the total "area" under the graph.

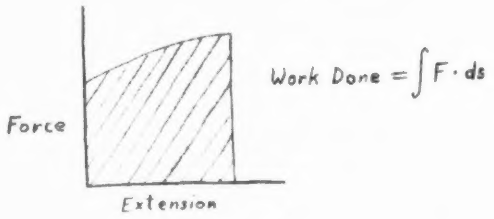


Fig. 3

Cathode-Ray Oscilloscope

The cathode-ray oscilloscope (CRO) is one of the most versatile laboratory instruments. This note can only summarize its different capabilities and functions in the lab and some of its uses as a teaching aid. The approximate numerical values given in this note refer to a typical inexpensive 'scope such as the Heathkit Model IO-12 ($126.95, wired from Heath, Benton Harbor, Michigan).

Functions of the oscilloscope

The CRO is a voltmeter. It can measure voltages down to about 10^{-2} volts (depending on the amplifier). It can measure short voltage pulses (down to about 10^{-6} sec). Because it has a high input impedance, it draws little current from the voltage source being measured.

As well as being used to measure voltages, the CRO is useful as a null detector. Examples are:

a) a phototube (illuminated by a pulsed light source) is connected through an amplifier to a CRO and the reverse voltage across the phototube is increased until the pulses on the CRO trace disappear, indicating that the "stopping voltage" for the photoelectrons has been reached.

b) an ultrasound detector or microphone is connected to the CRO and moved through an interference or standing-wave pattern until the signal falls to zero, indicating that the detector is at a node (point of zero intensity).

The CRO can be used to show the wave form of a voltage signal (sinusoidal, square, saw-tooth, etc.) and to measure the phase difference between two signals. It can be used to measure time intervals (10^{-1} to 10^{-6} sec) and frequency (10 to 10^6 cycles per second).

These and other functions make the CRO a valuable tool for "trouble shooting" in the lab, in the repair of radio and TV sets and in electronics work generally.

The oscilloscope as a teaching aid

The CRO also has very many applications in the teaching of physics, some of which are listed here.

Electricity: demonstration of the effect of capacitance and inductance in a circuit; phase relationships between voltages across different elements in an LCR circuit; oscillations in tuned circuits.

Sound: demonstrations of the wave forms of pure and impure tones, beats.

Simple harmonic motion: addition of two sine curves to slow amplitude modulation (beats) and Lissajous figures; measurement of phase and frequency; Fourier synthesis.

Electronics: display of the function and characteristics of devices such as diodes, transistors, vacuum tubes, etc.

Time measurements: time-of-flight measurement of projectiles, pulse of sound, etc.; display of pulses from Geiger counter.

Detailed notes on some of these demonstrations appear in the section "Examples of the Use of a CRO in Teaching Project Physics," and in the appropriate sections of the Teacher Guide.

The CRO can also be used to set up some very effective attention-getting displays, corridor demonstrations, science fair projects and so on. Examples of interesting traces are given in Figs. 1, 2 and 3.

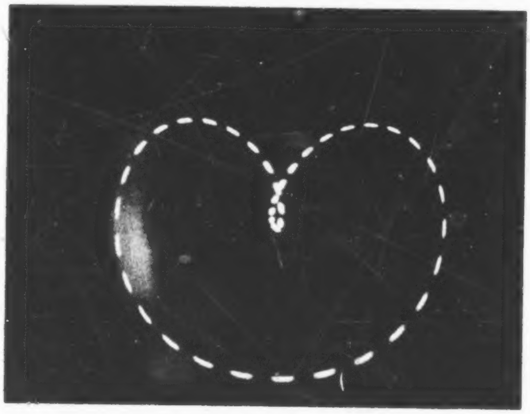


Fig. 1

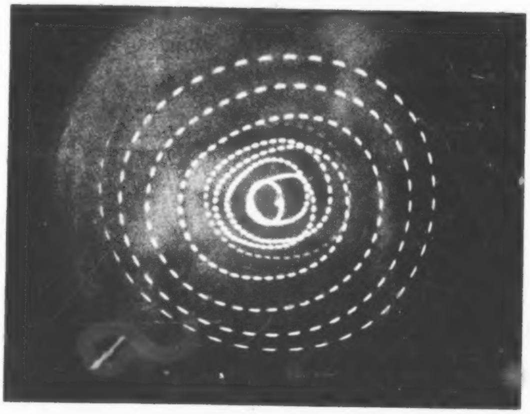


Fig. 2

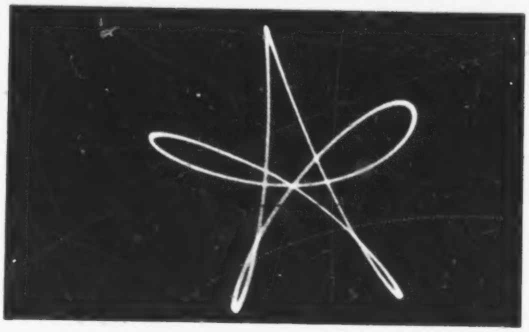


Fig. 3

Operation

These notes are necessarily of a very general nature. Refer to the manufacturer's instruction manual for detailed notes on the operation of a particular oscilloscope.

The ON/OFF switch is often combined with the INTENSITY control. Wait a minute or two after turning on before trying to get a trace on the screen. Adjust intensity and the FOCUS knob until a bright sharp spot or line is obtained. Its position on the screen can be varied by means of the VERT(ICAL) POS(ITION) and HOR(IZONTAL) POS(ITION) controls. It is bad practice to leave the 'scope turned on with a high intensity stationary spot—it may burn a hole in the phosphor coating of the screen.

Vertical deflection: voltage measurement

The cathode-ray tube itself is sensitive to both dc and ac voltages, but its sensitivity (displacement of the spot per volt of potential difference between the deflecting plates) is low—typically of the order of 0.2 mm per volt. Amplifiers are therefore added to increase the sensitivity. In most simple oscilloscopes these are ac amplifiers, and so these oscilloscopes cannot be used for dc signals.* A dc oscilloscope has a dc/ac switch which must be set to the appropriate position.

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*In ac 'scopes it is sometimes possible to bypass the amplifier and apply a signal directly to the tube, thus getting a deflection for a dc input. This usually involves removing a panel at the back or side of the instrument to expose the appropriate terminals. PROCEED WITH GREAT CAUTION: THESE TERMINALS MAY BE AT VOLTAGES AS HIGH AS 1500 V. Be sure to unplug the instrument before you expose the terminals. Because of the low sensitivity of the cathode-ray tube itself, the deflection will probably be small.

A voltage signal to be measured is applied between the VERT INPUT terminal and GROUND terminal. (Make sure that the connection to ground is consistent with the circuit or device providing the signal: i.e., beware of "crossed grounds.") Voltages applied here deflect the beam up and down on the screen. This terminal is sometimes referred to as the Y INPUT, and the deflection as Y DEFLECTION. The amplification of this signal is controlled by two knobs which may be called VERT INPUT, VERT GAIN, VERT ATTENUATOR, VERT AMPLIFIER, etc. Usually, one knob provides coarse control in three or more steps (e.g., 1X, 10X, 100X) and the other gives fine control. In more expensive oscilloscopes these controls are calibrated in volts per centimeter deflection of the spot on the screen. With simpler 'scopes it is necessary to calibrate the sensitivity at a given setting by applying a signal of known voltage and measuring the deflection. Such a calibrating signal may be provided at one of the terminals on the 'scope itself; on the Heath 10-12, for example, the 1 volt P-P terminal provides a 60-cycle signal with a peak-to-peak amplitude of 1 volt. (Note that the legends 100X, 10X, 1X may refer to how much the input signal is attenuated rather than to how much it is amplified, so the 1X is the range of highest sensitivity.)

Pickup

Sometimes you may find that a signal is seen on the oscilloscope face even if no obvious voltage is applied to the oscilloscope input. To see some of the characteristics of this "pickup", try the following procedure. Set the FREQ SELECTOR to about 10/sec, and turn up the VERT GAIN to the maximum setting. Attach one end of a short length of wire to the VERT INPUT leaving the other end unconnected. An approximately 60-cycle sinusoidal trace will appear on the oscilloscope. Its amplitude increases if you touch the end of the wire, or use a longer piece of wire.

The wire is acting as an antenna and is picking up the 60-cycle electromagnetic field that exists, to a greater or lesser extent, in the vicinity of any 60-cycle current, and is particularly strong near, transformers fluorescent lamps, etc. Although the ac voltage due to the varying field is small, the large amplification and high input impedance of the oscilloscope can result in an appreciable trace amplitude.

Connect a resistor (R∿1 megohm) between the "antenna" and the ground terminal of the CRO. The amplitude of the signal decreases, but is still appreciable. If the value of R is decreased the pick-up becomes smaller.

Because of this spurious "pick-up" signal, shielded cable must be used to connect the CRO to high impedance, low voltage sources. The same considerations apply to all high-gain amplifiers. (Note that the phototube supplied by Project Physics—R∿5 megohms— is mounted in a grounded metal box, and shielded cable is used to connect it to the amplifier.

Horizontal deflection: measurement of time, frequency, etc.

With the HORIZ/FREQ SELECTOR (or SWEEP SELECTOR) set to EXT(ERNAL), a signal applied to the HORIZ INPUT (X input) terminals causes the beam to move left or right across the tube face (horizontal or X deflection). As with the VERT INPUT the signal is amplified and except on dc oscilloscopes a steady (dc) voltage does not produce a deflection. The amplification is controlled by the HOR(IZONTAL) GAIN or HOR(IZONTAL) AMP(LITUDE) knob.

When the FREQ SELECTOR or SWEEP SELECTOR is in the LINE SWEEP position, a 60 cycle-per-second sinusoidal voltage is applied to the horizontal deflection plates; if there is no vertical deflection, the spot will move back and forth across the screen in simple harmonic motion. Note that the deflection is not linear with time in this setting. If another sinusoidal voltage is applied to the vertical deflection plates, the resultant motion of the spot will be the combination of two perpendicular SHM's—i.e., straight line, circle, ellipse, Lissajous figure, depending on the relative amplitude, phase and frequency of the two signals. With the SWEEP SELECTOR at LINE SWEEP, the PHASE knob is used to shift the phase of the sweep voltage with respect to the input signal. The traces shown in Figs.4 and 5 were both made with the selector on LINE SWEEP and 60 cps signal on the vertical plate: the phase is shifted 90° between Fig. 4 and Fig. 5.

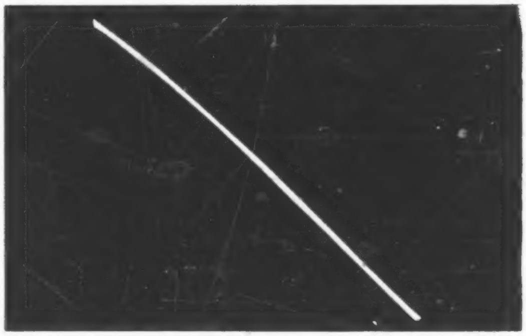


Fig. 4

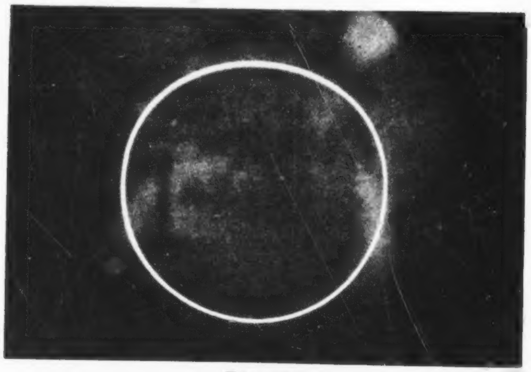


Fig. 5

For other settings of the HOR/FREQ SELECTOR or SWEEP SELECTOR control, an internal circuit applies a varying voltage to the plates that control the horizontal position of the spot. This voltage has a saw-tooth wave form: . The spot moves across the screen from left to right at a uniform rate while the voltage is increasing, and very rapidly flies back to its starting position when the voltage drops to its minimum value. In this setting deflection is linear with time. (Automatic "retrace blanking" reduces the intensity of the spot so that it is not seen as it flies back to the left of the screen.) The sweep frequency is controlled by two knobs. The HOR/FREQ SELECTOR, or SWEEP SELECTOR, provides coarse control in steps. Typically, one setting will cover a "decade" of frequencies, e.g., 10 - 100, 100 - 1000, etc., cycles per second. The FREQ VERNIER or SWEEP VERNIER gives fine control within these ranges. On expensive oscilloscopes, these controls may already be calibrated. Otherwise, they can be calibrated by the following procedure. A signal of known frequency is applied to the VERT INPUT. By counting the number of cycles of the known-frequency signal on the trace one can establish the sweep frequency. (E.g., if there are exactly n cycles of a 60 cycle-per-second signal on the trace, then the sweep frequency is $\frac{1}{n} \times 60$ per second; see Fig. 6.) For low sweep rates, a 60-cycle signal can be used. On the Heath oscilloscope IO-12, changing the FREQ SELECTOR one step will change the sweep rate by approximately a factor of ten, e.g., from 20 per second to 200 per second. For more accurate calibration at higher sweep rates, use a calibrated audio oscillator (signal generator) to provide a signal of known frequency. Some expensive oscilloscopes have a built-in oscillator that can be used to apply known frequency signals to the vertical deflection plates. Others have an output terminal which gives a 1-volt 60-cycle signal.

The length of the trace is controlled by the HOR GAIN knob; this does not affect the sweep frequency (number of sweeps per second)—one full sweep still represents the same time interval—but it does, of course, change the sweep rate (centimeters per second), and one centimeter will represent a different time interval.

Synchronization of the horizontal sweep frequency with the signal applied to the vertical input is important. If the two are synchronized, then the same pattern will be repeated for successive sweeps, and what appears to be a stationary trace will be obtained on the screen (as in Fig. 6).

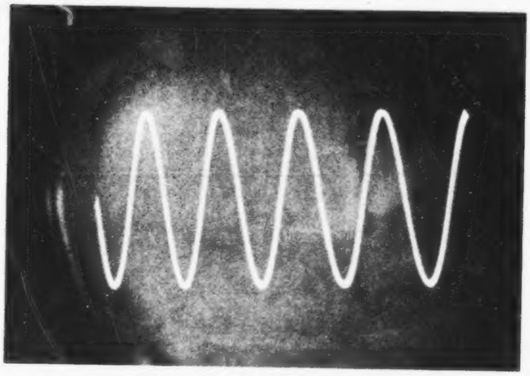


Fig. 6

But if the signal and sweep frequencies are not synchronized, then the traces obtained for successive sweeps of the screen will not coincide (Fig. 7):

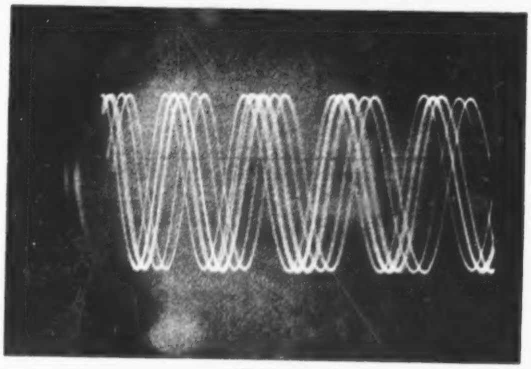


Fig. 7

Synchronization is achieved by fine adjustment of the FREQ VERNIER CONtrol until the sweep frequency is an exact fraction of the signal frequency. By setting the SYNC SELECTOR to INT+ or INT- the start of the sweep can be synchronized with either the positive or negative slope of the input signal (Figs. 8a, b).

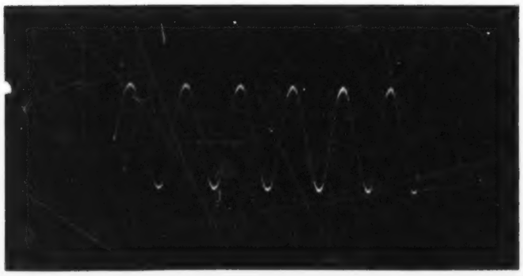


Fig. 8a

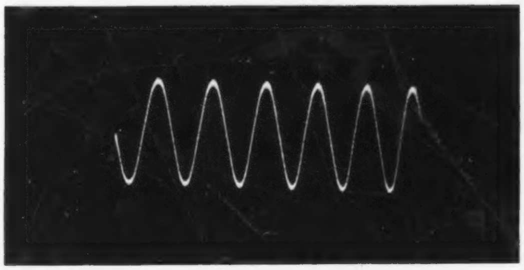


Fig. 8b

The sweep can also be synchronized with a signal applied to the EXT(ERNAL) SYNC terminal, by setting the SYNC SELECTOR knob to EXT SYNC. Adjust the EXT SYNC AMPLITUDE control until the sweep is synchronized with the signal. The EXT SYNC amplitude setting has no effect unless the SYNC SELECTOR is set to EXT SYNC.

There may also be a "LINE" setting of the SYNC SELECTOR. In this position, the horizontal sweep is synchronized with the (60-cycle) line frequency.

A useful feature present on some oscilloscopes is a "trigger." (The Heath IO-12 does not have this feature.) The horizontal sweep can be triggered by a signal applied from an external circuit to the trigger input. Until the triggering signal is applied, the spot remains stationary. This is particularly useful, for example, in time-of-flight measurements. If one wants to measure the time interval between two signals (e.g., the interruption of two beams of light to two photocells), it is desirable (though not essential) that the two signal pulses occur on the same horizontal sweep. This can be achieved by triggering the sweep on the rise of the first signal pulse. If the CRO has no trigger facility, then it may happen that the first signal will occur towards the end of one sweep and the second signal will occur on the next sweep. This makes measurement of the time interval between the two signals more difficult.

Some, but by no means all, oscilloscopes have a two-beam display: there are two Y inputs and it is possible to apply different signals to the two beams. This makes it very easy to compare the amplitudes and frequencies of two different signals. In reality, there is only one electron beam which is switched up and down so rapidly that two apparently continuous beams are seen (Fig. 9). The sweep rate for the two beams must be the same, but the amplifications of the two signals can be adjusted independently.

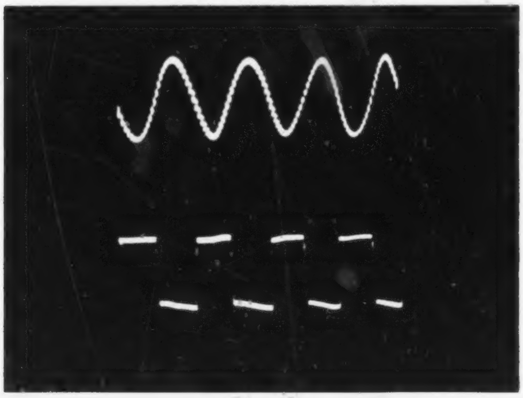


Fig. 9

On a "two-beam oscilloscope" the switching is done internally. External switching circuits are available that enable one to make a two-beam display of two independent signals on a regular oscilloscope (e.g., Heathkit Electronic Switch ID-22, $23.95 unassembled).

Intensity modulation (Z modulation)

Some oscilloscopes have an input terminal that is connected (through a capacitor) to the intensity-control grid of the cathode-ray tube. This terminal is commonly marked Z-AXIS. On some oscilloscopes it is necessary to remove the back panel to uncover this terminal. Always unplug the oscilloscope before removing the panel.

The potential of the grid (with respect to the cathode) controls the intensity of the electron beam, and hence the brightness of the spot or trace on the screen. It is this grid potential which is adjusted by the INT(ENSITY) control knob. (If the grid is made more positive, the spot becomes brighter.) If a varying voltage is applied to the grid, the beam intensity will be modulated at the frequency of the applied signal. Typically, about 10 volts is required for complete blanking of the trace.

Intensity modulation may be used to provide accurate time markers on the trace. (And it is the same intensity modulation, by the way, that creates the light and dark areas in the picture on a TV screen.)

Photography of CRO traces

The use of fast (3000-speed) Polaroid film makes it possible to photograph the trace. A close-up auxiliary lens to give an approximately 1:1 object-to-image ratio is necessary. If possible, remove the camera back and insert a ground glass or other focussing screen in the plane of the film: with the shutter open, adjust the camera position to sharp focus. A rigid support for the camera is needed, of course. Turn up the oscilloscope intensity control until a bright trace is obtained. However, if the intensity is increased too far on some oscilloscopes, the whole screen may begin to glow faintly and there will be a loss of contrast. If there is a colored screen or filter mounted in front of the oscilloscope face, it may help to remove it. Background illumination should be low, but it is certainly not necessary to work in a darkroom.

The appropriate aperture and time settings can quickly be found by trial and error. Don't forget that if the shutter speed is faster than the sweep rate only part of the trace will be photographed; (e.g., at 1/50 sec exposure, you cannot photograph a complete 1/30-second trace).

The photographs used to illustrate this note were taken with the experimental model 002 Polaroid Land Camera, using the clip-on auxiliary lens.

Examples of the Use of a CRO in Teaching Project Physics

a) The ultrasound transducers used in the wave experiments in Unit 3 have a very sharp resonance at 40 kilocycles; before attempting anv experiment with them, the oscillator driving the source transducer must be carefully tuned to the resonant frequency. Set up the equipment as shown in Fig. 10 with the receiver transducer a few centimeters in front of the source. Set the CRO to a sweep rate of about 10 kilocycles per second. Slowly adjust the frequency control on the audio oscillator until the trace on the oscilloscope screen "peaks" to a maximum signal.

In the experiments themselves, the amplitude of the trace on the oscilloscope screen is used to estimate the effectiveness of various materials as reflectors and absorbers of ultrasound, and to locate the positions of nodes (zero amplitude) and antinodes (maximum amplitude) in various interference and standing wave patterns.

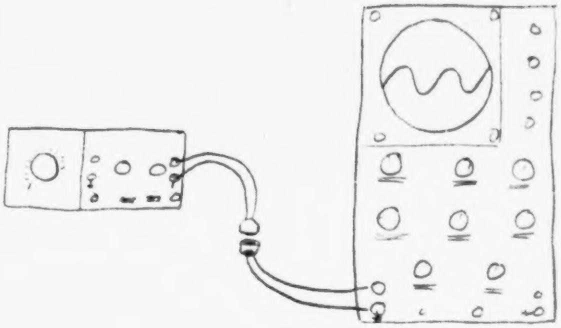


Fig. 10

b) The oscilloscope is used as a current meter—or, more accurately, as a null detector—in the investigation of the photoelectric effect (one of the Unit 5 experiments). The output of the phototube is fed (via an external amplifier) to the oscilloscope. As the counterpotential across the phototube is increased, the photocurrent, and thus the amplitude of the trace on the CRO, decreases. The experiment consists of finding what "stopping voltage" is needed to reduce the photocurrent to zero for light of different frequencies.

c) (Not especially relevant to Project Physics) The CRO can be used to make quantitative measurements of ac voltages and currents, and to compare the peak-to-peak voltage with the reading given by an ac voltmeter, which is the root-mean-square voltage ($\frac{1}{2}V_{p-p} = \sqrt{2}\,V_{RMS}$).

To measure ac current, connect the oscilloscope across a known resistance (non-inductive) and use I = V/R to calculate current.

Wave form display

The CRO can be used to show the difference between sinusoidal and square waves; to show half- and full-wave rectification of ac and the effect of a smoothing capacitor. It can also be used to show the wave forms of the sounds produced by various musical instruments, by students' voices, etc.; use microphone as detector. (A small speaker can also be used as a microphone—amplification may be necessary.)

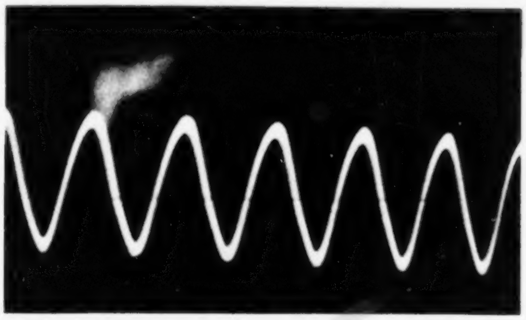


Fig. 11 Recorder plays a high C

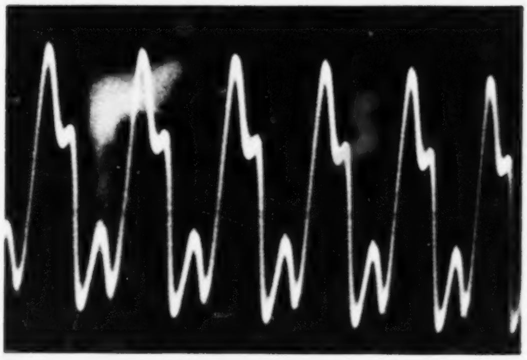


Fig. 12 Harmonica plays a high C

Two or more audio oscillators can be set to fundamental and harmonic frequencies to synthesize tones approaching those of various musical instruments. The higher frequencies must be set to exact multiples of the fundamental to get a stable trace.

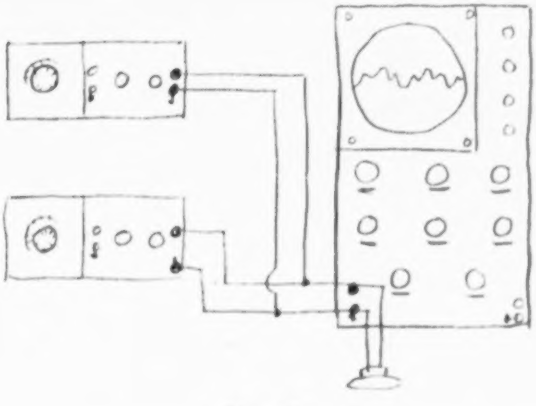


Fig. 13

To demonstrate the formation of beats, set the two oscillators to frequencies that are only slightly different.

Specific examples of wave-form display in Project Physics work:

a) In E/D 14.13 (electron-beam tube) the oscilloscope can be used to show that the electron-beam tube, like a vacuum diode, is a rectifier. Even if an ac voltage is applied between filament and plate, the current is dc (half-wave rectified), corresponding to electrons moving from filament to plate. (A two-beam display would be useful here.)

b) To show the action of the transistor switch used in various standing-wave demonstrations in Units 4 and 5.

c) To show the damped oscillations in an LCR circuit (Demonstration on Induction, Resonance—Unit 4).

d) To "see" the signal broadcast by a radio station (Demonstration on Induction, Resonance—Unit 4).

Time Measurements

a) Timing moving objects

Use two photocells in series and two light beams. The phototube units (PV100) and light sources from the Millikan equipment supplied by Project Physics can be used. Notice that no voltage supply is needed for the phototubes. This arrangement can be used to time falling objects, bullets, etc. Sweep rate must be known, of course. A rough idea of the speed of the object will make it easier to choose a suitable sweep rate (and distance between photocells).

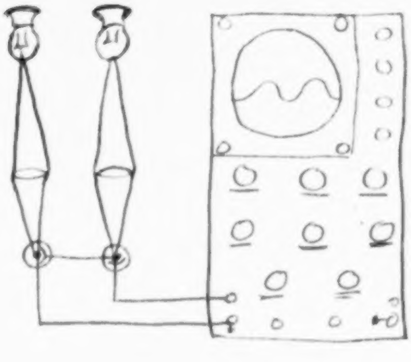


Fig. 14

In some situations, the photocell can be replaced by a simple switch that is momentarily closed by the object as it moves past. (For instance, a steel ball making contact between two pieces of aluminum foil as it passes.)

Some care is needed in interpreting the trace. The signals from the two phototubes (or switches) will have slightly different shapes due to differences in illumination, etc. Establish which signal is from which tube by intercepting first one light beam, then the other. Now examine carefully the trace record obtained when the moving object crosses both light beams or switches. If the first signal (i.e., signal due to interruption of first light beam) occurs towards the beginning of the sweep and is followed by the second signal, then there is no special problem: the distance between the two signals represents the time between the two events. But it can happen that the first signal occurs towards the end of one sweep and the second signal occurs on the next trace: in this case the sum of the distance from the first signal to the end of the trace plus the distance from the beginning of the trace to the second signal represents the time interval between the events.

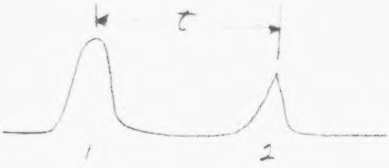


Fig. 15(a)

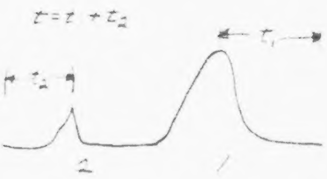


Fig. 15(b)

If a triggered sweep circuit is available, this complication need not occur. The triggering circuit can be used to start the sweep just as the object crosses the first beam, or closes the first switch.

b) Stroboscope calibration

A 60-cycle signal is used as a reference. See the notes on calibration of xenon stroboscope in the Teacher Guide.

Frequency measurement

The precision of a frequency measurement depends upon the accuracy of the reference source available. If the unknown frequency is a simple multiple or sub-multiple of 60 cycles, then 60-cycle line frequency, which is usually very closely controlled, can be used.

Set the HORIZ SELECTOR to LINE SW(EEP) and the SYNC SELECTOR to LINE, or apply a 60-cycle signal from a stepdown transformer to the HORIZ INPUT.

Apply the signal whose frequency is to be measured to the VERT INPUT. Adjust the HORIZ and VERT gains if necessary. Figures 16 through 19 are typical of the patterns that can be obtained.

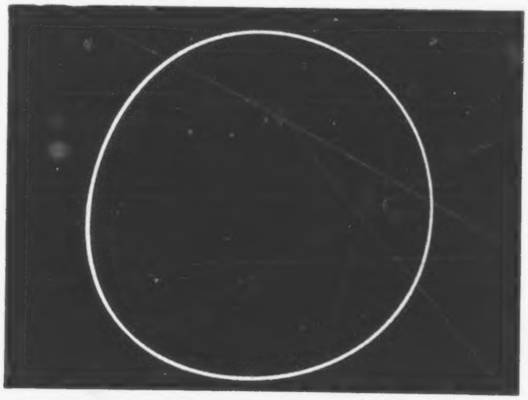


Fig. 16

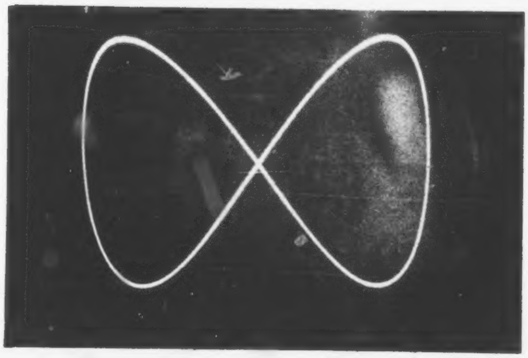


Fig. 17

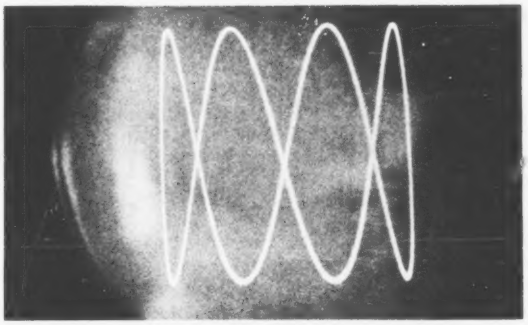


Fig. 18

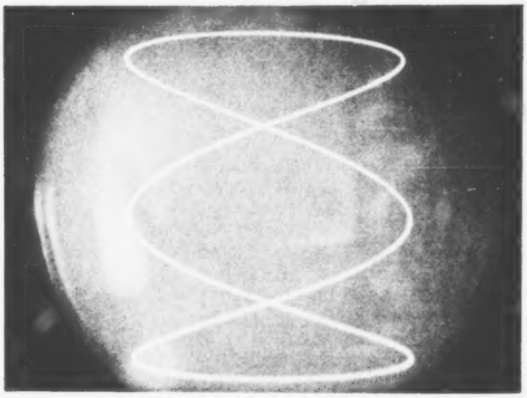


Fig. 19

Only if there is a simple whole number ratio between f_{vert} and f_{horiz} will stationary figures of this type be obtained.

The pattern observed depends on the relative phase of the two signals as well as their frequency ratio. The circle shown in Fig. 16 is obtained from two perpendicular sinusoidal signals 90° ($\pi/2$) out of phase. If the phase difference between the two signals is 0° or 180° (π radians), the resultant trace will be a straight line. Intermediate values of phase difference will give ellipses. The PHASE knob can be used to vary the phase difference between the signal applied to the Y plates and the line frequency sweep.

If the two frequencies are not equal, then the phase difference will vary continuously. The trace will change from straight line (0°) to ellipse (45°) to circle (90°) to ellipse (135°) to a straight line perpendicular to the original one (180°) and through ellipse, circle, ellipse, back to the original straight line. The frequency at which this change occurs is equal to the frequency difference between the two signals; e.g., if the two signals are 60 and 61 cycles, the trace pattern will go through one complete cycle of transformations in one second.

This technique can be used to calibrate an oscillator against a 60-cycle signal, at frequencies that are, or are very close to, multiples or submultiples of 60/sec. But if the problem is to measure a frequency which does not happen to be equal or close to a multiple or submultiple of 60 cycles, then this method cannot be used. Instead, it is necessary to use a variable-frequency oscillator whose calibration is accurately known as a reference.

Demonstrations of complex motions

In the previous section on frequency measurement by means of Lissajous figures, two independent signals were applied—one to the Y and one to the X input.

It is also possible to produce circular and elliptical traces using only one ac voltage by making use of the fact that in an RC circuit there is a 90° phase difference between the voltage across the resistor and the voltage across the capacitor (Fig. 20).

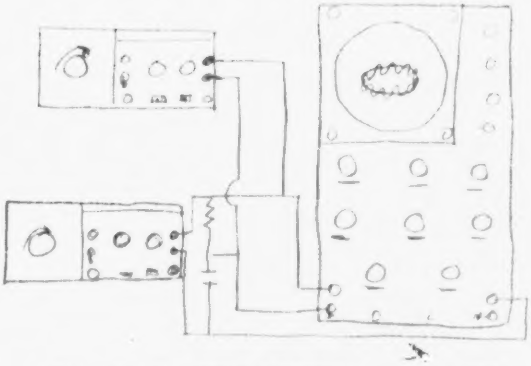


Fig. 20 The HORIZ SELECTOR is set to EXT.

Note that the mid-point of the RC circuit is connected to the ground terminal of the oscilloscope; it is important that neither output terminal of the oscillator be grounded. If both the oscilloscope and the oscillator are connected to the line by a three-wire cable and three-pin plug, you may have to use a three-to-two adapter plug to isolate the oscillator from ground.

The trace will be a circle or an ellipse, depending on the two voltages and the horizontal and vertical gains.

Suitable values of R and C for 1000 cycles per second are 1000 ohms and 0.1 microfarads. (Note that as the frequency is increased the impedance of the capacitor drops, the voltage drop across it drops, and what was a circle becomes elliptical.)

More complex patterns can be made if two oscillators are available. For example, the trace shown in Fig. 3 at the beginning of this article was produced by the circuit shown in Fig. 21 (the HORIZ SELECTOR is set to EXT).

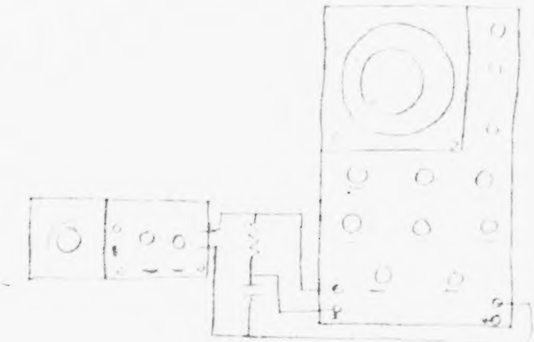


Fig. 21

Set up a circular or elliptical trace as described above, at say 60 cycles. Then apply a higher frequency, say 1200 cycles, sine- or square-wave voltage to the VERT INPUT.

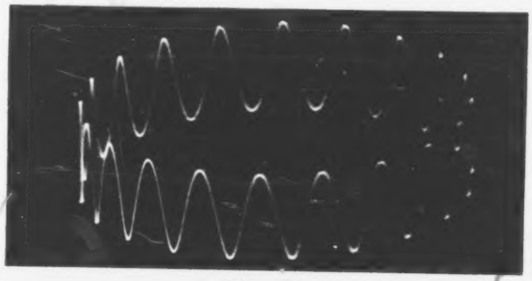


Fig. 22

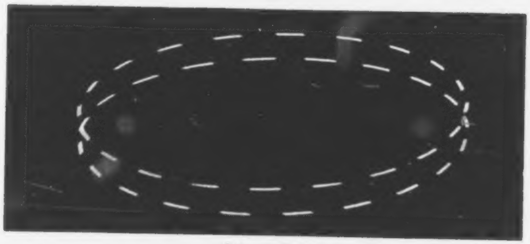


Fig. 23

Intensity modulation

1. Time markers. Set up circular, elliptical and epicycle traces as described in the section "Demonstrations of complex motions." Apply a sinusoidal signal to the Z-axis to provide intensity modulation at a frequency that is at least ten times higher than the frequencies applied to the X- and Y-inputs to provide at least ten time markers per cycle. This modulation frequency must be adjusted carefully: only when it is an exact multiple of the trace frequency will a stationary pattern be obtained. These time markers show that the spot moves around the circle with constant speed. In the ellipse it moves most quickly when it is close to the center; but note that this motion, unlike planetary motion, is symmetrical about the center of the ellipse. (So in this case the "equal areas" law can be applied to the motion about the center, not to motion relative to a focus. The same is true for the motion of a conical pendulum.)

2. "Television." Set the HORIZ FREQ to about 15 kilocycles. Apply a 60-cycle sinusoidal signal of a few volts peak to peak to the Y input (this can be from an audio oscillator, a step-down transformer, or the 60-cycle calibration signal provided by the oscilloscope itself). Adjust the horizontal and vertical gain to obtain a square or rectangular area that fills most of the tube face.

Apply an ac voltage of about 10 volts peak to peak (e.g., from the Project Physics oscillator unit) to the Z axis. If the frequency of this signal is a few times greater than the sweep frequency, a sweep pattern of vertical bars will be formed as the trace is blanked out several times in each sweep. If the modulation frequency is several times less than the sweep frequency, then a pattern of horizontal stripes is formed. Stabilize the pattern by setting the SYNC SELECTOR to EXT, connecting the EXT SYNC terminal to the Z-axis oscillator, and turning up the EXT SYNC AMP control until the pattern "freezes."

With two oscillators at different frequencies it is possible to combine the vertical bars and the horizontal stripes to form a checkerboard pattern.

Bibliography

"How to Understand and Use Your Oscilloscope," Heath Company, Benton Harbor, Michigan. $5.00.

"Working with the Oscilloscope," Welch Scientific Company, Skokie, Illinois. $4.95.

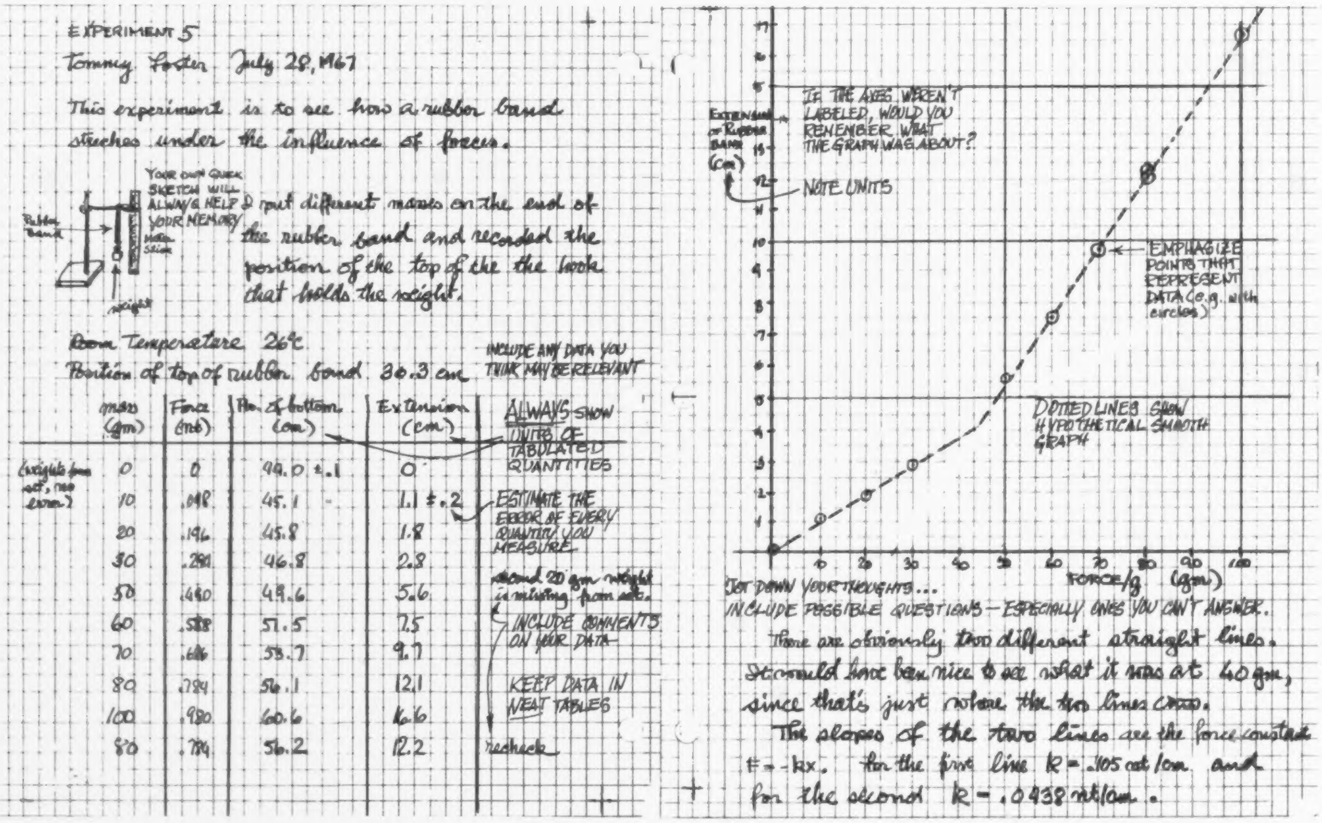
C. W. Thorp, "Use of Cathode-ray Oscilloscope in Schools," Physics Education (London), Vol. 2, p. 86.

Experiment Notes (Commentary on the Student Handbook)

In the following section, we have reprinted a column from the Student Handbook on the inner column of each page. In the outer column next to it you will find suggestions about how to run the experiments, answers to questions, and other information.

The film loop notes are collected in a section at the end.

Teachers are encouraged to add their own notes to experiments and loops.



EXPERIMENT 5

Tommy Foster July 28, 1967

This experiment is to see how a rubber band streches under the influence of forces.

Rubber band
meter stick
weight

YOUR OWN QUICK SKETCH WILL ALWAYS HELP YOUR MEMORY

I put different masses on the end of the rubber band and recorded the position of the top of the the hook that holds the weight.

Room Temperature 26°C

Position of top of rubber band 30.3 cm

INCLUDE ANY DATA YOU THINK MAY BE RELEVANT

	mass (gm)	Force (nt)	Pos. of bottom (cm)	Extension (cm)
(weights from set, not even.)	0	0	44.0 ±.1	0
	10	.098	45.1	1.1 ±.2
	20	.196	45.8	1.8
	30	.280	46.8	2.8
	50	.490	49.6	5.6
	60	.588	51.5	7.5
	70	.686	53.7	9.7
	80	.784	56.1	12.1
	100	.980	60.6	16.6
	80	.784	56.2	12.2

ALWAYS SHOW UNITS OF TABULATED QUANTITIES

ESTIMATE THE ERROR OF EVERY QUANTITY YOU MEASURE

Second 20 gm weight is missing from set.

INCLUDE COMMENTS ON YOUR DATA

KEEP DATA IN NEAT TABLES

recheck

IF THE AXES WEREN'T LABELED, WOULD YOU REMEMBER WHAT THE GRAPH WAS ABOUT?

NOTE UNITS

EMPHASIZE POINTS THAT REPRESENT DATA (e.g. with circles)

DOTTED LINES SHOW HYPOTHETICAL SMOOTH GRAPH

JOT DOWN YOUR THOUGHTS...
INCLUDE POSSIBLE QUESTIONS — ESPECIALLY ONES YOU CAN'T ANSWER.

There are obviously two different straight lines. It would have been nice to see what it was at 40 gm, since that's just where the two lines cross.

The slopes of the two lines are the force constant F = -kx. For the first line k = .105 nt/cm and for the second k = .0438 nt/cm.

Pages from a student's lab notebook. The table is used to record both observed quantities (mass, scale position) and calculated quantities (force, extension). The graph shows at a glance that the extension of the rubber band changes as the force is increased in a non-linear way (that is, a different rate 0 to 5 cm and above 5 cm).

Experiment 1 Naked Eye Astronomy

Although most of the salient facts about the motion of the sun, stars, moon and planets are given in Chapter 5 of the text, it would be a great loss if students' knowledge of them remained purely theoretical. There is no substitute for the students' own experience in making this sort of observation for themselves. For some students this may well be the first time that their attention has been guided to the beauty of the night sky; or that they have understood the change in position of the sun during the year and the moon during the month. At the least they will come to appreciate the skill and patience of early astronomers working with the same sort of primitive instruments. And some students will hopefully be excited enough to continue their observations beyond the outlines suggested.

The activity is unusual in that it continues over several weeks; on the other hand, the time required for each observation can be quite short. Start early in the year.

Encourage all students to make these elementary observations.

Preliminary

Do you know how the sun and the stars, the moon and the planets appear to move through the sky? Do you know how to tell a planet from a star? Do you know when you can expect to see the moon during the day?

The Babylonians and Egyptians knew the answers to these questions over 5,000 years ago. They found them simply by watching the ever-changing sky. Thus astronomy began with simple observations of the sort you can make with your unaided eye.

Although astronomy is one of the oldest branches of science, it is one of the most active today. These naked-eye observations are the basis for the work done by the giant telescopes and space probes of today.

You know that the earth appears to be at rest while the sun, stars, moon and planets are seen to move in various paths through the sky. The problem is to describe, in quite general terms, what these paths are, and how they change from day to day, from week to week and from season to season.

Because some of these changes occur slowly, you will need to watch carefully and keep a careful record of your observations over a period of at least four to six weeks.

Observing the Sun

(CAUTION: NEVER look directly at the sun; it can cause permanent eye damage. Do not depend upon sunglasses or fogged photographic film for protection. It is safest to make sun observations on shadows.)

1. In what direction does the sun set? Always make your observation from the same observing position. If you don't have an unobstructed view of the horizon, note where the sun disappears behind the buildings or trees in the evening.

2. At what time does the sun set, or disappear below your artificial horizon?

3. Try to make these observations about once a week. Draw a simple sketch showing the prominent objects on the horizon and the position of the setting sun.

4. Repeat the observation a week later. Have the position or time of sunset changed? Does it change in a month? Try to continue these observations for at least two months.

5. If you are up at sunrise, you can record its time and position, too. (Check the weather forecast the night before to be reasonably sure that the sky will be clear!)

6. How does the length of the day, from sunrise to sunset, change during a week? A month?

Observations

Below there are more detailed suggestions for observations to be made on the sun, the moon, the stars and the planets. Choose at least one of these objects and later compare notes with classmates who concentrated on others.

Choosing references

Before you can locate the positions of objects with respect to each other you must choose some fixed lines or planes to which all measurements may be referred. For example, establish a north-south line, and then with a protractor measure positions of all objects in the sky around the horizon with reference to this line. Such angles around

Suggest that one student (or pair) concentrate on making observations of either the sun, or the stars, or the moon, or the planets. Different groups can later share their observations. No student should feel he must attempt all the suggested observations (although he may do so).

Of course, observing conditions vary greatly and in smoggy urban areas they are distinctly bad. Even in good areas there will be bad nights.

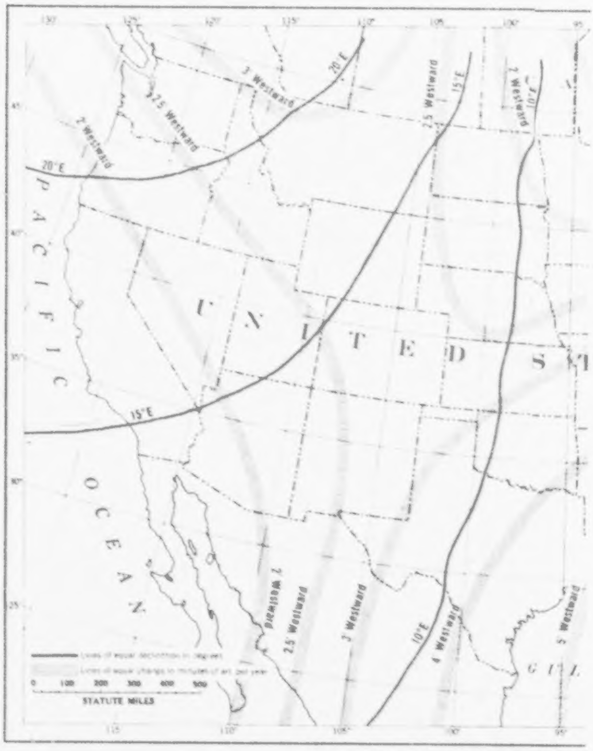
A planetarium visit can be used as a supplement to (or in poor viewing areas as a substitute for) personal observations. Contact the nearest planetarium and explain briefly what the course is about. Most planetarium directors will be very willing to put on a special show for your class that emphasizes the celestial motions important in Unit 2. A suggested program is given at the end of these notes.

the horizon are called azimuths and are measured from the north point through east (90°) to south (180°) and west (270°) around to north (360°, or 0°).

A horizontal plane is the second reference. This plane can be used even when the true horizon is hidden by trees, buildings, hills or other obstructions. The angle between the line to a star and the horizontal plane is called the altitude.

Establishing a reference

The north-south line can be established in several different ways. A compass is used to establish magnetic north, which may not be the same as true north. The magnetic north pole toward which your compass points, is more than 1000 miles from the geographic north pole, so in most localities the compass does not point true north. The angle between magnetic north and true north is called the angle of magnetic declination. At some places

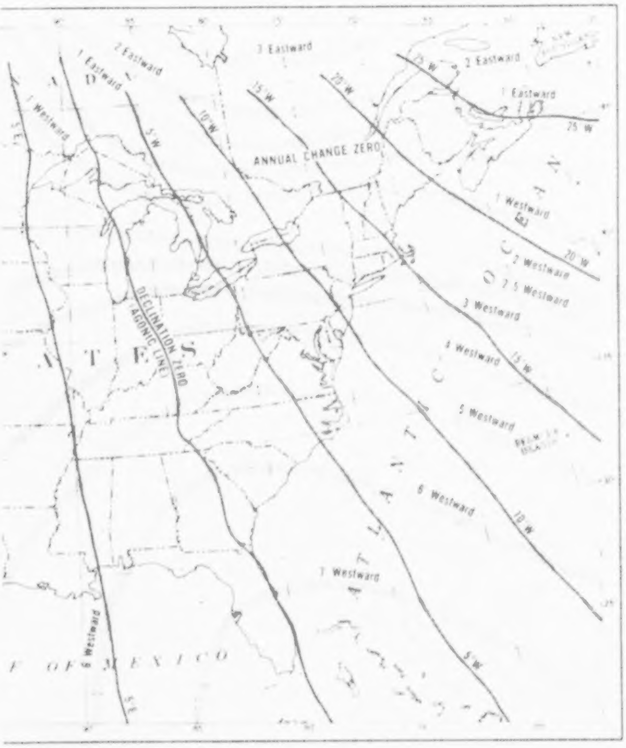

130°
125°
120°
115°
110°
105°
100°
95°
20°E
20°E
15°E
15°E
10°E
10°E
3′ Westward
2.75 Westward
2′ Westward
2.5 Westward
2′ Westward
2′ Westward
2.5 Westward
3′ Westward
4′ Westward
5′ Westward
45°
40°
35°
30°
25°
P A C I F I C
O C E A N
U N I T E D S T
G U L
Lines of equal declination in degrees
Lines of equal change in minutes of arc per year
0 100 200 300 400 500
STATUTE MILES
115°
110°
105°
100°
95°

Distribution of magnetic declination t

Fig. 1.

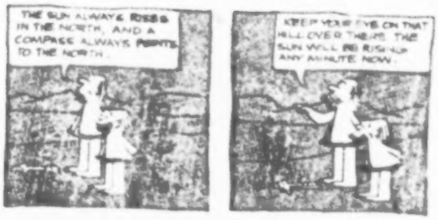
the magnetic declination is zero and the compass points toward true north. On a map of the U.S.A. these points lie along a wiggly line which runs through western Michigan, Indiana, eastern Kentucky, Tennessee, across Alabama and along the eastern side of Florida. At places east of this line, the compass points west of true north; at places west of the line, the compass points east of true north. In the far northwestern or northeastern United States (e.g., Portland, Oregon or Portland, Maine) the difference between magnetic north and true north is nearly 20°. You can find the angle of declination for your area from the map (Fig. 1).

The North Star (Polaris) is also used to establish the north-south line. It is the one star in the sky that does not move much from hour to hour or with the seasons, and it is almost due north of an observer anywhere in the northern hemisphere.

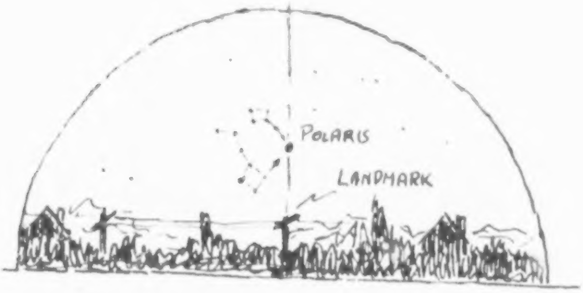

3 Eastward
2 Eastward
1 Eastward
1 Westward
2 Eastward
1 Eastward
25°W
20°W
15°W
10°W
5°W
ANNUAL CHANGE ZERO
1 Westward
20°W
2 Westward
2.5 Westward
3 Westward
4 Westward
15°W
5 Westward
DECLINATION ZERO
(AGONIC LINE)
6 Westward
10°W
7 Westward
5°W
6 Westward
5°E
A T E S
F OF MEXICO
A T L A N T I C
O C E A N

or 1965 (derived from C&GS Chart 3077)

An observing night early in the school year at which the teacher, or if possible a local (amateur) astronomer, is present will help get the students started. It is much easier to pick out a constellation, or planet, when someone is pointing to it in the sky, than to recognize it for oneself from a map.

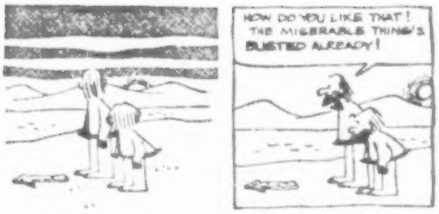

THE SUN ALWAYS RISES IN THE NORTH, AND A COMPASS ALWAYS POINTS TO THE NORTH.
KEEP YOUR EYE ON THAT HILL OVER THERE. THE SUN WILL BE RISING ANY MINUTE NOW.

You may use a prominent constellation (star group) to locate Polaris. First, find the "Big Dipper" which on a September evening is low in the sky and a little west of north. The two stars forming the side of the dipper opposite the handle are known as the "pointers," because they point to the North Star. A line passing through them and extended upward passes very close to a bright star—the last star in the handle of the "Little Dipper." This bright star is the Pole Star, Polaris. On September 15 at 8:30 p.m. these constellations are arranged about as shown in the diagram:


POLARIS
LANDMARK

Imagine a line from Polaris straight down to the horizon. The point where this line meets the horizon is due north of you.

Now that you have established a north-south line, either with a compass or from the North Star, note its position with respect to fixed landmarks, so that you can use it day or night.

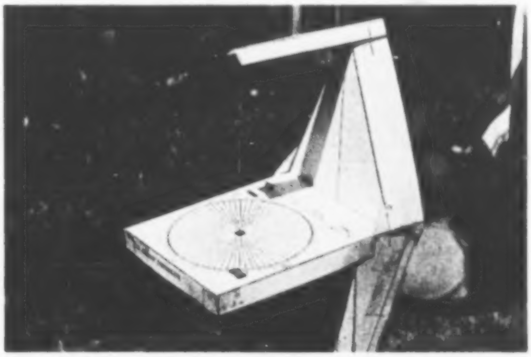

HOW DO YOU LIKE THAT! THE MISERABLE THING'S BUSTED ALREADY!

A. Sun

You can use the "shadow theodolite" to make observations of the altitude of the sun (height of sun above horizon) and its azimuth (angle between your north point and a line to the sun). Follow the assembly instructions packed with the shadow theodolite parts; work carefully—the accuracy of your instrument depends upon the precision with which you assemble it.

Set the theodolite so that the zero line on the horizontal table points north-south. When the plumb line passes through the hole without touching it, the table is horizontal. This will be easier to do if you support the instrument on the top of a post or wall.

Look for the shadow of the plumb line on the table. Read off its position in degrees: this the sun's direction east or west of the south point. To find the true azimuth add 180° to the reading. Example: a reading of -30° means that the sun is 30° east of south at the time of observation. Azimuth = -30° + 180° = 150°.



Azimuth changes steadily (15°/hour).

Noon (sun due south, and highest in the sky) is NOT likely to be exactly 12 o'clock. You may be on daylight saving (or summer time), in which case noon is at about 1 p.m. But even on standard time you are not likely to be located on the central meridian of your time zone. Places near those central meridians are given in Table 1. If you are east of the central meridian for your time zone, the sun will cross your local meridian 4 minutes early for each 1° eastward. Similarly, if you are west of your time zone central meridian, the sun will transit 4 minutes later for each 1° of longitude westward.

TABLE 1

Some Places Near the Central Meridians of Time Zones

Zone	Mid-longitude	Places Near Mid-longitude
Eastern	75° W	Philadelphia
Central	90° W	Memphis, St. Louis, New Orleans
Mountain	105° W	Denver
Pacific	120° W	Lake Tahoe

Now rotate the theodolite until the thread's shadow is on the zero line. Look for a bright rectangle on the vertical plate caused by sunlight passing through one of the windows in the top plate. Make sure the plate is still horizontal; read off the position of the bright area on the scale. This is the sun's altitude.

Some things to observe:

Record the date and time of all your observations.

Observations to be made during one day

(1) Sun's azimuth at various times during the day. Keep a record of azimuth and time of observation. Does the azimuth angle change steadily during the day, or is the sun's apparent motion more rapid at some times than at others? How fast does the sun move, in degrees per hour?

(2) When is the sun due south?

(3) How does the sun's angular altitude change during the day? When is it greatest?

Observations to be made over an extended period

Try to make these observations about once a week for a period of at least a month or two. Continue for longer if you can. Don't worry if you miss some observations because of poor weather.

(1) Altitude of sun at noon—or some other convenient hour. On what date is the noon altitude of the sun a minimum? What is the altitude angle on that date?

(2) Try to use your theodolite to make similar observations of the moon (at full moon) too.

B. Moon

1. Observe and record the position and shape of the moon on successive evenings through as much of its cycle as possible.

I'VE BEEN STUDYING THE SUN ALL DAY!
WHAT DID YOU LEARN?
ZAK
NEVER TO STUDY THE SUN ALL DAY!

If you miss a night, just record the existence of the gap in the data. Make sketches showing the relative positions of moon and sun. Show the moon's phase. If the sun is below the horizon when you can see the moon, you will have to estimate the sun's position.

2. Can you locate the position of the moon against the background of fixed stars and plot its position on a sky map? Sketch the phase of the moon on the Constellation Chart (SC-1) supplied by your teacher.

3. What is the full moon's maximum altitude? How does this compare with the sun's maximum altitude on the same day? How does it vary from month to month?

4. At full moon you may be able to use the shadow theodolite, described in section A, on sun observations, to determine the moon's altitude and azimuth. Try it.

5. There will be a total eclipse of the moon on October 18, 1967. Consult Table II of the Unit 2 Handbook Appendix for the dates of lunar eclipses in other years.

This should show that the moon moves about 360° in a month. By plotting the position and shape of the moon on the constellation chart SC-1 they may be able to confirm how the moon's phase depends on its position relative to the sun. (Sun's position at ten-day intervals is given along ecliptic on SC-1.)

Students could also use the sighting device described in the equipment notes on p. 112 to measure the moon's elevation.

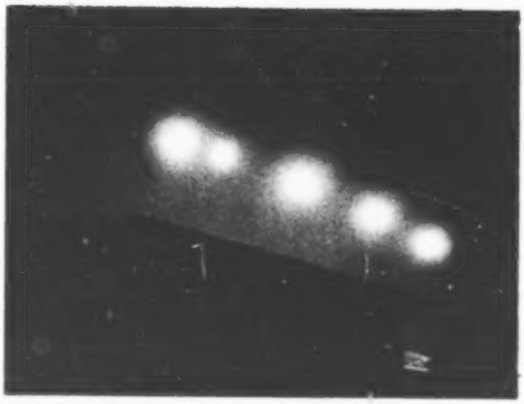
In spring and autumn the full moon follows a path close to the sun's across the sky. But in winter the full moon is far north while the sun is far south. (Fig. 2.) In summer their relative positions are reversed. These relations can be shown with the sky on a flask, with an armillary sphere, or our approximation to it, or in a planetarium.

Try to arrange an observation night. Enlist the help of a local (amateur) astronomer if you can.

Figure 4 of the student's notes shows the stars near Polaris. Not all will be visible above the horizon throughout the year. The "Star and Satellite Path Finder" (supplied by Project Physics, extras obtainable from Edmund Scientific, Barrington, New Jersey for 50 cents each) shows which stars are above the horizon at latitude 40° N at a particular date and time.

The Constellation Chart SC-1 shows the stars in a band 60° N and S around the celestial equator. This includes all the stars high in the sky. The curved line across the middle of the chart is the ecliptic—the sun's path throughout the year and its position at 10-day intervals is marked on the ecliptic.

Relative to the sun, the stars move about 30° westward per month. Different stars appear in the southern sky as the seasons change (e.g., Orion is prominent in winter but is not seen in summer because the sun is in that part of the sky.) See the "Star and Satellite Path Finder."



This multiple exposure picture of the moon was taken with a Polaroid Land Camera by Rick Pearce, a twelfth-grader in Wheat Ridge, Colorado. The time intervals between successive exposures were 15 min, 30 min, 30 min, and 30 min. Each exposure was for 30 sec using 2000-speed film.

C. Star

1. On the first evening of observation locate some bright stars that will be easy to find on successive nights. Look for star groups that will be easy to relocate. Later you will identify some of these groups with constellations that are named on your star map (Fig. 2). Record how much the stars have changed their positions after an hour; after two hours.

2. Take a photograph (several minutes' exposure) of the night sky to show this motion. Try to work well away from bright street lights, and on a moonless night. Include in the picture some of the horizon for reference. Prop up your camera so it won't move during your time exposures. Use a small iris opening (large f-number) to reduce fogging of your film.

3. When viewed at the same time each night, are the positions of the star groups constant in the sky from month to month? Do any new constellations appear after one month? After 3 or 6 months?

A time exposure photograph of Ursa Major ("The Big Dipper") taken with Polaroid Land Camera on an autumn evening in Cambridge, Massachusetts.

D Planet and Meteor

In October and November of 1967 Venus will be bright and should be easy to see in the eastern sky before sunrise. Jupiter is also visible in the morning sky; Mars is in the evening sky in the autumn of 1967. Saturn will be above the horizon all night, but is not as bright as the other three planets mentioned.

See Table II of the Unit 2 Handbook Appendix for the positions of planets in other years.

1. If you can identify a planet, check its position in the sky relative to the stars at two-week intervals. The planets are located within a rather narrow band, along which the moon and sun move. In what direction does the planet move against the star background?

2. Consult the Celestial Calendar and Handbook and the monthly magazine Sky and Telescope for more details on the positions of planets, when they are close to the moon, etc.

3. Look for meteor showers each year around November 5 and November 16, beginning around midnight. The dates of meteor showers in other months are given in Table II of the Unit 2 Handbook Appendix. Moonlight interferes with meteor observations whenever the moon is between first and third quarter.

Because sun, moon, planets stay in the same narrow band, we can conclude that they all move in nearly the same plane; that is, the planetary system is essentially "flat."

In "normal" motion planets move eastward against the stars; in retrograde motion they move westward. Consult the Celestial Calendar to find out when the different planets are in retrograde motion.

Post the month's Celestial Calendar with events of note—eclipses, conjunctions, etc. marked.

ADDENDUM TO NAKED EYE ASTRONOMY

Although coordinate systems for locating objects in the sky are not an important aspect of this study, teachers may wish an explanation of the various systems used.

Celestial Coordinate Systems

On Earth: The latitude-longitude system is used to locate objects on the earth's surface. The equator of the earth is established as a great circle along the earth's surface halfway between the north and south poles and perpendicular to the earth's polar axis. Meridians are a set of great circles passing

through the poles and are perpendicular to the equator. The local meridian (your north-south line) establishes your east-west location. The meridian passing through Greenwich, England, is called the prime meridian and has an assigned longitude of 0° . Places west of the prime meridian up to halfway around the earth (to the international date line) have longitudes west. Places east from Greenwich around to the international date line have have longitudes east. Maximum longitudes are therefore 180° E and 180° W.

Latitudes are distances measured north and south from the equator to the poles (90° away). Latitudes are also the angular distance between a place and the equator as one might see it from the earth's center (see Fig. 3).

In the sky: One convenient way to establish a position of a star or other heavenly object is to use the altitude-azimuth system (see Fig. 4). The coordinates in this system are:

(1) Altitude: the angle of the object above the observer's local horizon.
(2) Azimuth: the direction in the horizontal plane measured eastward from true north.

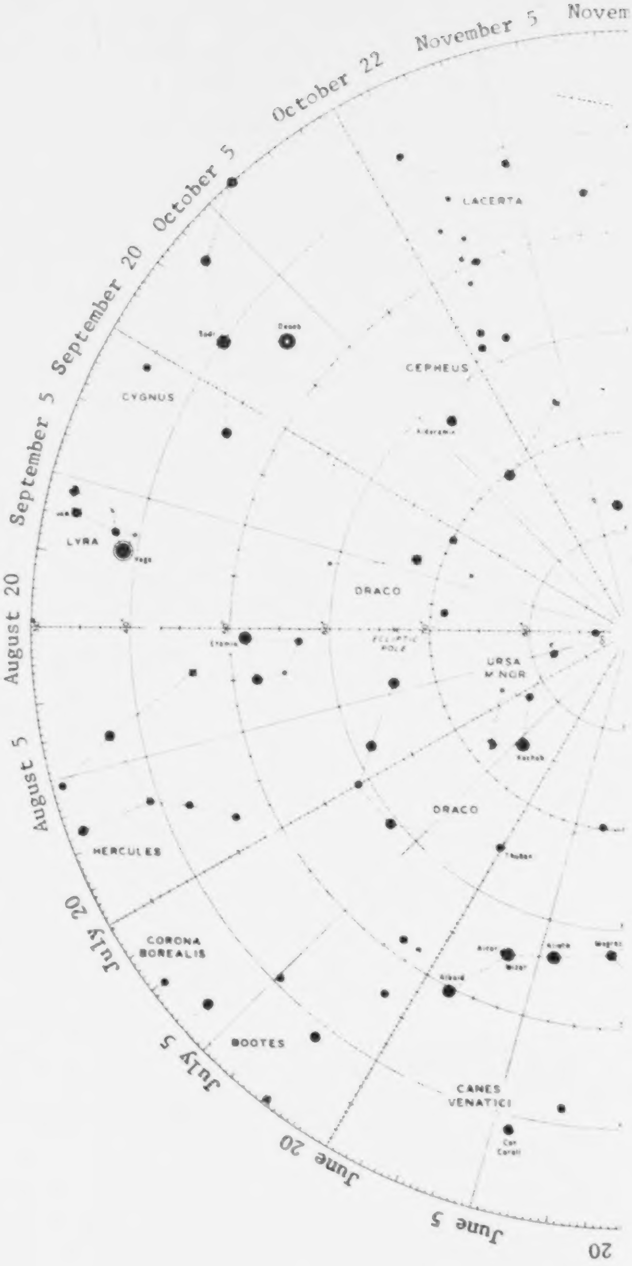
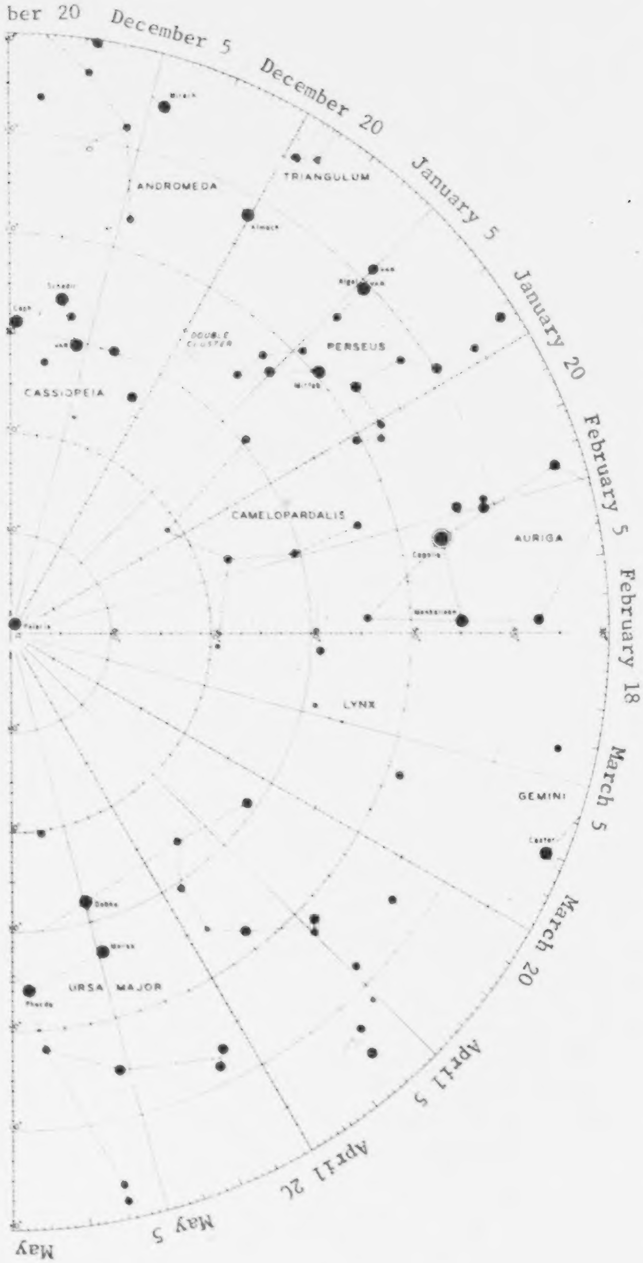

November 5
November 5
October 22
October 5
September 20
September 5
August 20
August 5
July 20
July 5
June 20
June 5
20
LACERTA
CEPHEUS
CYGNUS
LYRA
DRACO
ECLIPTIC POLE
URSA MINOR
DRACO
HERCULES
CORONA BOREALIS
BOOTES
CANES VENATICI

Fig. 2. Star map


ber 20
December 5
December 20
January 5
January 20
February 5
February 18
March 5
March 20
April 5
April 20
May 5
May
Mirach
ANDROMEDA
TRIANGULUM
Almach
Schedir
Caph
CASSIOPEIA
DOUBLE CLUSTER
PERSEUS
Algol
Mirfak
CAMELOPARDALIS
AURIGA
Capella
Menkalinan
Polaris
LYNX
GEMINI
Castor
Dubhe
Merak
URSA MAJOR
Phecda

Such a system is local. No two observers (even a few miles apart) have at the same moment the same coordinates for the same star. Also, as the earth turns, a star's position on this system constantly changes.

For this reason, astronomers long ago devised a coordinate system attached to the celestial sphere. This is sometimes referred to as the equatorial system and the elements measured are called right ascension and declination.

Imagine that we extended the earth's axis to the so-called celestial sphere. Also, extend the plane of the equator until it intersects the celestial sphere. Great circles passing through the North Celestial Pole and crossing the celestial equator at right angles are called hour circles. These are similar to meridians on the earth's surface.

One hour circle passes through the Vernal Equinox (see text Unit 2, p. 9) and is the reference from which right ascension is measured. The right ascension of a star is the time interval between the hour circle of the Vernal Equinox being overhead and the hour circle of the star. Since it takes 24 hours for the celestial sphere to rotate through 360° one hour is equivalent to 15°.

Declination establishes the distance of a star along an hour circle north or south of the celestial equator. Declinations are like latitudes on the earth's surface. A star having a declination of 40° N passes overhead at places having a latitude of 40° N.

Stars remain relatively fixed with respect to their coordinates in the right ascension-declination system.

REFERENCES

1. A Star Atlas, A.P. Norton, 15th ed., Sky Publishing Corp., 50 Bay State Road, Cambridge, Mass., 02138.

2. New Handbook of the Heavens, Bernhard, Bennett, Rice, Signet Science Library, McGraw-Hill Book Co., Inc., 501 Madison Ave., New York, New York, 60 cents.

3. Elementary Astronomy (Streve, Lynd and Pillans, Oxford University Press, New York, has a good description of the various coordinate systems.

Even if you were exactly on the central meridian for your time zone, only rarely would noon occur at 12 o'clock. Each day the sun moves east among the stars, but not at an exactly even rate because the earth's orbit is elliptical, not circular. Your students will understand this when Kepler's Second Law is discussed in Chapter 7. Even a uniform motion of the sun along the ecliptic would result in uneven days because the sun's annual path also has a north-south component. So, our clocks run on a fictitious average day (Mean Solar Time) based on the length of a year. The real sun (the one you see) gains and loses on Mean Solar Time. The difference is called the Equation of Time and may amount to over 16 minutes.*

See Unit 2 text, Figure 5.16 (on page 7) for an example.

For many more experiments that can be done with the Shadow Theodolite get "Experiments in Astronomy" $2.00 from G. Wootan, Inc., 36 Brighton Road, Worcester, New York.

The new moon is close to the sun; full moon is 180° from the sun; quarter moon is 90° from the sun (Figure 1, page 140).

Students cannot make observations at the same time every night. New moon to first quarter moon can only be seen in the late afternoon and evening; and third quarter to new moon can only be seen in morning. (Yes, the moon can often be seen while the sun is up.)

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*New Handbook of the Heavens, page 176.

PROJECT PHYSICS PLANETARIUM PROGRAM

(MORRISON PLANETARIUM, SAN FRANCISCO,
15 NOVEMBER 1966) OUTLINE PREPARED

FOR

This is an outline of the major phenomena which would be most useful and appropriate to Harvard Project Physics students for observation in the planetarium.

1 THE CURRENT NIGHT SKY
   a. both as of about 9 p.m. and as of just before dawn (many of our students are on their way to school before dawn and do observations then).
   b. major constellations and bright stars (first as they are seen under excellent viewing conditions, then as they appear in the typical hazy, lighted sky at night in the metropolitan area, as from a park or other reasonably dark place.)
   c. how to find Polaris by using the Big Dipper and by using Casseopeia, or any other ways.
   d. planets visible now and how to locate and identify them; for example, by their proximity to the moon on certain nights and times.
   e. location of the sun and the moon against the starry field (and how the location of the sun with respect to the stars can be determined).

2. MOTIONS IN THE HEAVENS
   a. motion of the circumpolar stars; of the southern stars; of the celestial sphere as a whole.
   b. motion (change in position) of the sun: (1) along the horizon over a period of about six weeks or more; (2) higher and lower in the sky at its highest point; (3) how these are really two manifestations of the same shift in orientation of its path across the sky; (4) across the sky (celestial sphere) as seen against the pattern of the stars (and how this position of sun against stars can be determined, when the sun blots out the stars, and they cannot both be seen at the same time—except in the planetarium).
   c. motion of the moon: (1) through a full month, slowly, one day at a time, showing only the moon at, say, moonrise, so the shift in position with respect to the stars is very clearly visible, and point it out; (2) through a month quickly, just running the moon projector, and if possible pointing out some of the irregularity of its motion.
   d. planetary motion, emphasizing (1) retrograde motion of some planets, and (2) the maximum angular displacement from the sun, of Mercury and Venus.

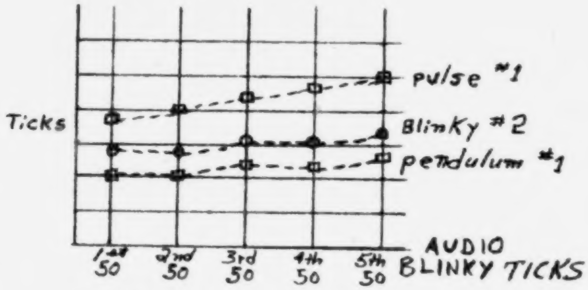
3. CELESTIAL COORDINATE SYSTEMS
   a. celestial longitude and latitude (or right ascension and declination) with particular attention to the reference points in the sky, such as where the 0^{o} position is (the vernal equinox) and how this is located and used for other measurements.
   b. azimuth and altitude (local) measurements, with the drawbacks of this type of system as compared to the coordinate system fixed in the sky and not depending on the local position of the earthly observer.
   c. using the constellations to locate the positions of planets, so that when a student reads that Mars is in Leo, he knows what this means.

PART I

Students will compare a variety of recurrent phenomena with a "standard clock" such as class blinky, metronome, etc. Recurrent phenomena that might be used include another blinky, pendulum, mass on spring, dripping burette, the human pulse, or crickets (tape recorded?)....

The mention of "time" should be avoided in this part, because the students' notion of absolute time will confuse the issue of relative regularity. The exclusion of actual clocks needn't seem forced—the point is to investigate first the regularity found naturally in the world, and then to investigate contrived measurement standards.

When students have completed the measurements on their own tapes, the information can be pooled on a master graph. It might look something like this:



If light from outdoors is allowed to fall on the blinky bulb, the rate will change by 4% or more (even if the light comes through the window). It might be a good idea to intentionally cause the rate to change during the run. The relativity of regularity would be emphasized, since the "good clocks" would show common curved records on the graph. (However, since we want the students to accept the blinky as a reasonably good clock, the disturbance should be accounted for afterwards. If one or two blinkies are used as test phenomena, the explanation will be more convincing.)

Experiment 2 Regularity and Time

The idea of regularity is very important in science. But how many natural phenomena occur regularly? Clocks will be left out of the first part of this lab because you know that clocks are built for the purpose of being regular.

Part I

Students and teacher will agree on a number of phenomena to check for regularity. One of these phenomena will be chosen as a "standard clock" and all the others will be compared to it by using the strip chart recorder.

One lab partner marks each "tick" of the standard clock on one side of the strip chart recorder tape while the other lab partner marks each "tick" of some other phenomenon. After a long run has been taken, you can inspect the tape to see how the regularities of the two phenomena compare. Run for about 300 ticks of the standard. For each 50 ticks of the standard, find on the tape the number of ticks of the other phenomenon, estimating to 1/10 of a tick. Record your results in a table something like this:

STANDARD CLOCK	YOUR CLOCK
First 50 ticks	____ ticks
Second 50 ticks	____ ticks
Third 50 ticks	____ ticks
Fourth 50 ticks	____ ticks

The results for the different periods are almost certain to be different; but is the difference a real difference in regularity, or could it come from your recording or measuring being a little off? If you think that the difference is larger than you would expect from human error, then which of the two phenomena is not regular?

Part II

In this part of the lab, you will compare the regularity of some devices specifically designed to be regular. The standard this time will be the time recording provided by the telephone company. To get two periods of time, you will have to make three calls to the telephone time station, for example, 7 p.m., 7 a.m., and 7 p.m. again. Agreement should be reached in class the day before on who will check wall clocks, who wristwatches, etc. Wait for the recording to announce the exact hour and watch your clock. Tabulate your results something like this:

TELEPHONE TIME STATION

"7 p.m. exactly"	
	12:00:00 hr
"7 a.m. exactly"	
	12:00:00 hr
"7 p.m. exactly"	

ELECTRIC WALL CLOCK

7: :	
	: :
7: :	
	: :
7: :	

PART II

With some concrete experience with relative regularity, students can more profitably consider the regularity of clocks. If a telephone or short-wave receiver is available in class, then all watches and wall clocks can be brought to the class and the experiment performed over two 24-hour periods. If not, then students will have to call the telephone time station from their homes. Two 12-hour periods might be more appropriate for students at home—7 a.m. - 7 p.m. - 7 a.m. or 8 a.m., etc., depending on school hours and local custom. (Since the power company tries to keep the total number of cycles for a 24-hour period constant and may suffer considerable variation in rate during that period, two 12-hour periods should be more revealing than two 24-hour periods for synchronous electric clocks.) Some students should be given the responsibility of finding out from the telephone company where they get their time.

Notice that there are really two problems: (1) Is your clock regular compared to the telephone company's clock? (2) If so, does your clock run at the same rate as the telephone clock? (Since the rate of most watches can be adjusted, it is the first question which is most important for watches. For electric clocks, on the other hand, the second question is more of a problem because the electric company forces a 24-hour match.) It should be emphasized to students that in physics the point is not so much whether their clocks are fast or slow, but whether they run at a constant rate.

The student should become familiar with kinds of variation in measurement. While it would be possible formally to introduce significant figures, estimates of average deviations, etc., it is the intention here only to make students comfortable with variation in its simplest terms—refinements can be introduced as the need arises in the laboratory exercise which will be done later.

The steps outlined below have been planned to start the student on familiar ground where he firmly believes the variations to be in the things measured, through situations where the variation is in the measuring process, to situations where the source of variation is uncertain.

You may wish to give these classifications of variation to the students after they have finished making the measurements but before the discussion; or you may feel it will be more valuable if these or similar categories are discovered through discussion. A few examples follow.

A. Situations where the variation is unequivocally due to differences in the things being measured:

    Students' heights or weights
    Family size
    Number of pieces of candy, raisins, etc. in different boxes.

B. Variation unequivocally due to changes in the thing being measured:

    Temperature of a beaker of warm water
    Weight of a chunk of dry ice
    Weight or length of a burning candle

Experiment 3 Variations in Data

The success which physics has had in contributing to scientific knowledge is due in no small part to man's ability to measure. Yet every measurement is to some extent uncertain. The numbers resulting from measurements are not simply the same as the numbers used for counting. That is, numbers read from measuring instruments are not exact in the sense that one or two is exact when one counts objects. If the number of chairs or people in a room is counted at a certain time, an exact value is obtained; but if the width of this sheet of paper is measured, the value found is known only within a margin of uncertainty.

The uncertainty in measurement of length mentioned above is just one of the causes of variation in data. This experiment should point out how some of the variations in data arise.

Various stations have been set up around the room. At each station there is some measurement to be made. Each of you will write your results on the board in order to compare and discuss them. Some interesting patterns should emerge. To see these patterns it is important that your measurements are not influenced by anyone else's—therefore, you shouldn't talk about how you measured or what results you obtained until everyone is through.

Keep a record of your observations. This tabulation form is convenient:

Type of Measurement	Remarks	Measurement

C. Variations unequivocally due to the process of measuring:

- Separation of blinky dots on a photograph using ruler
- Separation of blinky dots on photograph using magnifier
- Diameter of a piece of wire measured with a ruler
- Diameter of wire using micrometer or magnifier
- Diameter of a puck

D. Sources of variation uncertain:

- Rotation rates of students' phonograph turntables
- Height as measured in the morning compared to height at night

These classifications are not the only ones possible. One important class of variation not really covered in them is the statistical variation of random events, e.g., background radiation count. This will be considered in more detail in Unit 6. For the lab work in this course, Class C - Variations due to the process of measurement - is the most important, and is emphasized in some of the experiments.

The purpose of this laboratory is not to achieve unanimous agreement on the sources of variation in the measurements, but to make students aware of the issue and of how critical the issue can be in experiments. It is important for students to realize that variation problems are not confined to school laboratories; the best of scientists with the most expensive equipment are often faced with having to interpret variation.

Preparation in advance

The stations around the room must be set up before the class starts. Suggestions for stations are listed below. About ten or twelve stations will be needed if students are to gain a variety of experiences.

If each student makes every measurement, one period (50 min) will be needed. From one-half to a full period will be needed to write the results on the board and for discussion.

Station suggestions

More ideas are listed than can probably be used and you may have other ideas which you wish to substitute. Variety is the keynote, of course.

Diameter of a marble with vernier caliper
Large, steel ball with vernier caliper
Temperature of a beaker of water colder than room temperature
Same, at room temperature
Same, warmer than room temperature
Empty beaker mounted as in the case of the above beakers with thermometer

Note: Students may realize that all thermometers do not read exactly alike under like circumstances. Teacher should anticipate this and have carefully selected the thermometers to reduce or eliminate this.

Length of metal cylinder with ruler
Diameter of puck with common calipers and ruler
Separation of blinky dots on photograph with magnifier, and with ruler
Diameter of a piece of wire affixed to a 3" x 5" card with ruler
Time of fall of object from indicated height with stopwatch
Fill a small bottle with water and measure volume with graduated cylinder
Weight of a chunk of dry ice
Rotation rate of a slowly turning wheel with stopwatch
Length or weight of a burning candle
Select one plastic sphere from a lot, measure diameter with magnifier and discard sphere
Measure line voltage
Measure voltage of dry cell

All students should visit every station but they do not have to start at the same point in the cycle.

One can study uniform motion in a variety of ways. For example, two students working together can

a) photograph a puck sliding on beadie-covered ripple tank.
b) photograph a glider coasting on a level air track.
c) photograph a toy tractor pushing a blinky.
d) measure motion of an object in a film loop projected on the blackboard, or

one student alone can measure

e) a transparency showing what is asserted to be uniform motion.
f) a strobe photograph such as the momentum-conservation collision photos or the photo on page 13 of the text.

If not enough apparatus is available for the whole class to do the same experiment perhaps the class can be broken up into small groups, each group using a different method.

Instructions for operating the Polaroid camera and for using the rotating disc stroboscope are found elsewhere in this book.

In the student guide we describe the experiment as done by method (a) above. Other methods differ only slightly and in obvious ways. The procedure for measuring the data from any of the methods is identical.

Experiment 4 Uniform Motion

In this experiment, just as in Sec. 1.3, you will record the successive positions of a moving object. You do this in order to find its velocity at several points during its motion. Then you will try to decide if the velocity remained constant.

This decision may be harder than you expect, since your experimental measurements can never be exact; therefore there will always be ups and downs in your final results. Your problem will be to decide whether the ups and downs are due partly to real changes in velocity or due entirely to uncertainty in measurement.

If the velocity turns out to be constant, we have an example of uniform motion. Such motion is described in Sec. 1.3 in the text, which you should read carefully before doing this experiment.

Doing the experiment

The set-up is shown in Fig. 1. You will see that it takes two people. You can get similar results by yourself if you gather data by one of the other methods mentioned in the next section below.

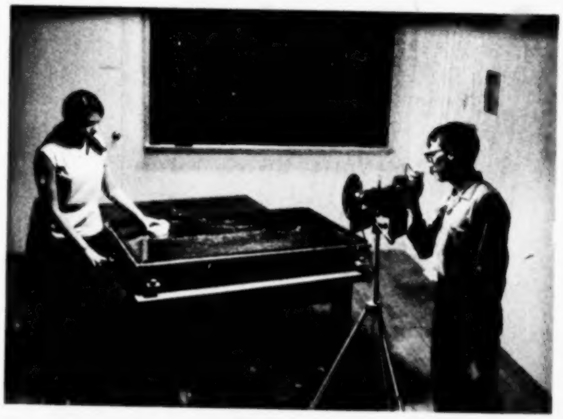


Fig. 1.

The set-up in Fig. 1 uses for the moving object a disc like the one illustrated in Sec. 1.3 of the text. It is made of metal or plastic instead of dry ice and it slides with almost no friction at all if the surface it slides on is smooth and free of grit or dust. Make sure the surface is quite level, too, so that the disc will not start to move once it is at rest.

Set up the Polaroid camera and the stroboscope equipment according to your teacher's instructions. Unlike the picture in the book, no ruler is necessary. Instead you will use a ruler of your own to measure the photograph.

Either your teacher or a few trials will give you an idea of the camera settings and of the speed at which to launch the disc, so that the images of your disc are clear and well-spaced in the photograph. One student operates the camera while his companion launches the disc. A "dry run" or two without taking a picture will probably be needed for practice before you get a good picture. A good picture is one in which there are at least five sharp and clear images of your disc far enough apart for easy measuring on the photograph.

Other ways of getting data

Instead of discs sliding on a table, you can photograph other objects, such as a glider on a level air track or a blinky (steadily flashing light) pushed by a toy tractor. Your teacher will explain their use. Excellent photographs can be made of either one.

If you do not use a camera at all or if you work alone, then you may measure a transparency or a movie film projected on the blackboard. Or you may simply work from a previously prepared photograph such as the one in Sec. 1.3.

Whether an air puck is used or a puck sliding on beadies it should have a large white X or a white-painted rubber stopper for easy reference in the photograph. Since the puck will probably rotate it must be at the center, not on the edge, of the puck.

We assume students have studied the text through Sec. 1.4, in which case they will end their write-ups after the section, If we measured more precisely.

If they have studied graphs (Sec. 1.5), however, it may be desirable to have them go on to subsequent sections in which they graph their data.

In either case they can answer Questions 8-10.

Drawing Conclusions

Whatever method you have used, the next step is to measure the spaces between successive images of your moving object. For this, use a ruler with millimeter divisions and estimate the distances to the nearest tenth of a millimeter. List each measurement in a table like Table 1.

As your time unit, use the time needed for the moving object to go from one position to the next. Of course it is the same time interval in all cases. Therefore if the velocity is constant the distances of travel will all be the same, and the motion is uniform.

Q1 How will you recognize motion that is not uniform?

Q2 Why is it unnecessary to find the time interval in seconds?

Table 1

Distance traveled in each time interval	0.48 cm	0.48	0.48	0.48	0.48	0.48

Here in Table 1 we have data that indicate uniform motion. Since the object traveled 0.48 cm during each time interval, the velocity is 0.48 cm per unit time.

It is more likely that your measurements go up and down as in Table 2.

Table 2

Distance traveled in each time interval	0.48 cm	0.46	0.49	0.50	0.47	0.48

Q3 Is the velocity constant in this case?

Since the distances are not all the same you might well say, "No, it isn't."

Or perhaps you looked again and said, "The ups and downs are because it is difficult to measure to 0.01 cm with the ruler. The velocity really is constant as nearly as I can tell."

Which statement is right?

Look carefully at your ruler. Can you read your ruler accurately to the nearest 0.01 cm? If you are like most people you read it to the nearest 0.1 cm (the nearest whole millimeter) and estimated the next digit.

In the same way, whenever you read the scale of any measuring device you should read accurately to the nearest mark and then estimate the next digit in the measurement. This means that your value is the estimated reading plus or minus no more than half a scale division.

Suppose you assume that the motion really is uniform, and that the slight differences between distance measurements are due only to the uncertainty in reading the scale. What is then the best estimate of the constant distance the object traveled between flashes?

To find the "best" value of distance you must average the values. The average for Table 2 is 0.48 cm, but the 8 is doubtful.

If the motion recorded in Table 2 really is uniform, the distance traveled in each time interval is 0.48 cm plus or minus 0.05 cm, written as 0.48 ±0.05 cm. The 0.05 is called the uncertainty of your measurement. It is commonly half a scale division for a single measurement.

It is worth emphasizing that any statement in science is valid only down to the limits of the uncertainty of the measurements on which the statement is based. Hence more precise measurements may possibly disprove it. In this sense physics is not an exact science.

The velocity evidently is not constant in Table 3 since the displacements vary by more than the precision of the measurements.

Now we can return to our big question: is the velocity constant or not? Because the numbers go up and down you might suppose that the velocity is constantly changing. Notice though that the changes of data above and below our average value of 0.48 cm are always smaller than the uncertainty, 0.05 cm. Therefore, the ups and downs may all be due to your difficulty in reading the ruler to better than 0.05 cm—and the velocity may, in fact, be constant.

Our conclusion is that the velocity is constant to within the uncertainty of measurement, which is 0.05 cm per unit time. If the velocity goes up or down by less than this amount we simply cannot reliably detect it with our ruler.

If we measured more precisely

A more precise ruler might show that the velocity in our example was not constant. For example if we used a measuring microscope whose divisions are accurate to 0.001 cm to measure the same picture again more precisely, we might arrive at the data in Table 3.

Table 3

Distance traveled in each time interval	0.4826 cm	0.4593	0.4911	0.5032	0.4684	0.4779

Q4 Is the velocity constant when we measure to such high precision as this?

The average of these numbers is 0.4804, and they are all presumably correct to half a division, which is 0.0005 cm. Thus our best value is 0.4804 ±0.0005 cm.

Drawing a graph

If you have read Sec. 1.5 in the text you have learned that your data can be graphed. If you have never drawn graphs before, your data provide an easy example to start with.

Just as in the text example on page 19, lay off time intervals along the horizontal axis. Your units are probably not seconds; they are "blinks" if you used a stroboscope, or simply "arbitrary units," which means here the equal time intervals between positions of the moving object.

Likewise the total distances traveled should be laid off on the vertical axis. The beginning of each scale is in the lower left-hand corner of the graph.

Choose the spacing of your scale divisions so that your data will, if possible, spread across the entire page.

The data of Table 2 are plotted here as an example (Fig. 2).

Q5 In what way does the graph of Table 2 show uniform motion? Does your graph show uniform motion too?

If the motion of your object is uniform, find the value of the uniform velocity from your graph. Describe how you found it.

Q6 What does a graph look like if the motion is not uniform?

If your motion is not uniform, review Sec. 1.7 of the text and then from your graph find the average speed of your object over the whole trip.

Q7 Is the average speed for the whole trip the same as the average speeds between successive measurements?

Additional Questions

Q8 Could you use the same methods to measure the speed of a bicycle? A car? A person running? (Assume they are moving uniformly.)

If the results are to be graphed it may be worth taking two runs at different speeds in order to show how the two resulting graph lines differ in slope.

a) 1 mi/hr
b) as small as 1 mi/hr

Q9 The speedometer scale on many cars is divided into units 5 mi/hr in size. You can estimate the reading to the nearest 1 mi/hr.

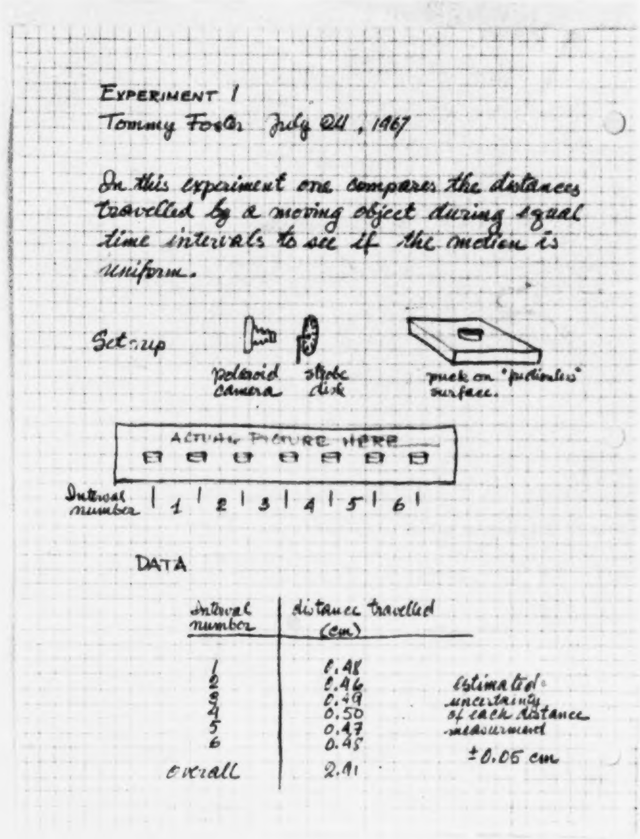
a) What is the uncertainty in a speed measurement?

b) Could you measure reliably velocity changes as small as 2 mi/hr? 1 mi/hr? 0.5 mi/hr? 0.3 mi/hr?

Q10 Sketch the shape of a distance-time graph of

a) an object that is slowly gaining speed.

b) a bullet during the second before and the second after it hits a brick wall.



EXPERIMENT 1
Tommy Foster July 24, 1967

In this experiment one compares the distances travelled by a moving object during equal time intervals to see if the motion is uniform.

Set-up

polaroid camera

strobe disk

puck on "frictionless" surface.

ACTUAL PICTURE HERE

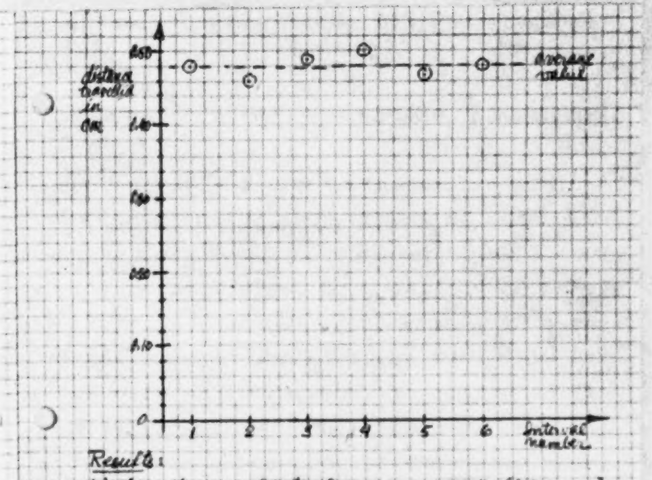
Interval number 1 2 3 4 5 6

DATA

interval number	distance travelled (cm)
1	0.48
2	0.46
3	0.49
4	0.50
5	0.47
6	0.48
overall	2.91

estimated uncertainty of each distance measurement
±0.05 cm

Fig. 2.



Results:

Within the uncertainty of my measurements (±0.05 cm) the speed of the puck was constant at 0.48 cm/unit time. As a check on my data I measured the overall distance travelled by the puck and found it to be 2.81 cm. The sum of the six intervals is 2.88 cm, which agrees with the measured value to within the ±0.05 cm uncertainty.

Answers to questions in the handbook:

Q1. By a change in the separations of the puck by more than the uncertainty.

Q2. Because we weren't asked to find the actual speed.

Only one phase of Galileo's investigation has been selected for this laboratory. A full description can be found in Dialogues Concerning Two New Sciences, the "Third Day." See also the references to the Crew-de Salvio translation reproduced in part in Chapter 2 of Unit 1. There is a Dover Publishing Company reprint of this enjoyable book. A careful modern repetition of this experiment is described by Settle, Thomas B., An Experiment in the History of Science, in Science, Vol. 133, January 6, 1961. Historians and philosophers of science are still hotly debating whether or not Galileo took actual experimentation very seriously, and whether he actually did some of the experiments he described in such graphic detail.

Accelerated Motion

In Chapter 2 you have been reading about Galileo's interest in accelerated motion. Scientists are still interested in accelerated motion today. In the following experiments you learn to measure acceleration in a variety of ways, both old and new.

If you do either of the first two experiments you will try to find, as Galileo did, whether d/t^2 is a constant for motion down an inclined plane.

The remaining experiments are measurements of the value of the acceleration of gravity, a_g—the value that $2d/t^2$ would approach as an inclined plane is made more and more nearly vertical. Perhaps you would like to try one of them.

Experiment 5 A Seventeenth-Century Experiment

This experiment is similar to the one discussed in the Two New Sciences by Galileo. It will give you first-hand experience in working with tools similar to those of a seventeenth-century scientist. You will make quantitative measurements of the motion of a ball rolling down an incline, as described by Galileo. From these measurements you should arrive at a suitable definition of acceleration—the major purpose of the exercise. It is also possible to calculate the value of a_g (acceleration due to gravity), which you should try to do.

The reasoning behind the Experiment

You have read in Sec. 2.6 how Galileo discussed his belief that the speed of free-falling objects increases in proportion to the time of fall—that is, that they have uniform acceleration. But since free fall was much too rapid to measure, he assumed that the speed of a ball rolling down an incline increases in the same way as an object in free fall does, only more slowly. Its average speed could now be measured.

But to see if the accelerations of the ball were the same from point to point required a knowledge not of average speed but of instantaneous speed at each point, and even a ball rolling down a low incline still moved too fast to measure the speed at a point at all accurately. So he worked out the relationship $\frac{a}{2} = \frac{d}{t^2}$, an expression for acceleration in which speed has been replaced by the total time and total distance rolled by the ball. Both these quantities can be measured. Be sure to study Sec. 2.7 in which this derivation is described. If Galileo's original assumptions were true, this relationship would hold for both freely falling objects and rolling balls. Since total distance and total time are not difficult to measure, seventeenth-century scientists now had a secondary hypothesis they could test by experiment. And so have you. Section 2.8 of the text discusses much of this.

Apparatus

The apparatus which you will use is shown in Fig. 1. It is similar to that discussed by Galileo.

At least two students are needed for each set-up: one to handle the rolling ball and the other to operate the water clock and to record data. By splitting the jobs further as many as four can be usefully employed.

An inclined plane about six feet long is needed. It should have a groove or channel down one edge in which a ball runs very smoothly.

If only one inclined plane is available it can be operated by one or two students while the rest of the class is arranged in ones or twos nearby operating water clocks.

If a good wooden incline is not available, perhaps a piece of metal channel iron or two metal rods clamped together can be made to serve—although this violates the seventeenth century spirit of the experiment!

Distances marked on the incline should be arbitrary, but be chosen to work well with the rate of flow of the water clock. (We find that 12 marks 6 inches apart serve well.) Students should not convert to present-day standards of length, but merely record them as 1, 2, 3, etc., units of length.

The right size of tube and of collecting vessel for the water clock has to be found by trial and error. The flow should last at least three or four seconds without overflowing the collector. The clock does not work as well if it started and stopped with a pinchcock on a rubber exit tube below the funnel.

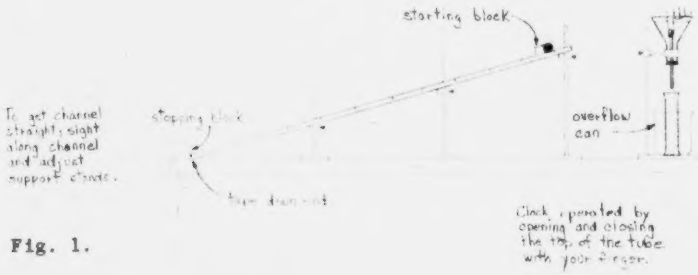

starting block
To get channel straight, sight along channel and adjust support stands.
stopping block
tape down end
overflow can
Clock operated by opening and closing the top of the tube with your finger.

Fig. 1.

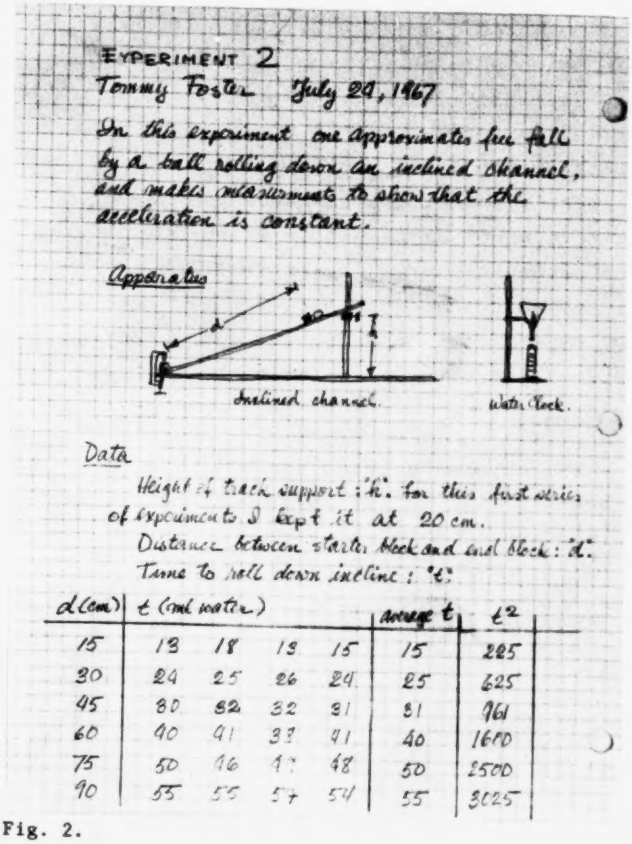
It is recommended that the students record a minimum of four trials for each distance and that the average value for the time be used in the calculations. For longer distances fewer trials may be used, if the times seem to be in close agreement. Four different distances (e.g. 3, 6, 9, 12) should be sufficient for each angle of the ramp. It is probably not practical for any one group to attempt to take measurements at more than two different angles of inclination. An exact adjustment of the height of the channel is not critical. The results for heights over 30 centimeters may show considerable scatter; this depends upon the skill of the students and quirks of the equipment.

You will let a ball roll various distances down a channel about six feet long and time the motion with a water clock.

Some early water clocks are illustrated on page 56 of your textbook. The way yours works is very simple.

Since the volume of water flowing into the cylinder is proportional to the time of flow you can measure time in milliliters of water. Start and stop the flow with your finger over the upper end of the tube inside the funnel. Be sure to refill the "clock" to about the same point for each trial, for its rate of flow changes slightly with the level of the water. Whenever you refill it let a little water run through the tube to clear out the bubbles.

It is impossible to release the ball with your fingers without giving it a slight push or a pull. Dam it up, there-



EXPERIMENT 2

Tommy Foster July 29, 1967

In this experiment one approximates free fall by a ball rolling down an inclined channel, and makes measurements to show that the acceleration is constant.

Apparatus

d
h
Inclined channel.
Water clock.

Data

Height of track support: "h". For this first series of experiments I kept it at 20 cm.

Distance between starter block and end block: "d".

Time to roll down incline: "t".

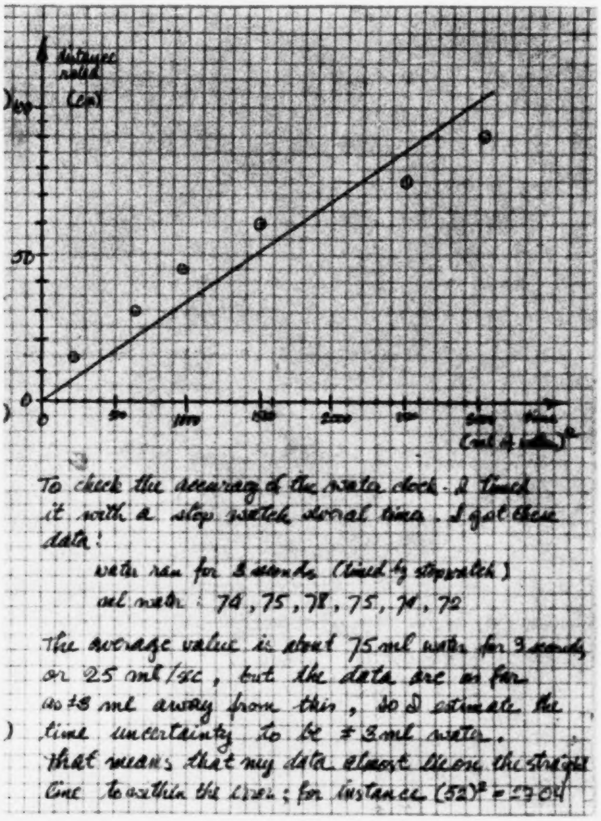
d (cm)	t (ml water)				average t	t^2
15	13	18	13	15	15	225
30	24	25	26	24	25	625
45	30	32	32	31	31	961
60	40	41	37	41	40	1600
75	50	46	47	48	50	2500
90	55	55	57	54	55	3025

Fig. 2.

fore, with a ruler, and release it by quickly moving the ruler away from it down the plane. The end of the run is best marked by the sound of the ball hitting the stopping block.

A brief comment on recording data

You can find a good example of a way to record your data in Fig. 2. We should emphasize the need for neat, orderly work. Orderly work looks better and is more pleasing to you and everyone else. It may also save you from extra work and confusion. If you have an organized table of data, you can easily record and find your data. This will leave you free to think about your experiment or calculations rather than worry about which of two numbers on a scrap of paper is the one you want, or whether you made a certain measurement or not. A few minutes' preparation before you start work will often save you



To check the accuracy of the water clock I timed it with a stop watch several times. I got these data:

water ran for 3 seconds (timed by stopwatch)
ml water: 74, 75, 78, 75, 74, 72

The average value is about 75 ml water for 3 seconds or 25 ml/sec, but the data are as far as ±3 ml away from this, so I estimate the time uncertainty to be ± 3 ml water. that means that my data almost lie on the straight line to within the error; for instance $(52)^2 = 2704$

It is a good idea at this early stage in the course to insist firmly on a neat data table. If students are always required to record all their data in ink directly into their final report it does wonders to develop clear careful thinking as the year goes on. Mistakes, of course, will be made—and simply crossed out neatly—but no "copying over" allowed!

In graphing the results plot (time)^2 along the horizontal axis. Not only is this conventional but also when d is plotted along the vertical axis the resulting slope is equal to twice the acceleration.

It is useful to plot d vs. t first to show that one does not get a straight line. Point out that there is no way of recognizing with the unaided eye any curve except a circle and a straight line. Only by plotting in such a way as to generate one of these shapes can we identify the relationship between d and t.

If a student suggests that the d-t curve does look like a parabola and is therefore a d-t^2 relationship, challenge him to show that it isn't a d-t^3 relationship, both of which may have the same general form as he can verify by trial.

It may be useful to have a pair of students do the following, twentieth-century version of Galileo's experiment photographing a glider sliding down a tilted air track. This yields more precise data, which may be reassuring when the final conclusion of the experiment is being discussed.

Possible extensions

After each group has completed its investigation, one of two possible procedures is recommended:

1) Each group may report its findings orally, and comparisons may be made during a discussion period.
2) Composite findings may be tallied on the board, and all students, using these combined results, could plot the entire family of curves for the different inclinations.

The point to make when comparing results is that the linear relation between d and t^2 appears to hold (within the variation expected) for the rolling ball, at least for small angles of inclination of the channel. As an aside, for any given angle of inclination the distance intervals rolled down the incline, in successive units of time, will go as 1:3: 5:7...

an hour or two of chasing around checking in books and with friends to see if you did things right.

Some operating suggestions

You should measure times of descent for several different distances, keeping the inclination of the plane constant and using the same ball. Repeat each descent four times, and average your results. Best results are found for very small angles of inclination (the high end of the channel raised less than about 30 cm). At greater inclinations the ball tends to slide as well as to roll. With these data you can check the constancy of d/t^2.

Then if you have time, go on to see if Galileo or Aristotle was right about the acceleration of objects of various sizes. Measure d/t^2 for several different sizes of balls, all rolling the same distance down a plane of the same inclination.

If you try to find the acceleration of an object in free fall, a_g, you should measure the time a ball takes to descend the full length of the plane at various carefully measured angles. Use angles up to the steepest for which you can measure the times of descent. From these data you can extrapolate to free fall (90°). You might want to use a stopwatch here instead of a water clock.

From data to calculations

Galileo's definition of uniform acceleration (text, page 49) was "equal increases in speed in equal times." Galileo expected that if an object actually moved in this way the total distance of travel should be directly proportional to the square of the total times of fall.

Q1 Why does this follow from his definition? (See Sec. 2.7 in the text if you cannot answer this.)

When you have collected enough data, plot a graph of the distances rolled (vertical axis) against the squared times for each inclination.

Q2 What must your graph look like if it is to support Galileo's hypothesis?

Q3 Does your graph support the hypothesis?

You have been using a water clock to time this experiment because that was the best timing device available in Galileo's time. How accurate is it? Check it against a stopwatch or, better yet, repeat several trails of your experiment using a stopwatch for timing.

Q4 How many seconds is one milliliter of time?

Extension

Review Sec. 2.7. There you learned that $a = 2d/t^2$.

Use this relation to calculate the actual acceleration of the ball in one of your runs.

If you have time you might also try to calculate a_g from your results. This is a real challenge. Your teacher may need to give you some help on this.

Additional Questions

Q5 Does the acceleration depend upon the size of the ball? In what way does your answer refute or support Aristotle's ideas on falling bodies?

Q6 Galileo claimed his results were accurate to 1/10 of a pulse beat. Do you believe his results were that good? Did you do that well?

Q7 Galileo argues that in free fall, an object tends to accelerate for as long as it falls. Does this mean that the speed of an object in free fall would keep increasing to infinity?

The value of a_g found by extrapolating data from this experiment will be too low because the ball's increasing kinetic energy as it descends is only partially in the form of energy of motion along the plane. The rest is in the form of energy of rotation. Second, friction is a very large factor in reducing the acceleration. Both these important effects are reduced or eliminated by photographing a rider descending an air track.

The analysis for a rolling ball is as follows:

Some of the better students should use their data to calculate the value of a_g, the acceleration of gravity. But this is not simple. The acceleration of a rolling object has two components: 1) the rather straightforward linear acceleration a, which can be approximated using $a = 2d/t^2$ (this takes into account friction), and 2) the rotational acceleration which is both more complicated and harder to visualize.

Neglecting friction, the linear acceleration can be found by breaking the gravitational acceleration up into two vectors, one along the surface of the track and one perpendicular to it. This will give a value of

$$a = \frac{\text{height of channel}}{\text{channel length}} \times a_g.$$

The rotational acceleration can be derived in a number of ways. We shall derive it kinematically here. At a height h the ball has a Potential Energy P.E. = ma_gh. This will be converted into linear kinetic energy $1/2mv^2$ and rotational kinetic energy $1/2I\omega^2$. Here ω is the angular rotation defined by $\omega = 2\,\alpha\theta$ (where α is $\frac{a_\ell}{\text{radius}}$ and θ is $\frac{d}{\text{radius}}$ which gives $\omega^2 = 2\ \frac{a_\ell}{\text{radius}} \times \frac{d}{\text{radius}}$.

$$I = 2/5m\ \text{radius}^2$$

We also define I as the rotational

inertia of a sphere of mass, m. Upon substituting these quantities into the rotational kinetic energy component, we get

$$1/2I\omega^2 = 1/2\cdot 2/5m\ \text{radius}^2 \times 2\ \frac{a_\ell d}{\text{radius}^2}$$

$$= 2/5ma_\ell d$$

for the rotational kinetic energy in linear terms. Since the total kinetic energy, rotational plus linear, must equal the potential energy,

$$\text{P.E.} = \text{K.E. rot.} + \text{K.E. lin.}$$

we get

$$\cancel{m}a_g h = 2/5\cancel{m}a_\ell d + 1/2\cancel{m}v^2.$$

If we substitute $v^2 = 2ad$ into this equation we get $a_g h = 7/5a_\ell d$, the acceleration due to gravity is

$$a_g = 7/5a_\ell\ \frac{\text{channel length}}{\text{height of channel}}.$$

The above equation can be given to the students in order to calculate the acceleration of gravity. If they do not see the need for including rotational kinetic energy, the equation may be made plausible by indicating that while a sliding block can use all of the energy from the earth's field to slide down, a rotating ball absorbs some energy in spinning. The students can then compare the value of a_g found here with that found in the uniform acceleration and free fall demonstrations.

The acceleration does not depend on the size of the ball, which refutes Aristotle's assertion (text p. 40) that it does.

Galileo was right if (a) one can neglect air resistance, (b) the force of gravity continues to be significant throughout an enormous distance of fall, and (c) there is no ultimate speed limit at which an object can travel. Galileo, who lived in the 17th century, was not aware of limitations on any of these three conditions except perhaps the first one. In fact none of the statements, (a), (b) and (c), are correct in the light of subsequent knowledge.

Experiment 6 A Twentieth-Century Version of Galileo's Experiment

In Sec. 2.9 of the text you read about some of the limitations of Galileo's experiment.

In the modern version with improved clocks and planes you can get more precise results, but remember that the idea behind the improved experiment is still Galileo's idea. More precise measurements do not always lead to more significant conclusions.

The apparatus and its use

For an inclined plane use the air track. For timing the air track glider use a stopwatch instead of the water clock.

Otherwise the procedure is the same as that used in the first version above. As you go to higher inclinations you should stop the glider by hand before it hits the stopping block and is damaged.

Instead of a stopwatch your teacher may have you use the Polaroid camera to make a strobe photo of the glider as it descends. A piece of white tape on the glider will show up well in the photograph. Or you can attach a small light source to the glider. You can use a magnifier with a scale attached to measure the glider's motion recorded on the photograph. Here the values of d will be millimeters on the photograph and t will be measured in an arbitrary unit, the "blink" of the stroboscope.

Plot your data as before on a graph of t^2 vs. d.

Compare your plotted lines with graphs of the preceding cruder seventeenth-century experiment, if they are available. Explain the differences between them.

Q1 Is d/t^2 constant for an air-track glider?

d/t^2 should be constant for an air track glider to a high degree of precision, illustrating that the glider's acceleration is practically constant on an inclined plane.

Q2 What is the significance of your answer to the above question?

As a further challenge you should, if time permits, predict the value of a_g, which the glider approaches as the air track becomes vertical. To do this, of course, you must express d and t in familiar units such as meters or feet, and seconds. The accepted value of a_g is $9.8\ \text{m/sec}^2$ or $32\ \text{ft/sec}^2$.

Q3 What is the percentage error in your measurement? That is, what percent is your error of the accepted value?

Percentage error =

$$\frac{\text{accepted value - measured value}}{\text{accepted value}} \times 100$$

so that if your value of a_g is $30\ \text{ft/sec}^2$

percentage error =

$$\frac{32\ \text{ft/sec}^2 - 30\ \text{ft/sec}^2}{32\ \text{ft/sec}^2} \times 100$$

$$= \frac{2}{32} \times 100 = 6\%.$$

Notice that you cannot carry this out to 6.25% because you only know the 2 in the fraction 2/32 to one digit. You cannot know the second digit in the answer (6%) until you know the digit following the 2. This would require a third digit in the measurements of 30 and 32.

Q4 What are some of the sources of your error?

Students who are going to predict a_g may have had enough trigonometry to appreciate that the acceleration is $a = a_g \sin\theta$ where θ is the angle between the plane and the level table top. Since $\sin\theta$ = height of plane/length of plane, $a = a_g \times$ height of plane, h/length of plane, ℓ. Or

$$a_g = a\ell/h.$$

Students who have not learned about sines can be given the formula for a_g.

The quantities on the right are all easy to measure.

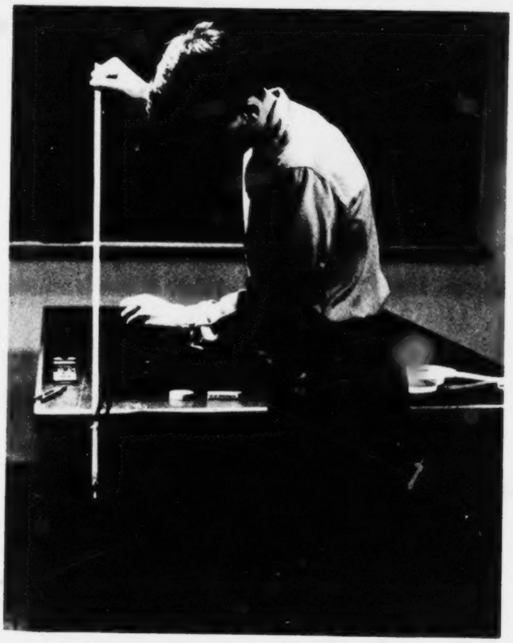
The major sources of error are the slight amount of air friction on the glider as it descends the air track and the small errors in stopwatch timing or photograph measuring. With care the results should agree with the accepted value to within 5%.

Experiment 7 Measuring the Acceleration of Gravity

a) a_g by direct fall*

In this experiment you measure the acceleration of a falling weight when you drop it. Since the distance of fall is too small for air resistance to be important, and since other sources of friction are very small, the acceleration of the falling weight is very nearly a_g.

In any direct measurement of a_g a falling object has to be timed accurately as it falls through several precisely measured distances. Ordinarily the distance of fall must be kept small in order to avoid the appreciable air resistance encountered at high speeds. But a short fall is usually too brief to time accurately without elaborate equipment. In this experiment with very simple equipment these two limitations cause an error of less than 2%.



How to do the experiment

The falling object is an ordinary laboratory hooked weight of at least 200 gm. (Friction has too great an effect on the fall of lighter weights.) The weight is suspended from a few feet of ticker tape as shown above. Reinforce the tape by doubling a strip of masking tape over one end and punch a hole in the reinforcement one centimeter from the end. With careful handling you can support at least a kilogram weight.

When the suspended weight is allowed to fall, the tape dragged behind it is to have equal time intervals marked on it by a vibrating tuning fork.

If a recording timer is available it may be more convenient than a tuning fork for marking the moving tape. The experiment is otherwise the same.

Clamp the timer at the edge of a table in such a way that the paper passes freely through it vertically

---

*Adapted from Brinckerhoff and Taft, Modern Laboratory Experiments in Physics, by permission of Science Electronics, Inc.

The tuning fork must have a frequency between about 100 vibrations/sec and about 400 vibrations/sec. In order to mark the tape the fork must have a tiny felt cone (cut from a marking pen tip) glued to the side of one of its prongs close to the end. Such a small mass affects the fork frequency by much less than 1 vibration/sec. Saturate this felt tip with a drop or two of marking pen ink, set the fork in vibration and hold the tip very gently against the tape.

The falling tape is most conveniently guided in its fall by two thumbtacks in the edge of the table. The easiest procedure is to have an assistant hold the weighted tape straight up until you have touched the vibrating tip against it and said "go." After a few practice runs you will become expert enough to mark several feet of tape with a wavy line as the tape is accelerated past the stationary vibrating fork.

Analyzing your tapes

Mark with an A one of the first wave crests that is clearly formed near the beginning of the pattern. Count 10 intervals between wave crests, and mark the end of the tenth space with a B. Continue, marking every tenth wave with a letter throughout the length of the record, which must be at least 40 waves long.

At A the tape already had a velocity of v_0. From this point to B the tape moved in a time t a distance we shall call d_1. The distance d_1 is described by the equation of free fall:

$$d_1 = v_0 t + \frac{a_g t^2}{2} .$$

In covering the distance from A to C the tape took a time exactly twice as long, 2t, and fell a distance d_2 described (on

and clears the edge of the table. Since it is difficult to measure the frequency of the clapper accurately when operated by 1.5 volts d.c. you must short-circuit the breaker gap inside the timer (using a short length of wire with a small battery clip on each end) and operate the timer on 60 cycles a.c. Be careful that your short circuit connections do not interfere in any way with the free motion of the clapper. You can provide the necessary low-voltage from a bell-ringing transformer (or in some models from the 6 volt a.c. tap of your power supply). Use in series with it a small rheostat such as the one used to control a ripple tank wave generator.

It is important that the current be adjusted until the vibrator action is loud and firm and regular. A skipped beat or two can completely spoil your results—and occasionally does. Since you are overloading the coils of the timer you should leave the current on as briefly as possible.

The clapper is now vibrating at either 60 or 120 cycles per second. To discover which, you need merely pull a few feet of tape through the timer by hand at a speed sufficient to resolve the dots being made by the clapper, and count the number of dots made in approximately one second. The choice between 60 and 120 cycles will be obvious; and no other frequencies are possible.

To measure a_g hold the weighted tape in the timer, start the timer, and release the tape. The series of carbon-paper dots on the tape can then be analyzed in the same way as the waves formed by the tuning fork.

substituting 2t for t) by the equation:

$$d_2 = 2v_0t + \frac{4a_gt^2}{2}.$$

In the same way the distances AB, AE, etc., are described by the equations:

$$d_3 = 3v_0t + \frac{9a_gt^2}{2}$$

$$d_4 = 4v_0t + \frac{16a_gt^2}{2}$$

and so on.

All of these distances are measured from A, the arbitrary starting point. To find the distances fallen in each 10-wave interval we must subtract each equation from its successor, getting:

$$AB = v_0t + \frac{a_gt^2}{2}$$

$$BC = v_0t + \frac{3a_gt^2}{2}$$

$$CD = v_0t + \frac{5a_gt^2}{2}$$

$$DE = v_0t + \frac{7a_gt^2}{2}.$$

From these equations you can see that the weight falls farther during each time interval. Moreover, when we subtract each of these distances, AB, BC, CD,... from the subsequent distance we find that the increase in distance fallen is a constant. That is, each subtraction $BC - AB = CD - BC = DE - CD = a_gt^2$. This quantity is the increase in the distance fallen in each successive 10-wave interval and hence is an acceleration. Our formula agrees with our knowledge that a body falls with a constant acceleration.

From your measurements of AB, AC, AD, etc., tabulate AB, BC, CD, DE, etc., and in an adjoining column, the resulting values of a_gt^2. The values of a_gt^2 should all be equal (within the accuracy of your measurements). Why? Make all your measurements to as many significant figures as are possible with the equipment—neither more nor less.

Find the average of all your values of $a_g t^2$, the acceleration in centimeters/(10-wave interval)^2. We want to find the acceleration in cm/sec^2. If we call the frequency of the tuning fork n per second, then the length of the time interval t is 10/n seconds. Replacing t of 10 waves by 10/n seconds then gives us the acceleration, a_g, incm/sec^2.

The ideal value of a_g is close to 9.8 m/sec^2, but a force of friction of about 15 gms impeding a falling kilogram is sufficient to reduce the observed value to 965 cm/sec^2, an error of about 1.5%.

Q1 What errors would be introduced by using a tuning fork whose vibrations are slower than about 100 vibrations per second?

Q2 Higher than about 400 vibrations per second?

Q3 Is a_g the same everywhere (a) on the earth's surface? (b) in the solar system?

b) a_g from a pendulum

An easy way to find a_g is to time the back-and-forth oscillations of a pendulum. Of course the pendulum is not falling straight down, but the time it takes for a round-trip swing still depends on a_g. The time T it takes for a round-trip swing is

$$T = 2\pi\sqrt{\frac{\ell}{a_g}}\ .$$

In this formula ℓ is the length of the pendulum. If you measure ℓ with a ruler and T with a clock, you should be able to solve for a_g.

You may learn in a later physics course how to derive the formula. Scientists often use formulas they have not derived themselves, as long as they are confident of their validity.

Although this is an indirect method for measuring a_g, it is probably the simplest method that can be considered accurate.

The derivation of the equation for T draws upon concepts of simple harmonic motion which students at this stage are unable to follow. Most first-year college texts in general physics give the derivation.

The practical considerations are very simple.

The clamp that holds the top of the pendulum suspension must not have rounded edges to its jaws, for if it does the suspension will, in effect, be shortened slightly as its sideways motion wraps the top few millimeters around the rounded edges. The clamp must also be very rigid; any back-and-forth wobble will increase the period.

Since the formula is only correct for very small amplitudes of swing (certainly no more than 10°), the timing should be done with the smallest swings that can still be seen after 20 round trips.

If 20 round trips lasting 12.0 seconds are timed with starting and stopping errors of 0.2 seconds each, the total timing error is 0.4 seconds. Since this error is shared among 20 swings, the timing error per swing is only 0.02 seconds. Since each swing takes 12.0/20 = 0.60 seconds = T, the uncertainty in T due to timing is 3%. This is very large indeed compared with other possible sources. To reduce it, time a larger number of swings—say 50, whereupon the same error in timing leads to only about 1.3% uncertainty in T.

Making the measurements

The formula is derived for a pendulum with all the mass concentrated in the bob. Hence the best pendulum to use is one whose bob is a metal sphere hung up by a fine thread. In this case you can be sure that almost all the mass is in the bob. The pendulum's length, ℓ, is the distance from the suspension to the center of the bob.

Your suspension thread can have any convenient length. Measure ℓ as accurately as possible, in either feet or meters.

Set the pendulum swinging with small swings. The formula doesn't work with large swings, as you can test for yourself later.

Time at least 20 complete round trips, preferably more. By timing many trips instead of just one trip you make the errors in starting and stopping the clock a smaller fraction of the total time being measured. Why is this desirable?

Divide the total time by the number of swings to find the time of one swing, T.

Repeat your measurement at least once as a check.

Finally, substitute your measured quantities into the formula and solve it for a_g.

If you measured ℓ in meters, the accepted value of a_g is 9.80 meters/sec^2.

If you measured ℓ in feet, the accepted value of a_g is 32.1 ft/sec^2.

Finding errors

You probably did not get the accepted answer.

Which of your measurements to you think was the least accurate?

If you think it was your measurement of length and you think you might be off by as much as 0.5 cm, change your value of ℓ by 0.5 cm and calculate once more the value of a_g. Has a_g changed enough to account for your error? (If a_g went up and your value of a_g was already too high, then you should have altered your measured ℓ in the opposite direction. Try again!)

If your possible error in measuring ℓ is not enough to explain your difference in a_g, try changing your total time by a few tenths of a second—a possible error in timing. Then you must recalculate T and thence a_g.

If neither of these attempts works (nor both taken together in the appropriate direction) then you almost certainly have made an error in arithmetic or in reading your measuring instruments. It is most unlikely that a_g in your school differs from the above values by more than one unit in the third digit.

Find your percentage error by dividing your error by the accepted value and multiplying by 100:

$$\%\ \text{error} = \frac{\text{accepted value - your value}}{\text{accepted value}} \times 100$$

$$= \frac{\text{error}}{\text{accepted value}} \times 100$$

With care your value of a_g should agree within about 1%.

Q1 How does the length of the pendulum affect your value of T? of g?

Q2 How long is a pendulum for which T = 2 seconds? This is a useful timekeeper.

The length of a pendulum whose T is 1 sec. is 24.8 cm. Remember that T is the time for a round trip. The pendulum that takes 1 sec to swing one way only is going to be 99.4 cm long.

More values are in Table 1.

Table 1

Period of various pendulums

ℓ	T
20 cm	0.75 sec
40	1.26
60	1.54
80	1.78
100	1.98

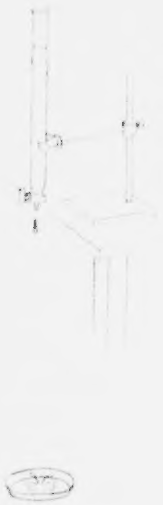
c) slow motion photography (film loop)

With a slow-motion movie camera you could photograph an object falling along the edge of a measuring stick. Then you could determine a_g by projecting the film at standard speed and measuring the distance the object fell in successive frames of the film.

This procedure has been followed in Film Loops 1 and 2, and detailed directions are given for their use in the film loop notes on page 17.

d) falling water drops

You can measure the acceleration due to gravity, a_g, with a burette and a pie plate.



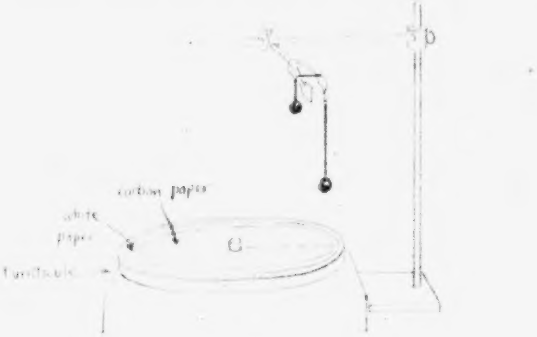
Put a metal tray or pie plate on the floor and set up the burette so that the tap is at least a meter above the plate. Fill the burette with water. Open the tap slightly so that water drops fall steadily onto the plate. Carefully adjust the tap so that one drop hits the plate just at the same instant that the next begins to fall. You can do this most easily by watching the drop on the tap while listening for the previous one to hit the plate. The time that it takes a drop to fall to the floor is now equal to the time interval between one drop and the next. When you have adjusted the rate of drip in this way, find the time interval between drops, t. (To gain accuracy, you may want to count the number that fall in one minute, or if your watch has a second hand, by timing 100 drops.) You also need to measure the height from plate to tap, d.

You now know the time t it takes a drop to fall a distance s from rest. From this you can calculate a_g (since $d = \frac{1}{2} a_g t^2$ for objects falling from rest).

Can you adapt this method to something that can be done at home, e.g., in the kitchen sink?

e) falling ball and turntable

You can measure a_g with a record-player turntable, a ring stand and clamp, carbon paper, two balls and thin thread.



The distance between marks indicates the difference in fall times. The expression for a_g will be manageably simple only if the lower ball is immediately above the turntable.

Ball X and ball Y are draped across the prongs of the clamp. They are lined up along a radius of the turntable.

With the table turning, the thread is burned and each ball, as it hits the carbon paper, will leave a mark on the paper under it.

The angular distance between the marks and the speed of the turntable are used to determine free-fall time.

It is assumed that students have recently completed Sec. 3.7 on Newton's Second Law.

Purpose #1

In this experiment they familiarize themselves with the relationship between F_{net}, m, and a. In no sense do they prove or even verify the law.

Working from stroboscope photographs there will certainly not be enough time for a single student (or group of students) to take a series of data on "a" vs F_{net} and also on "a" vs m. If the task is distributed among several students or groups, however, two graphs can be drawn and each student can contribute a point or two to one of the graphs.

The graph of "a" vs F_{net} with m held constant will be a straight line through the origin.

The graph of "a" vs m with F_{net} held constant will be a hyperbola. The graph cannot be recognized as a hyperbola, however. Students should be challenged on this point to show that it is not, for instance, part of a circle, an ellipse, or a parabola. Only by finding how to convert it into a straight line (or a circle), which are identifiable

Experiment 8 Newton's Second Law

Newton's Second Law is one of the most important and useful laws of nature.

Review Sec. 3.7 to make sure you understand what it says.

This is not a law that you can prove before trying it, as you can prove that the path of a projectile will be a parabola. The law is simply a description, much as the conservation laws are that you will study in Unit 3.

This is not even a law that you can verify, in the sense that you can verify by experiment that a projectile's path really is a parabola. Newton's Second Law has to agree with experiments simply because the law is used to define the units of all Second Law experiments in such a way as to make them come out right.

So what can be the use of our doing an experiment?

Our experiment has two purposes.

First, just because the law is so important it is useful to get a feeling for the behavior of F, m and a. The first part of the experiment is devoted to doing this.

Second, the experiment is an excellent one in which to consider the effect of uncertainties of measurement. This is the purpose of the latter part of the experiment.

How the apparatus works

You are about to find the mass of a loaded cart on which you then exert a measurable force. If you accept Newton's Law as true then you can use it to predict the resulting acceleration of the loaded cart.

Arrange the apparatus as shown in Fig. 1. A spring scale is firmly taped to a dynamics cart. The cart, carrying a

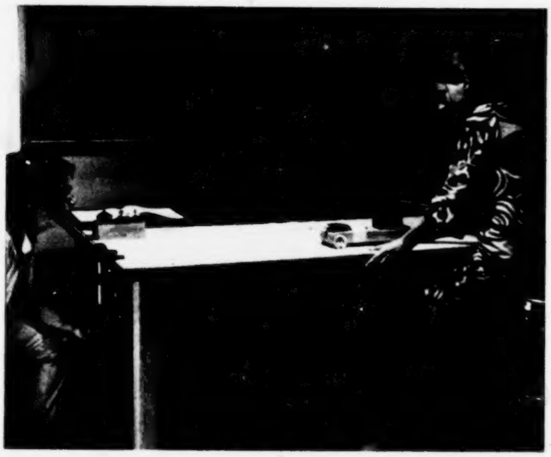


Fig. 1(a).

blinky, is pulled along by a cord attached to the hook of the spring scale. The scale therefore measures the force exerted on the cart.

The cord runs over a pulley at the edge of the lab table and from its end hangs a weight. The hanging weight can be changed so as to exert various tensions in the cord and hence various accelerating forces on the cart.

Measure the mass of the cart together with the blinky, the spring scale and any other weights you may want to include with it. This is the mass m being accelerated.

Now you are ready to go.

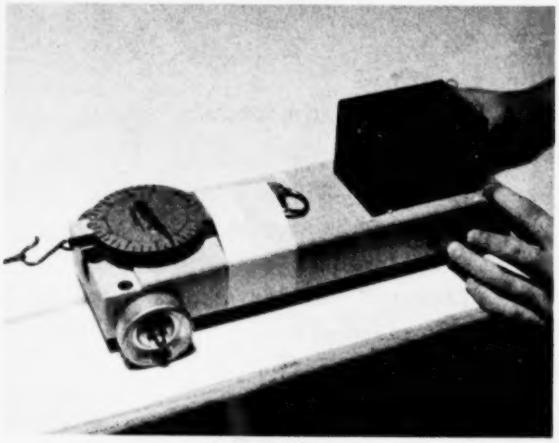


Fig. 1(b)

by simple inspection, can one then work backwards to discover the original shape.

Thus in the case of "a" vs m, a graph of 1/a vs m yields a straight line through the origin. All such lines must have the equation $y = kx$ or in this particular case

$$\frac{1}{a} = km$$

whence

$$ma = \text{constant}$$

which is the graph of a hyperbola.

Moreover this behavior is consistent with Newton's Second Law.

The liquid surface accelerometer

Using a calibrated accelerometer on his dynamics cart a single student (or group of students) may be able to gather all the data alone, if this seems desirable. Certainly he can work faster than he can from photographs.

The action of the accelerometer is described in the Student Handbook, Chap. 3, activity: "Accelerometers."

Begin the experiment by releasing the system and allowing to accelerate. Repeat the motion several times while watching the spring scale pointer. With a little practice you should be able to see that the pointer has a range of positions. The midpoint of this range is a fairly good measurement of the average force F_{av} (often written $\bar{F}$) producing the accelerations.

Record $\bar{F}$ in newtons.

Our faith in Newton's Law is such that we assume the acceleration is the same and is constant every time this particular $\bar{F}$ acts on the mass m.

Use Newton's Law to predict a.

Then measure a to see if your prediction agrees with reality.

To measure the acceleration a take a Polaroid photograph of the flashing blinky (or a strobe picture of a light source) mounted on the cart. As an alternative you might use a liquid surface accelerometer, described in detail elsewhere in this book. Analyze the record just as in the experiments on uniform and accelerated motion in order to find a.

This time, however, you must know the distance traveled in meters and the time interval in seconds, not just in blinks. To find the time interval, count the number of blinks in a minute and divide the total by 60.

Q1 Does $\bar{F} = ma$?

Q2 By what percent do your two values disagree?

Your teacher may ask you to observe the following effects without actually making numerical measurements.

1. Keeping the mass of the cart constant, observe how various forces affect the acceleration.
2. Keeping the force constant, observe how various masses of the cart affect the acceleration.

Q3 Do your observations support Newton's Second Law? Why do you think so?

Experimental errors

It is unlikely that your values of $\bar{F}$ and ma were equal.

This does not necessarily mean that you have done a poor job of taking data. There are at least two other possible reasons for the inequality.

a) You have not yet measured everything necessary in order to get an accurate value for each of your three quantities.

In particular, $\bar{F}$ means net, or resultant, force on the cart—not just the towing force that you measured. Friction force also acts on your cart opposing the accelerating force. You can measure it by reading the spring scale as you tow the cart by hand at constant speed. Do it several times and take an average, $\bar{F}_f$. Since friction, F_f, acts in a direction opposite to the towing force, F_T,

$$\bar{F}_{net} = \bar{F}_T - \bar{F}_{frict} \cdot$$

If $\bar{F}_{frict}$ is too small to measure, then $\bar{F}_{net} = \bar{F}_T$, which is simply the towing force that you wrote as $\bar{F}$ in the beginning of the experiment.

b) Another reason for the inequality of $\bar{F}$ and ma may be that your value for each of these quantities is based on measurements and every measurement is uncertain to some extent.

You should estimate the uncertainty of each of your measurements.

Uncertainty in $\bar{F}$

For example, your uncertainty in the measurement of $\bar{F}$ is the average reading of your spring scale (converted to newtons if necessary) plus or minus the range of uncertainty you marked on your

Purpose #2

A second major purpose of this experiment is the study of experimental errors.

It may be desirable to pursue this treatment in a subsequent laboratory or class period. The ideas developed here will be assumed in future discussions of experimental error.

The question is asked: "Does your measured value of F_{net} really equal your measured value of ma?" Since all three quantities are the results of measurements that have inherent uncertainties, the measurement of F_{net} will almost certainly not equal ma. This discrepancy does not necessarily mean disagreement with Newton's Second Law. It does mean that experimenters must consider uncertainties of measurement and the propagation of error.

If students do find that F_{net} is equal to ma, within the experimental uncertainty, then all is well. (Although if the uncertainty is large, they may justifiably point out that it is a poor experiment.) If the difference between the measured value of F_{net} and the calculated value of ma is greater than the experimental uncertainty, the most likely explanations (apart from miscalculation, or the use of inconsistent units) are probably:

a) the force measured by the spring scale is not the only force acting (friction), and

b) the spring scale has a significant error, or is inaccurately calibrated.

Discussion of error propagation

The student notes point out that the uncertainty in the difference between two measured quantities is the sum of the uncertainties in the two measurements. In symbols:

$$\Delta(x - y) = \Delta x + \Delta y$$

The same is true for the sum of two measurements, of course:

$$\Delta(x + y) = \Delta x + \Delta y$$

Students may ask about the uncertainty in a product. There is a transparency to show this, but if it is not available draw a rectangle with sides of length x and y (Fig. 1). The area of the rectangle is $x \cdot y$. This is how we compute the product of two numbers.

Draw a second rectangle, sides of length $x + \Delta x$, $y + \Delta y$ (Fig. 2).

The area of this rectangle is:

$$(x + y) + (x \cdot \Delta y) + (y \cdot \Delta x) + (\Delta x \cdot \Delta y)$$

If each side of the rectangle represents a measurement (x,y) and the uncertainty in the measurement $(\Delta x, \Delta y)$, then the uncertainty in the product is represented by the shaded area in the figure.

$$\Delta(x \cdot y) = (y \cdot \Delta x) + (x \cdot \Delta y) + (\Delta x \cdot \Delta y)$$

A little algebra will show that the percentage uncertainty in the product is equal to the sum of the percentage uncertainties in each measurement. Since Δx and Δy are both small, we can neglect the product $\Delta x \cdot \Delta y$, and write:

$$(x \cdot y) = (y \cdot \Delta x) + (x \cdot \Delta y)$$

The percentage uncertainty is:

$$\frac{\Delta(x \cdot y)}{x \cdot y} \cdot 100 = \frac{y \cdot \Delta x + x \cdot \Delta y}{x \cdot y} \cdot 100$$

$$= \frac{\Delta x}{x} \cdot 100 + \frac{\Delta y}{y} \cdot 100$$

(If we had used a rectangle $x - \Delta x$ by $y - \Delta y$, we would have gotten just the same result.)

paper tape (also converted to newtons). Thus if your scale reading ranged from 1.0 to 1.4 N then the average is 1.2 N.

The range of uncertainty is 0.2 N. Thus the value of $\bar{F}$ is 1.2 ± 0.2 N.

What is your value of $\bar{F}$?

Uncertainty in m

Your uncertainty in m is half the smallest scale reading of the balance with which you measured it. Your mass consisted of a cart, a blinky and a spring scale (and possibly an additional weight). Record the mass of each of these in kilograms, in some way such as follows.

$m_{cart} = 0.90 \pm 0.05$ kg

$m_{blinky} = 0.30 \pm 0.05$ kg

$m_{scale} = 0.10 \pm 0.05$ kg.

The total mass being accelerated is the sum of these masses. The uncertainty in the total mass is the sum of the three uncertainties. Thus, in our example,

$m = 1.30 \pm 0.15$ kg.

Even when you subtract measured values the uncertainty is still the sum of the uncertainties.

What is your value of m?

Uncertainty in a

Finally, consider the measurement of a. You found this by measuring $\Delta s/\Delta t$ for each of the intervals between the points on your blinky photograph.

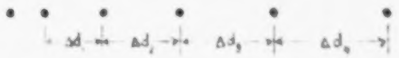


Fig. 2.

Suppose the points in Fig. 2 represent the record of blinky flashes. The distance between the points must be measured. Your table of data should be similar to Table 1.

The uncertainty in each value of $\Delta s/\Delta t$ is due primarily to the fact that the records of the blinky flashes are not true points. Suppose that the uncertainty in locating the distance between the centers of the dots is 0.01 cm as shown in the first column of Table 1. When we take the differences between successive values of the speeds, $\Delta s/\Delta t$, we get the accelerations, $\Delta v/\Delta t$, the speeds recorded in the second column. As we noted above, when a difference in two measurements is involved, we find the uncertainty of the difference (in this case, $\Delta v/\Delta t$) by adding the uncertainties of the two measurements. This results in a maximum uncertainty in acceleration of ± 0.2 cm/sec^2 as recorded in the table.

TABLE 1

AVERAGE SPEEDS	
$\frac{\Delta s_1}{\Delta t} = 2.5\ \frac{\text{cm}}{\text{sec}} \pm 0.1\ \frac{\text{cm}}{\text{sec}}$	
	$\frac{\Delta v_1}{\Delta t} = 0.9\ \frac{\text{cm}}{\text{sec}^2} \pm 0.2\ \frac{\text{cm}}{\text{sec}^2}$
$\frac{\Delta s_2}{\Delta t} = 3.4\ \frac{\text{cm}}{\text{sec}} \pm 0.1\ \frac{\text{cm}}{\text{sec}}$	
	$\frac{\Delta v_2}{\Delta t} = 0.6\ \frac{\text{cm}}{\text{sec}^2} \pm 0.2\ \frac{\text{cm}}{\text{sec}^2}$
$\frac{\Delta s_3}{\Delta t} = 4.0\ \frac{\text{cm}}{\text{sec}} \pm 0.1\ \frac{\text{cm}}{\text{sec}}$	
	$\frac{\Delta v_3}{\Delta t} = 0.8\ \frac{\text{cm}}{\text{sec}^2} \pm 0.2\ \frac{\text{cm}}{\text{sec}^2}$
$\frac{\Delta s}{\Delta t} = 4.8\ \frac{\text{cm}}{\text{sec}} \pm 0.1\ \frac{\text{cm}}{\text{sec}}$	
	Average $0.8\ \frac{\text{cm}}{\text{sec}^2} \pm 0.2\ \frac{\text{cm}}{\text{sec}^2}$

Comparing our results

We now have values of $\bar{F}$, m and a together with their uncertainties, and we should consider the uncertainty of ma. When we have discovered the uncertainty of this product of two quantities, we shall then compare the value of ma with the value of $\bar{F}$ and draw our final conclusions.

This is a general result: the percentage uncertainty in a product is equal to the sum of the percentage uncertainties in each measurement, however many terms there are in the product. Similarly, for a quotient, the percentage uncertainty is equal to the sum of the percentage uncertainties of all the terms.

While the simplicities outlined above are useful for our introductory exercise, a much more general approach to uncertainty and its analysis is required for most experimental situations. This generality is needed because a) there is a variety of mechanisms responsible for introducing uncertainties, and b) there are, of course, other kinds of functions through which uncertainties are to be propagated during the course of the year. Tables 1 and 2 provide a brief outline summary of these factors.

Table 1

Mechanisms Responsible for Uncertainties

1) Scale reading uncertainties. Finite space between marks on scales.
2) Object irregularities.
    a) Obvious variations which can be identified and have predictable effects.
    b) Perturbations requiring a statistical treatment of the final results (e.g., population surveys, radioactive disintegrations).
3) Systematic discrepancies introduced by
    a) Bias, due to poor experimental design
    b) Use of oversimplified theory.

Table 2

Propagation Rules to Calculate Maximum Uncertainties

1) Sums and differences

Add absolute uncertainties to obtain absolute uncertainty in result.

For example:

If $A = 2.51 \pm 0.01$
and $B = 3.33 \pm 0.02$
then $A + B = 5.84 \pm 0.03$.

2) Products and quotients

Add % uncertainties to obtain % uncertainty in result.

For example:

If $A = 2.51 \pm 0.01$ or $\pm 0.4\%$
and $B = 3.33 \pm 0.02$ or $\pm 0.3\%$,
then $AB = 8.36 \pm 0.7\%$

When two quantities are multiplied the percentage uncertainty in the product is the sum of the percentage uncertainties in each of the factors. Thus, in our example,

$$m \times a = 1.30 \text{ kg} \times 0.8 \frac{\text{cm}}{\text{sec}^2} = 1.04 \frac{\text{kg-m}}{\text{sec}^2}.$$

The percentage uncertainty in $a = 0.8 \pm 0.2$ cm/sec² is 25% (since 0.2 is 25% of 0.8). The percentage uncertainty in m is 11.5%. Thus the percentage uncertainty in ma is 25% + 11% = 36% and we can write our product as

$$ma = 1.04 \frac{\text{kg-m}}{\text{sec}^2} \pm 36\%$$

which is, to two significant figures,

$$ma = 1.0 \pm 0.4 \frac{\text{kg-m}}{\text{sec}^2} \text{ (or newtons).}$$

In our example we found from direct measurement that $\bar{F}_{net} = 1.2 \pm 0.2$ N.

Are these the same quantity?

Although 1.0 does not equal 1.2, since the range of 1.0 ± 0.4 overlaps the range of 1.2 ± 0.2 we can say that "the two numbers agree within the range of uncertainty of measurement."

An example of lack of agreement would be 1.0 ± 0.2 and 1.4 ± 0.1. These cannot be the same quantity since there is no overlap.

In a similar way, work out your values of $\bar{F}_{net}$ and ma.

Q4 Do they agree within the range of uncertainty of your measurement?

Q5 Is the relationship $F_{net} = ma$ an experimental fact? If not, what is it?

3) Power and roots

Multiply % uncertainty by power or root (exponent) to obtain percentage uncertainty in the result.

For example:

If A = 2.51 ± 0.01 or ± 0.4%, then A^2 = 5.80 ± 0.8%

Exercises could be invented to provide drill and practice on any of the items listed in the tables. But it is probably more appropriate to call attention to them as they are needed.

Remember not to give the whole bottle of medicine in one sitting—parcel it out in gentle doses over the whole year. The therapy takes time!

Experiment 9 Inertial and Gravitational Mass

Apparatus

Inertial balance, clamps, metal slug, wire, ring stand, cord, unknown mass, spring balance.

Procedure Notes

Weight is a measure of the gravitational force on an object. Mass is a measure of resistance of an object to changes in the state of motion, a measure of inertia.

The inertial balance is a simple device for measuring the inertial mass of different objects. The frequency of its horizontal vibration depends upon the inertial mass placed on the balance, since inertia is a resistance to any motion or change of motion.

1. Measure the period of the balance alone by measuring the time for as many vibrations as you can conveniently count.

2. Select six identical objects of mass such as six C-clamps. Measure their period using first one, then two, then three, etc., of the clamps on the balance.

3. Measure the period of an unknown mass supplied by the instructor and record this result.

4. Calibrate the C-clamps by measuring their actual masses on a scale.

5. Discover whether or not gravity plays a part in the operation of the inertial balance. Load it with the iron slug. This can be done by inserting a wire through the center hole of the slug and letting the slug rest on the platform. Measure its period. Now lift the slug slightly so that it no longer rests upon the platform, support it from a ringstand and again measure the period.

Written Work

1. Plot the period, T, against the mass used in each case.

2. Arrange the data in orderly fashion.

3. Locate the value of the unknown mass on the prepared graph and compare with the actual measured value.

4. Compare the data obtained when the metal slug was supported by the platform and when it was free. Is inertia related to or dependent upon gravity?

5. Include a sketch of the apparatus.

6. Summarize briefly what you have learned from this exercise.

In the short version of this experiment students can stop after recording the path of the ball (before section, Analyzing your data). By this point they have plotted the trajectory for themselves, which may be sufficient.

However, another important pay-off of this experiment is an understanding of the principle of superposition and for this the students must go on to analyze their data. The principle can be made particularly clear if the horizontal displacements are graphed against time squared. Both graphs should be straight lines, as would be expected of the two motions if they took place separately.

Experiment 10 Trajectories—I

When a ball rolls off a table top, as in Fig. 1, we know it will eventually hit the floor and that before it does it will travel some distance horizontally. There are a number of paths it might follow in doing this.

In this experiment your problem is to find out just what path the ball does travel. You will probably then be able to find a mathematical description with which you can make useful and accurate predictions.

How to use the equipment

If you are setting up the equipment for the first time follow the manufacturer's instructions for assembling it.

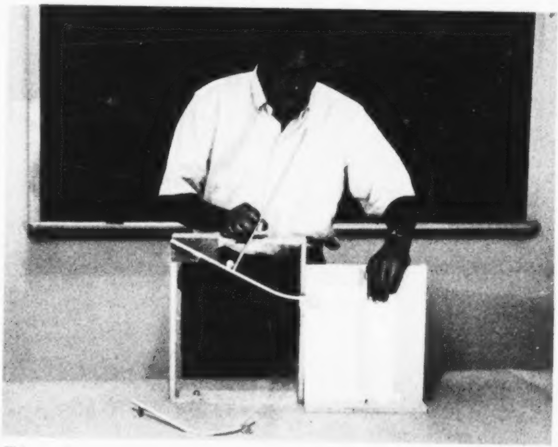


Fig. 1.

The apparatus (Fig. 1) consists of one or two ramps down which you can roll a steel ball. Adjust one of the ramps (perhaps with the help of a level) so that the ball leaves it horizontally. Set the vertical impact board on the plotting board so that a ball launched from the ramp will hit it.

Attach a piece of carbon paper to the side of the impact board facing the end of the ramp, with the carbon side facing the ramp. Then tape a piece of translucent onion skin paper over the carbon paper.

Tape a larger piece of paper, preferably squared graph paper, to the plotting board with its left-hand edge behind the end of the launching ramp.

Release the ball from various points on the ramp until you find one from which the ball falls close to the bottom right-hand corner of the plotting board. Mark the point of release on the ramp. Now when you put the impact board in its way, the ball hits it and leaves a mark, recording the point of impact between ball and board, that you can see through the onion skin paper. (Make sure that the impact board doesn't move when the ball hits it; steady it with your hand if necessary.) Transfer the point to the plotting board by making a mark on it just next to the point on the impact board.

Repeat this for other positions of the impact board to record more points on the ball's path. Move the board back equal distances every time and always release the ball from the same spot on the ramp. Continue until the ball does not hit the impact board any longer.

To release the ball do not hold it in your fingers—it is impossible to let go of it in the same way every time. Instead dam it up with a ruler held at a mark on the ramp and release the ball by lifting the ruler quickly.

Try releasing the ball several times (always from the same point) for the same setting of the impact board. Do all the impact points exactly coincide?

Now remove the impact board, release the ball once more and watch carefully to see that it moves along the points marked on the plotting board.

By observing the path the ball follows you have completed the first goal of the experiment.

The curve traced out by your plotted points is called the trajectory of the ball.

You may want to stop here, though you will find it useful to go further and explore some of the properties of your trajectory.

Analyzing your data

To help you analyze the trajectory, draw a horizontal line on the paper at the level of the lip of the launching ramp. Then remove the paper from the plotting board and draw a smooth continuous curve through the points.

If it is true that an object moves equal distances in equal times when no net force acts on it, then you can assume that the ball will move horizontally at constant speed.

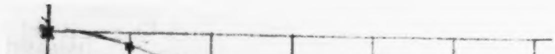


Fig. 2.

Draw vertical lines through the points on your graph (Fig. 2). Make the first line coincide with the lip of the launching ramp. Because of your plotting procedure these lines should be equally spaced. If the horizontal speed of the ball is uniform, these vertical lines are drawn through positions of the ball separated by equal time intervals. This means that the ball travels across the first measured space and reaches the second vertical line one unit of time after leaving the ramp; it reaches the third line after two time units, the fourth after three, and so on.

Now consider the vertical distances fallen in each time interval. Measure down from your horizontal line the vertical fall to each of your plotted points. Record your measurements in a column and alongside them in a parallel column record the corresponding horizontal distance measured from the first vertical line.

Q1 What would a graph look like on which you plot horizontal distance against time?

Earlier in your work with accelerated motion you learned how to recognize uniform acceleration (see Secs. 2.5-2.8 in the text and Experiment 5). Use the data you have just collected to decide whether the vertical motion of the ball was uniformly accelerated motion.

Q2 What do you find?

Q3 Do the horizontal and the vertical motions affect each other in any way?

Write the equation that describes the horizontal motion in terms of horizontal velocity, v_h, the horizontal distance, d_h, and the time of travel, t.

Q4 What is the equation that describes the vertical motion in terms of the distance fallen vertically, d_v, the vertical acceleration, a_g, and the time of travel, t?

The graph of horizontal distance against time is a straight line beginning at the origin.

If vertical motion is uniformly accelerated then a plot of total vertical distance fallen against the square of the horizontal distance (actually t^2) will be a straight line.

Another way to show this is to measure the change in distance, Δd, (the additional distance fallen) in each time interval. Δd should increase by the same amount in each time interval.

The horizontal and vertical motions have no effect on each other.

Horizontal distance, $d_h = v_h t$

Vertical distance, $d_v = a_g t^2/2$

To find a single equation for the trajectory see the derivation on pp. 99-100 of the text.

Testing your model

A good test of such a model is to use it to make predictions which can be checked experimentally. Suppose you put the ramp near the edge of a table so that the ball would land on the floor. Could you predict for a given release point where it would land?

Here is one way you can do this; you may be able to think of others.

There will be some factors in the two equations for the ball's motion that you do not know (the times, the ball's initial horizontal speed). But you can combine the equations to get rid of these unknowns, and use just the vertical and horizontal distances that you measure on your plot. Then for the new vertical distance (height of the launch point above the floor) you can predict the horizontal distance that the balll will travel. Put down a sheet of paper at this point and mark your predicted spot. Before you test your prediction try to estimate how close to your mark you think the ball will actually land.

Extensions

There are many other things you can do with this apparatus. Here are some of them.

Q5 What do you expect would happen if you repeated the experiment with a glass marble of the same size instead of a steel ball? Try it!

Q6 What will happen if you next try to repeat the experiment starting the ball from a different point on the ramp?

Q7 What do you expect if you use a smaller or a larger ball starting always from the same reference point on the ramp?

All balls of whatever size or mass should have nearly the same acceleration, though very small or lightweight balls are slowed by roughness of the plane or by air resistance.

Q8 Plot the trajectory that results when you use a ramp that launches the ball at an angle to the horizontal.

Q9 In what way is this curve similar to your first trajectory?

Q10 Find a_g from this experiment using a procedure similar to that described on page 5

Q11 What other changes in the conditions of this experiment can you suggest? Consult your teacher before testing them.

If the ball has an upward velocity at the time of launching, separate analysis of the upward and the downward leg will show that each of them is a parabola as in the earlier case.

This experiment, like the preceding one (Trajectories - I) assumes student mastery of Secs. 4.2 and 4.3.

This experiment develops the same insights as Trajectories - I but in less detail. It also requires less time and less equipment.

Experiment 10 Trajectories — II

This experiment tests the equation for projectile motion by using it to predict the landing point of a ball launched horizontally from a table top. You will determine the speed, v_0, of the ball as it leaves the table and, knowing the height of the table above the floor and a_g, you will use the equation to predict where on the floor the ball will land.

The equation is based on certain assumptions. If it correctly predicts the landing point, then the assumptions are evidently valid.

The assumptions

If the ball continues to move horizontally with velocity v_0 (Assumption 1), the horizontal distance x at time t from launch will be given by the equation

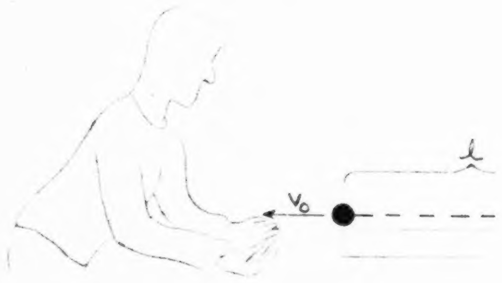
$$x = v_0 t \qquad (1)$$

where v_0 is the horizontal velocity at launch.

Similarly, if the ball moves vertically with uniform acceleration (Assumption 2), the vertical distance fallen at time t will be given by this equation.

$$y = \tfrac{1}{2} a_g t^2 \qquad (2)$$

Measuring v_0



$v_0 = \frac{\ell}{t}$

F

Solving equation (1) for t,

$$t = \frac{x}{v_0} \qquad (3)$$

Substituting this into equation (2), we have

$$y = \tfrac{1}{2}a_g\left[\frac{x}{v_0}\right]^2 = \frac{a_g x^2}{2v_0{}^2}\,. \qquad (4)$$

This is the equation to test,by using it to predict the value of x where the ball strikes the floor. Solving equation (4) for x, we have

$$x^2 = \frac{2v_0{}^2 y}{a_g}$$

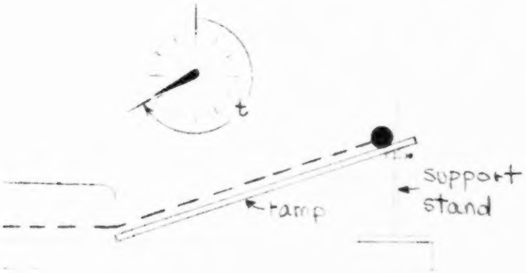
or,

$$x = \sqrt{\frac{2v_0{}^2 y}{a_g}} = v_0\sqrt{\frac{2y}{a_g}}\,.$$

From this last equation we can predict x if we know v_0, y and a_g.

The experiment

You can determine v_0 by measuring the time t that the ball takes to roll a distance ℓ along the table top (Fig. 1). Repeat the measurement a few times, always releasing the ball from the same place on the ramp, and take the average value of v_0.



ig. 1.

Q1 This is the same situation as that examined in the laboratory. If the slingshot is held at a distance y above the ground, the range, x, will be

$$x = v_o\sqrt{\frac{2y}{a_g}}$$

But what is v_o? To find this, shoot the same projectile vertically upward and time its flight. Since $v_o = a_g t$ for each half of the flight, the time, T, for the round trip will be twice t, or

$$T = 2t = \frac{2v_o}{a_g}$$

whence $v_o = \frac{a_g T}{2}$ and our expression for the range x becomes

$$x = \frac{a_g T}{2}\sqrt{\frac{2y}{a_g}} = T\sqrt{\frac{a_g y}{2}}$$

Q2 We assume that the ball is launched with the same horizontal velocity, v_h, and vertical velocity, v_v, as on the earth.

Consider the first half of the ball's flight on earth, in which it rises to the top of its trajectory. It will reach this high point in a time, t, defined by $v_v = a_g t$, whence $t = v_v/a_g$.

During this same time the ball is also travelling horizontally with a velocity v_h and will therefore have covered a horizontal distance, $d = v_h t$ which becomes on substitution $d = v_h \cdot v_v/a_g$.

Measure y and calculate x. Place a target on the floor at your predicted landing spot. How confident are you of your prediction? Since it is based on measurement there is some uncertainty involved. Mark any area round the spot to indicate the uncertainty.

Now release the ball once more, and this time let it roll off the table and land on the floor (Fig. 2).

If the ball actually does fall within the range of values of x you have estimated, then you have verified the assumptions on which your calculation was based.

Measuring

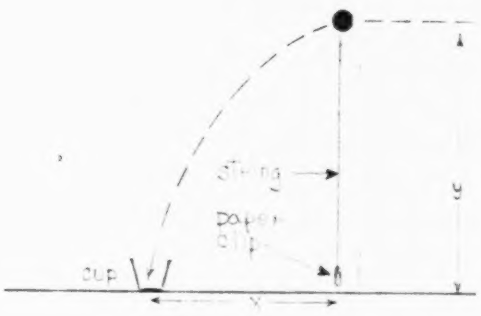


Fig.

Q1 How could you determine the range of a ball launched horizontally by a sling shot?

Q2 Assume you can throw a baseball 100 meters on the earth's surface. How far could you throw that same ball on the surface of the moon, where the acceleration of gravity is one-sixth what it is at the surface of the earth?

Q3 Will the assumptions made in equations (1) and (2) hold for a ping-pong ball? If the table was 1,000 meters above the floor, could you still use equations (1) and (2)? Why or why not?

x

For checking on the predicted x, place a cup at the predicted point of impact.

The ball will cover an additional equal distance during the descending half of its trajectory so its total range, R, on earth will be

$$R = 2d = 2v_h v_v / a_g .$$

On the moon a_g is only one-sixth as great as on the earth and hence R must be six times as great.

Q3 Assumptions (1) and (2) hold as long as we can ignore the effect of air resistance and as long as we assume the force of gravity is constant in magnitude and direction.

If the earth had no atmosphere, therefore, the answer to the question would be "yes," but in fact air resistance will reduce both the horizontal and the vertical distances travelled in a time t. The quickest way to appreciate this is to play "catch" with a ping pong ball.

2.

The instructions assume that the students have already studied the subject of circular motion through Sec. 4.7.

The next experiment, Circular Motion II, assumes that the students have not yet studied circular motion, and they discover $F = mv^2/R$ for themselves in the lab.

Whichever version of circular motion is used, the teacher should notice that it uses insights derived from the preceding work on Newton's Second Law (an acceleration, v^2/R, results from a force, F). It is also very important to notice that the work on circular motion is central to the study of satellite motion in Unit 2 and leads directly into Chapter 7.

The object of the lab is to predict the maximum radius at which an object can be located on a rotating platform as a function of the period, and the friction force. If their predictions are within 10% of the experimental results you can consider them a success.

Experiment 11 Circular Motion I

You may have had the experience of spinning around on an amusement park contraption known as the Whirling Platter. The riders seat themselves at various places on a large flat polished wooden turntable about 40 feet in diameter. The turntable gradually rotates faster and faster until everyone (except for the person in the center of the table) has slid off. The people at the edge are the first to go.

Q1 Why do the people fall off?

Study Sec. 4.6, where you learn that the centripetal force needed to hold a rider in a circular path is given by

$$F = mv^2/R.$$

Friction on a spinning table

On the rotating table the centripetal force is provided by friction. Friction is the centripetal force. On a frictionless table there could be no such centripetal force and everyone except a rider at the center would slide off right at the start.

Near the outer edge where the riders have the greatest velocity, v, the friction force, F, needed to hold them in a circular path is large. (The effect of R in the denominator also being large is more than cancelled out by the fact that v is squared in the numerator.)

Near the center where v is nearly zero very little friction force is needed.

Q2 Where should a rider sit on a frictionless table to avoid sliding off? Use the formula above to justify your answer.

Evidently, if you know the force needed to start a rider sliding across a motionless table top (the force of friction), you know the value of the centripetal force, mv^2/R, at which the rider would begin to slip when the table is rotating.

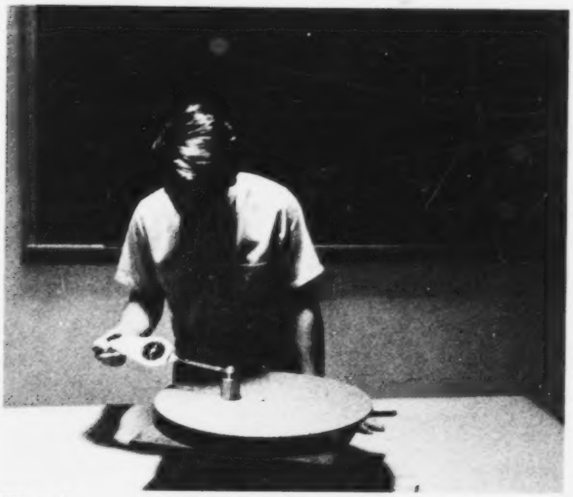


Fig. 1.

Studying centripetal force

Unfortunately you probably do not have a Whirling Platter in your classroom, but you do have a masonite disc which fits on a turntable. The object of this experiment is to predict the maximum radius at which a mass can be placed on the rotating turntable without sliding off.

If you do this under a variety of conditions, you will see for yourself how forces act in circular motion.

For these experiments it is more convenient to write the formula $F = mv^2/R$ in terms of the period, T, of the table instead of the value of v and R at some point on it. This is because it is easier to measure T and R than to measure v. We rewrite the formula as follows:

$$v = \frac{\text{distance travelled by a rider in one revolution}}{\text{time for one revolution}}$$

$$= \frac{2\pi R}{T}$$

Substitute this into the expression for centripetal force

$$F = \frac{mv^2}{R} = \frac{m}{R} \times \frac{4\pi^2 R^2}{T^2} = \frac{4\pi^2 mR}{T^2}$$

Then solve it for R.

$$R = \frac{FT^2}{4\pi^2 m}$$

You can measure all the quantities in this equation.

The friction force needed to get the object started differs from the sliding friction. You might want some students to investigate this curious difference.

Remember that R must be measured to the center of the mass on the turntable. Have the students mark the inner and the outer edges of the mass in the position where it begins to slip, and then later measure R to the midpoint between them. Also when we measure R to the center of mass we assume that R is several times larger than the radius of the weight. Typical data are in Table 1.

If there are no small spring balances (2.5 newtons) you can have the students measure the friction force by tilting the turntable up until the mass begins to slide down. The friction force can be determined from the vector diagram.

$$\sin\theta = F_f \text{ or}$$

$$F_f = \sin\theta$$

For small angles $\sin\theta = \theta$ and $\theta = \frac{h}{2R}$,

so $$F_f = \frac{Wh}{2R}.$$

The students can be asked to determine the frequency of the turntable. The table may not be turning at 33, 45 or 78 rpm. You may have to review the difference between frequency and period and also make sure that periods are expressed in seconds not minutes.

Q4 When the mass is made smaller it might seem that the radius, R, would have to be smaller since R appears to be proportional to m in the expression $F = mv^2/R$,

But F is also a function of m, and v is a function of R, so the answer to the question is not obvious.

In particular the centripetal force, F, is equal to the force of friction at the moment of slipping, which means that

$$F = kma_g$$

where k is the coefficient of friction and ma_g is the weight of the object on the turntable, assumed to be horizontal.

Also $v^2 = (2R/T)^2$.

Putting these expressions for F and v^2 into the centripetal force equation we get

$$kma_g = m \cdot \frac{4\pi^2 R^2}{RT^2}$$

and simplifying

$$R = \frac{ka_g T^2}{4\pi^2}$$

Since m has now vanished from the expression for R it follows that the value of R is independent of m. This means that for a given solution for R you can use any mass.

Your task is to predict the radius, R, at which a weight can be placed on the rotating table so that it will just barely remain and not slip.

Use a spring scale to measure the force, F, needed to make a mass, m, of 0.20 to 1.00 kg start to slide across the motionless disc (Fig. 1).

Then make a chalk mark on the table and time it for 100 revolutions to calculate T, the time for one revolution (or accept the given values of turntable frequency). Remember that T, the period, is 1/frequency.

Make your predictions of R for turntable frequencies of 33 rpm, 45 rpm and 78 rpm.

Then try it!

Q3 How great is the percentage difference between prediction and experiment in each case? Do you think this is reasonable agreement?

Q4 What effect would decreasing the mass have on the predicted value of R? Careful! Decreasing the mass has an effect on F also. Check your answer by doing an experiment.

Q5 What is the smallest radius you can turn a car in if you are moving 60 miles an hour and the friction force between tires and road is one third the weight of the car?

Q6 What happens to the period of an earth satellite as it falls into a lower orbit?

Q5 Changing the mass of the object should have no effect because the friction force should increase with the mass as will the centripetal force, and the two effects will cancel.

$$F_f = \frac{1}{3}\, ma_g = \frac{mv^2}{R}$$

$R = \frac{3v^2}{a_g}$, which does not contain m.

60 mph = 88 feet/sec = 27 m/sec.

$R = \frac{3 \times (27)^2}{9.8} = 224$ meters (pretty wide arc).

Q6 The expression derived in Q4 above gives the connection between R and the period, T. It shows that as R decreases so does T. More specifically, if R becomes one quarter as great, T becomes half as great. R is measured from the center of the earth in the case of an earth satellite.

Table 1

Typical data for brass weight on masonite turntable

Weight	Force to start slipping	R for slipping 16rpm	$33\frac{1}{3}$rpm	45rpm	78rpm
1000 gm	1.5 to 2N	no slip	21.5cm	9.7cm	3.0cm
500	0.9 to 1.1	no slip	20.9	9.3	2.5
300	0.5 to 0.6	no slip	14.5	7.8	2.2
200	0.4 to 0.5	no slip	21.0	9.7	2.5
100	0.2 to 0.3	no slip	19.2	10.0	3.2

Note: R includes the radius of the weight.

This experiment assumes that students have not studied text Sec. 4.6 in which the formula for centripetal force is derived. Instead the experiment leads to the students discovering that F is proportional to m, v^2, and 1/R.

Apparatus

The equipment is easy to assemble if no ready-made device is available. One needs a spring scale calibrated preferably in newtons or dynes, string, rubber stoppers for weights, a stick (such as a meter stick) around which the weighted string can be rotated, and an audible timing device. As a timing device use a metronome. Or a student with a watch can count out the time.

A scale calibrated in grams can be converted to a force scale in newtons by placing a piece of tape along one edge and marking the corresponding force units on it in newtons (1 newton = 102 grams weight; 1 kg. weight = 1 x 9.8 newtons).

If you use a glass medicine dropper tube for the bearing, be careful to tape it completely so that if it cracks it will not shatter. You can also use a plastic or metal tube.

Experiment 12 Circular Motion II

An earth satellite and a weight swung around your head on the end of a string are controlled by the same laws of motion. Both are accelerating toward the center of their orbit due to the action of an unbalanced force.

In the following experiment you discover for yourself how this centripetal force depends on the mass of the satellite and on its speed and distance from the center.

How the apparatus works

Your "satellite" is one or more rubber stoppers. When you hold the apparatus (Fig. 1) in both hands and swing the stopper around your head you can measure the centripetal force on it with a spring scale at the base of the stick. The scale should read in newtons or else its readings should be converted. Remember 1 N = the weight of 102 gms weight or 1 kg weight = 9.8 N.

You can change the length of the string so as to vary the radius, R, of the circular orbit, and you can tie on more stoppers to vary the satellite mass, m.

The best way to time the period, T, is to swing the apparatus in time with some periodic sound such as the tick of a metronome or have an assistant count out loud using a watch. You keep the rate constant by adjusting the swinging until you see the stopper cross the same point in the room at every tick.

Hold the stick vertically and have as little motion at the top as possible, since this would change the radius. Since the stretch of the spring scale also alters the radius it is helpful to move the scale up or down slightly to compensate for this.

Doing the experiment

The object of the experiment is to find out how the force read on the spring scale varies with m, with v and with R.

You should only change one of these three quantities at a time so that the effect of each one can be investigated independently of the others. It's easiest to either double or triple m, v and R (or halve them, etc. if you started with large values).

Two or three different values should be enough in each case. Make a table and record your numbers in it clearly.

Q1 How do changes in m affect F if R is kept constant? Write a formula that states this relationship.

Q2 How do changes in v affect F if m is kept constant? Write a formula to express this too.

Q3 Measure the effect of R and express it in a formula.

Q4 Can you put m, v and R all together in a single formula for centripetal force, F?

After you have committed yourself, check your formula by studying text Sec. 4.6.

An assumption

As the stoppers are swung in a circle at low speed the string is by no means horizontal, and the stoppers' distance from the vertical stick, R', grows less than R, the length of the string (Fig. 2). Students may wonder whether the centripetal force is determined by R or by R'. The answer is that R is still the correct length to measure, as the following analysis shows.

When the string sags the mass moves in a smaller circle whose radius is

$$R' = R \cos \theta$$

Its velocity becomes

$$v' = \frac{2\pi R'}{T} = \frac{2\pi R \cos\theta}{T} = v \cos \theta$$

and the centripetal force is reduced to

$$F' = F \cos \theta.$$

Substituting these expressions for R', v', and F' into

$$F' = \frac{mv'^2}{R'} \qquad (1)$$

gives us

$$F \cos \theta = m \frac{v^2 \cos^2 \theta}{R \cos \theta}$$

which simplifies to

$$F = \frac{mv^2}{R}. \qquad (2)$$

Thus formula (1) which describes the centripetal force when the string sags is really the same as formula (2) which students have been seeking to verify on the assumption that the string was horizontal.

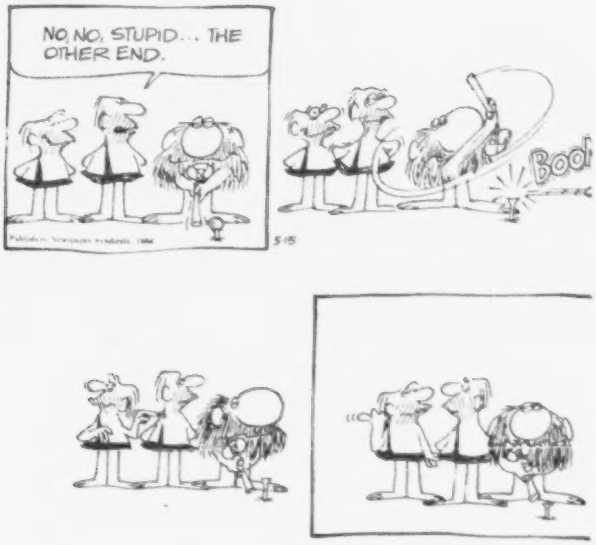
Discussion

Groups of students can be assigned different sets on conditions; then the data can be pooled.

With the data collected, the students can proceed to compare their data. As we are only concerned with developing the functional relation between the force and variables, the data can be treated as units (1, 2, 3) and the $4\pi^2$ constant dropped.

Because the error terms associated with the variables in this experiment range from very small (for the mass) to very large (for the period), a discussion of error and its estimation would be appropriate here.

A second topic would be to present the students with the height of a satellite orbit and its velocity (for Alouette I, $\overline{h}$ = 650 miles and $\overline{v}$ = 16,500 miles per hour) and ask them either how much faster it would have to go to boost its orbit 100 miles, or how much its orbit would be increased if it added 100 miles per hour to its velocity. This is a simplified problem quite similar to those which astronauts solve during maneuvering.

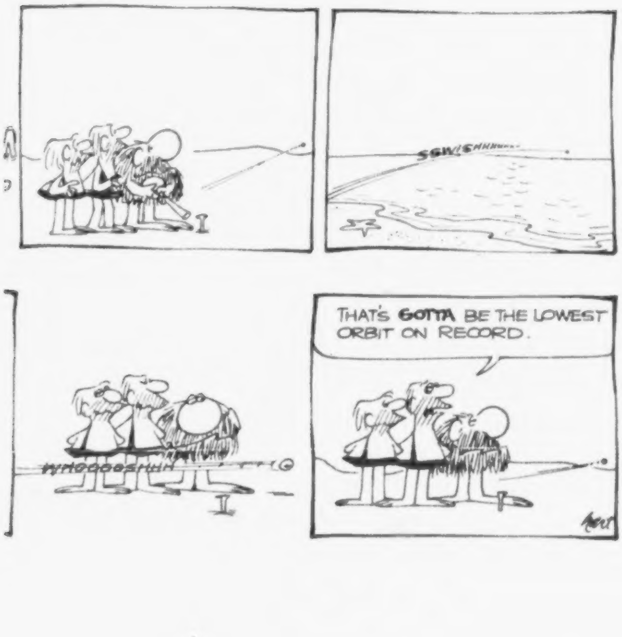

NO, NO, STUPID... THE OTHER END.
Boo
5-15

To solve the problem, one can assume that the gravitational force does not change appreciably over relatively small distances (like 100 miles). Set

$F_1 = \frac{mv^2_1}{R_1}$ for the initial orbit, and

$F_2 = \frac{mv^2_2}{R_2}$ for the final orbit. When

any three of the values are known, the fourth can be calculated. Radius of the earth is 3959 miles. So $R_1 = 3959 + \bar{h}$ miles.


SSWISHHH...
WHOOOOSHHH
THAT'S GOTTA BE THE LOWEST ORBIT ON RECORD.

An iron plug was inserted into the top of the bowling ball to allow for magnetic release of the ball.

Deviations of projector speed from the nominal 18 frames/sec are usually no more than ±1 frame/sec; but this is greater than 5%.

A formal proof of this statement is as follows: If the acceleration is constant during a time interval of duration T, the speed at the midtime is $v_m = v_1 + a(T/2)$. But the average speed is $\bar{v} = (v_1 + v_2)/2 = [v_1 + (v_1 + aT)]/2 = v_1 + a(T/2)$. Hence $v_m = \bar{v}$. The simplifying assumption that average speed equals instantaneous speed at the midtime is valid in uniformly accelerated motion for any size of time interval. The statement is true for an arbitrary motion only as the time interval approaches zero.

Film Loop 1 Acceleration Due To Gravity — Method I

A bowling ball in free fall was filmed in slow motion. The film was exposed at 3900 frames/sec, and is projected at about 18 frames/sec. The slow-motion factor is therefore 3900/18, or about 217. Your projector may not run at exactly 18 frames/sec, so for best results you should calibrate it by timing the length of the entire loop, which contains 3273 frames between punch marks. To find the acceleration of the falling body we need the instantaneous speed at two times; then we can use the definition

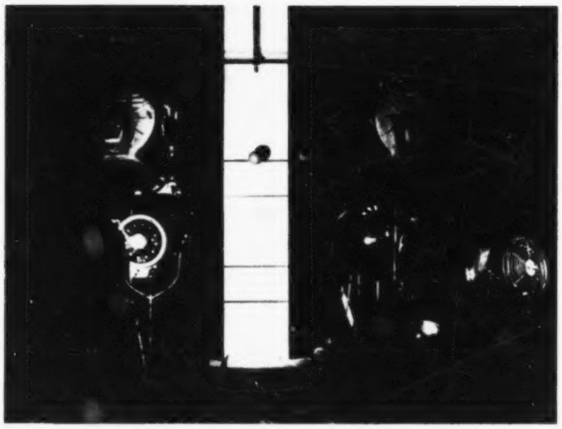
$$\text{acceleration} = \frac{\text{change in speed}}{\text{time interval}} .$$

We cannot measure instantaneous speed directly, but we can get around this by measuring the average speed during an interval. Suppose the speed increases steadily, as it does for a freely falling body. During the first half of any interval the speed is less than average, and during the second half of the interval the speed is greater than average. Therefore, for uniformly accelerated motion the average speed $\bar{v}$ equals the instantaneous speed at the midtime of the interval. We use this fact to find the values of instantaneous speed at the midtimes of each of two intervals. Then we can calculate the acceleration from

$$a = \frac{\bar{v}_2 - \bar{v}_1}{t_2 - t_1}$$

where $\bar{v}_1$ and $\bar{v}_2$ are the average speeds during the first and second intervals, and t_1 and t_2 are the midtimes of these intervals.

Two intervals 0.50 meter in length are indicated in the film. The ball falls 1 meter before the start of the first marked interval, so it has some initial speed as it crosses the first line. Use an ordinary watch (with a sweep second hand) to time the ball's



motion and record the times (by the watch) of crossing each of the four lines. From these times you can find the time (in apparent seconds) between the midtimes of the two intervals, and the times for the ball to move through each ½-meter interval. Repeat the measurements at least once, and find the average times. Now you can use the slow-motion factor to convert these times to real seconds, and then calculate the two values of $\bar{v}$. Finally, calculate the acceleration.

The film was made at Montréal, Canada. The value of the acceleration due to gravity there is, to 3 significant figures, 9.81 m/sec^2. When rounded off to ±1%, this becomes 9.8 m/sec^2. Try to decide from the internal consistency of your data (the repeatability of your time measurements) how precisely you should write your result.

An error of ±0.04 in the value of a would still give some "significance" to a final digit in a result such as 9.76 or 9.81 m/sec^2. This would require a student's measurements to be within half of one percent, which is very unlikely. A more reasonable expectation would be to obtain a to within perhaps ±0.1 m/sec^2 (i.e., to within 1%).

The Technicolor projectors are unlikely to have speeds in error by more than ±1 frame/sec; this is, however, more than 5%.

When Unit 3 has been studied, the student will see that this equation can also be derived from the law of conservation of energy. If the initial speed is v_i and the final speed is v_f, then

$$E_p + E_K = E_p + E_K$$

$$ma_g s + \tfrac{1}{2}mv_i^2 = 0 + \tfrac{1}{2}mv_f^2$$

$$v_f^2 - v_i^2 = 2a_g s$$

Film Loop 2 Acceleration Due To Gravity — Method II

A bowling ball in free fall was filmed in slow motion. The film was exposed at 3415 frames/sec, and is projected at about 18 frames/sec. For best results, you should calibrate your projector by timing the length of the entire film, which contains 3695 frames between punch marks.

If the ball starts from rest and acquires a speed v after falling through a distance d, the average speed is $\bar{v} = \frac{0 + v}{2} = \frac{1}{2}v$, and the time to fall this distance is given by $t = \frac{d}{v} = \frac{d}{\frac{1}{2}v} = \frac{2d}{v}$. The acceleration is given by

$$\text{acceleration} = \frac{\text{change of speed}}{\text{time interval}}$$

from which

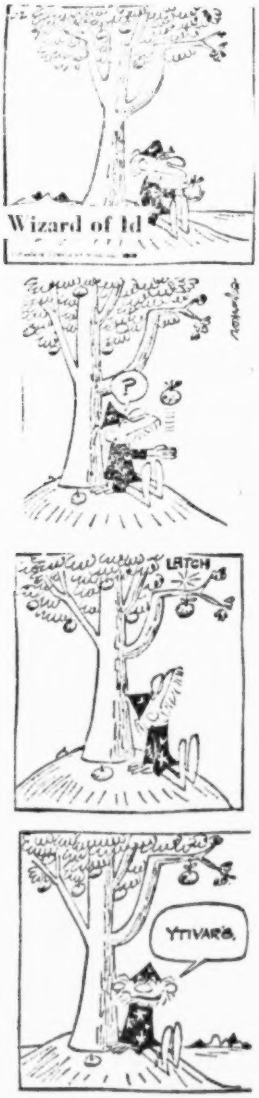
$$a = \frac{v}{2d/v} \text{ or } a = v^2/2d.$$

Thus to measure the acceleration we need to know the instantaneous speed v of the falling body at a known distance d below the starting point. All we can measure, of course, is an average speed over some small interval. In the film, small intervals of 20 cm are centered on positions 1 m, 2 m, 3 m and 4 m below the starting point. We make the approximation that the average speed is the instantaneous speed at the midpoint of the interval. Actually, the average speed is the instantaneous speed at the midtime, not the midpoint; but the error is negligible in our work because we are using such a short interval.

Determine the four average speeds by timing the ball's motion across the 20-cm intervals. Repeat each measurement several times to average out errors of measurement. Convert your measured times into real times, using the slow-motion factor. Compute the speeds, in m/sec, and then compute the value of

$v^2/2d$ for each value of d.

To analyze your results, make a table of calculated values of a, listing them in the order of increasing values of d. Is there any evidence for a systematic trend in the values? State the result of your experiment by giving an average value of the acceleration and an estimate of the possible error. The error estimate is a matter of judgment, and should be based on the consistency of your four measured values of the acceleration.



In theory, a systematic trend exists because of the approximation we made. The ball speeds up as it passes through any interval; at the midtime it is slightly above the midpoint of the interval. Hence each value of s should be decreased very slightly, and the effect is largest for small values of s where the speed changes by a larger fraction during the interval. In practice, the error is negligible and will not be observed. For the worst case, consider the motion of the ball from s = 0.90 m to s = 1.10 m. From $s = \frac{1}{2}a_g t^2$, the time to fall to 1.10 m is $t = 2s/a_g = 2(1.10)/9.80 =$ 0.47380 sec; the time to fall 0.90 m is $2(0.90)/9.80 = 0.42857$ sec. The midtime is thus at $t_m = 0.45118$ sec, and the displacement at the midtime is $s = \frac{1}{2}a_g t_m{}^2 = \frac{1}{2}(9.80)(0.45118)^2 =$ 0.9975 m. So we see that the value of s that corresponds to the measured $\overline{v}$ is 0.9975 m, only 0.0025 m (2.5 mm) above the midpoint of the interval. The error in s is 0.25%, and therefore the error in a_g is also only 0.25%. The percent error is even less for the measurements at s = 2 m, 3 m, and 4 m.

The student notes are somewhat more detailed than usual, because vector addition as such is not discussed very fully in the text.

The physical conditions were not ideal; you can easily verify that the river's speed is not as uniform as would be desirable. There is about a 25% variation between the speeds at the extreme left and the extreme right of the picture. The average speed at the middle of the frame should be used. Careful observation also shows that the direction of the river flow is a few degrees off from being perpendicular to the line connecting the markers; this is not mentioned in the student notes but is implicit in the illustrative vector diagrams of Fig. 4 and Fig. 5.

Film Loop 3: Vector Addition I – Velocity of a Boat

The head-to-tail method of adding vectors is illustrated in Fig. 1. Since velocity is a vector quantity (it has both magnitude and direction) we can study vector addition by using velocity vectors.

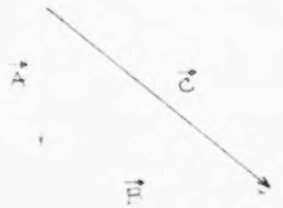


Fig. 1. $\vec{A} + \vec{B} = \vec{C}$

An easy way of keeping track of the order in which vectors are to be added is by using subscripts:

$\vec{v}_{BE}$ velocity of boat relative to earth

$\vec{v}_{BW}$ velocity of boat relative to water

$\vec{v}_{WE}$ velocity of water relative to earth.

Then

$$\vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE}.$$

In the film a motorboat is photographed from a bridge above a river. The operator tried to keep a steady speed relative to the water by keeping the throttle at a fixed setting. The boat heads upstream, then downstream, then directly across stream, and finally it heads at an angle somewhat upstream so as to move straight across. For each heading of the boat, a vector diagram can be drawn by laying off the velocities to scale, using a ruler and a protractor.

First project the film on a piece of graph paper and mark out lines along which the boat's image moves. Then measure speeds by timing the motion of the boat as it moves some predetermined number of squares. Repeat each measurement three

times, and use the average times to calculate the speeds. Express all speeds in the same unit, such as "squares per second," or "square per cm" where cm refers to measured separations between marks on the moving paper of a dragstrip. There is no need to convert the speeds to meters per second. Why is it a good idea to use a fairly large distance between the timing marks on the graph paper? A suggested procedure is to record data for each of the five scenes, and draw the vector diagrams after taking all the data.

1. Two blocks of wood are dropped overboard. Time the blocks; find the speed of the river. This is $\vec{v}_{WE}$, to be used in the vector additions to follow.

2. The boat heads upstream. Measure $\vec{v}_{BE}$, then find $\vec{v}_{BW}$ using a vector diagram (Fig. 2).

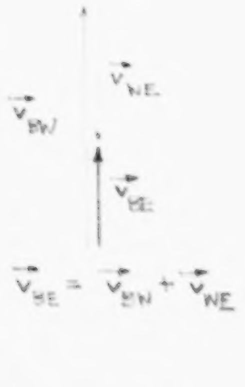


Fig. 2.

As with all measurements of speed using film loops, it is essential to repeat each time measurement several times to average out errors (or to allow one to discard an obviously wrong value).

Scene 1 is really superfluous, since the river speed can be measured well enough in the other scenes using patches of foam floating on the surface. However, the pieces of wood may be easier to see.

3. The boat heads downstream. Measure $\vec{v}_{BE}$, then find $\vec{v}_{BW}$ using a vector diagram (Fig. 3).

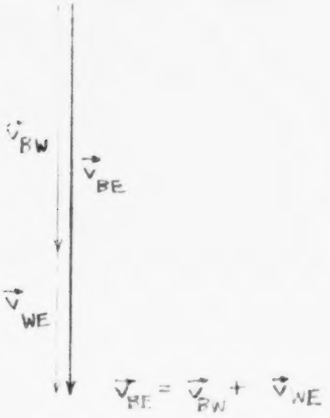

$\vec{v}_{BW}$
$\vec{v}_{BE}$
$\vec{v}_{WE}$
$\vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE}$

Fig. 3.

4. The boat heads across stream and drifts downward. Measure the speed of the boat and the direction of its path; these give you the magnitude and direction necessary to specify the vector $\vec{v}_{BE}$. Also measure the direction of $\vec{v}_{BW}$—this is the heading of the boat, the direction in which it points. A good way to record data is to refer everything to a set of axes with the 0° - 180° axis passing through the round markers anchored in the river. In Fig. 4 the numbers are deliberately falsified; record your own measurements in a similar diagram.

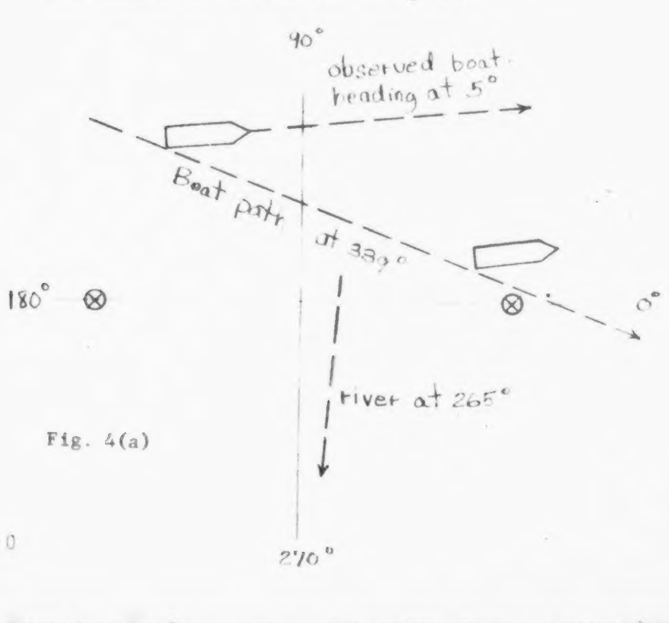

90°
observed boat heading at 5°
Boat path at 339°
180°
0°
river at 265°
270°

Fig. 4(a)

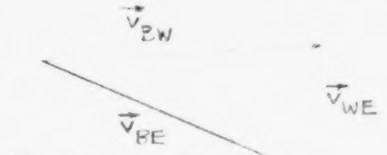
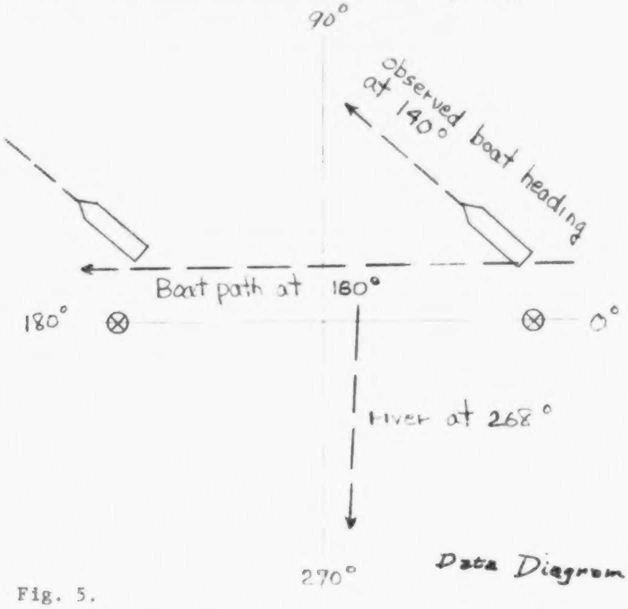


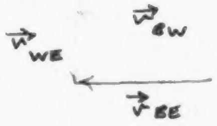
Fig. 4(b) $\vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE}$

5. The boat heads upstream at an angle, but moves across stream. Typical data might be similar to those in Fig. 5.



Data Diagram

Fig. 5.



$\vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE}$

Vector diagram

Checking your work

a) In parts 2, 3, 4 and 5 you have found four values of the magnitude of $\vec{v}_{BW}$. How well do these agree with each other? Can you suggest reasons for any discrepancies?

b) In part 4, you have found (by graphical construction) a direction for $\vec{v}_{BW}$, the calculated heading of the boat. How well does this angle agree with the observed boat heading?

c) In part 5, you have found a direction for $\vec{v}_{BW}$. How well does this angle agree with the observed boat heading?

As an indication of the consistency of results obtainable with this loop, we give some typical results of measurements by the techniques described. In the four scenes (using foam as reference points) the water speed was 2.0, 2.0, 2.1, and 2.0 units. On the same scale, the values of $\vec{v}_{BW}$ were 4.0, 3.9, 4.3 and 4.5 units. The agreement between calculated and observed boat heading was ±1° for scene 4, and ±5° for scene 5. Probable reasons for variations in $\vec{v}_{BW}$ were the inability of the operator to maintain exactly constant motor power, and some errors in steering.

This is a qualitative demonstration loop for repeated classroom use by the teacher. The concepts used are: (a) relative velocity and Galilean relativity (Unit 1); (b) principles of conservation of momentum and conservation of energy in elastic collisions (Unit 3). It is suggested that the teacher stop the projector near the start of the loop when a message on the screen asks "How did these events differ?" Encourage the students to describe the events they have just seen, without attempting to speculate on the ways in which the events were photographed. Then project the rest of the loop and initiate a discussion of relative motion and frames of reference. Come back to the loop when the conservation laws are studied in Unit 3.

In a technical sense, the word "event" implies knowledge of both places and times. A student walks from his home to school between 8:00 and 8:20 on Monday, and again between 8:00 and 8:20 on Tuesday. These are two different events. They are similar events, one being a repetition of the other. In the loop, three events not only occur during different time intervals, but also appear to be physically different. The student should be encouraged to describe what he sees--and the events do seem to require different descriptions.

The principle of Galilean Relativity is discussed on pp. 98-100 of the text of Unit 1: "Any mechanical experiment will yield the same results when performed in a frame of reference moving with uniform velocity as in a stationary frame of reference." In other words, the form of any law of mechanics is independent of the uniform motion of the frame of reference of the observer. Einstein broadened the principle to include all laws of physics, not just the laws of mechanics. Thus the Einstein relativity includes the laws of electromagnetism, which describe the propagation of light, as well as the mechanical laws of conservation of momentum and conservation of energy which are sufficient for our study of colliding carts.

Film Loop 4 A Matter Of Relative Motion

In this film loop mechanical experiments are performed in which two simple carts of equal mass collide. In the film, three sequences labeled Event A, Event B and Event C are photographed by a camera on a cart which is on a second ramp parallel to the one on which the colliding carts move. The camera is our frame of reference; this frame of reference may or may not be in motion. As photographed, the three events appear to be quite different. Describe these events, in words, as they appear to you (and to the camera). The question arises: could these three events really be similar events, viewed from different frames of reference?

Although to the observer Events A, B and C are visibly different, in each the carts interact similarly, and so could be the same event received from different reference frames. They are, in fact, closely similar events photographed from different frames of reference. The question of which cart is really in motion is resolved by sequences at the end of the film, in which an experimenter stands near the ramp to provide a reference object. But is this fixed frame of reference any more fundamental than one of the moving frames of reference? Fixed relative to what?

The algebra of this prediction involved simultaneous equations; the unknowns are V_1 and V_2 (Fig. 1).

Conservation of momentum:

$$0 + mv = mV_1 + mV_2 \qquad (1)$$

Conservation of energy:

$$0 + \tfrac{1}{2}mv^2 = \tfrac{1}{2}mV_1{}^2 + \tfrac{1}{2}mV_2{}^2 \qquad (2)$$

The masses m cancel out. Solve (1) for V_1, getting

$$V_1 = v - V_2 \qquad (3)$$

Substitute into (2): $v^2 = (v - V_2)^2 + V_2{}^2$, which simplifies to $(V_2 - v)V_2 = 0$. Hence $V_2 = v$ and, from (3), $V_1 = 0$.

If two equal carts approach each other with equal speeds, (Fig. 2), we have

$$mv - mv = mV_1 + mV_2 \qquad (4)$$

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 = \tfrac{1}{2}mV_1{}^2 + \tfrac{1}{2}mV_2{}^2 \qquad (5)$$

Solve (4) for V_2 and substitute into (5), getting $V_1{}^2 = v^2$, i.e., $V_1 = \pm v$. If $V_1 = +v$ there is no collision, so we use the - sign. Then $V_1 = -v$, and from (4) we get $V_2 = -V_1 = +v$.

The student may be able to get some clues as to what is "really" happening by closely observing the rolling wheels of the carts, and will perhaps see the apparent motion of wrinkles in the pale blue background cloth. The teacher should ask the student what he means by "really" happening—it should become clear that he is subconsciously identifying one frame of reference (the earth) as the "real" frame. The point of the film is that the other two (moving) frames are just as "real", and events taking place in them are described by the same laws of mechanics.

When using this loop in Unit 3, a discussion of the "laws" can be given. We are dealing with a collision. This is governed by the "law of mechanics" we call the law of conservation of momentum. In each event, momentum is conserved:

event	before collision	after collision	total momentum
A	$0 + (-mv)$	$(-mv) + 0$	$-mv$
B	$(+mv) + 0$	$0 + (+mv)$	$+mv$
C	$(+\tfrac{1}{2}mv) + (-\tfrac{1}{2}mv)$	$(-\tfrac{1}{2}mv) + (+\tfrac{1}{2}mv)$	0

The total momentum of the pair of carts has a different magnitude and direction in each of the three frames of reference, but the "law" or "principle" of conservation of momentum is equally valid in each frame of reference.

The collisions of the carts are, moreover, of the type called "perfectly elastic." In this type of collision, the "law of mechanics" which also applies is: kinetic energy is conserved. Again we find that this "law" is equally valid in the three frames of reference. The total kinetic energy in each case is indeed the same before and after collision. But its value needn't be the same in each frame of reference. It is $\tfrac{1}{2}mv^2$ in Event A, $\tfrac{1}{2}mv^2$ in Event B, and $\tfrac{1}{4}mv^2$ in Event C.

General reference: Text, Sec. 3.6. Sagredo's analysis of Event A is found in Dialogue on the Great World Systems.

Film Loop 5 Galilean Relativity I — Ball Dropped from Mast Of Ship

This film is a realization of an experiment described by Sagredo in Galileo's Two New Sciences:

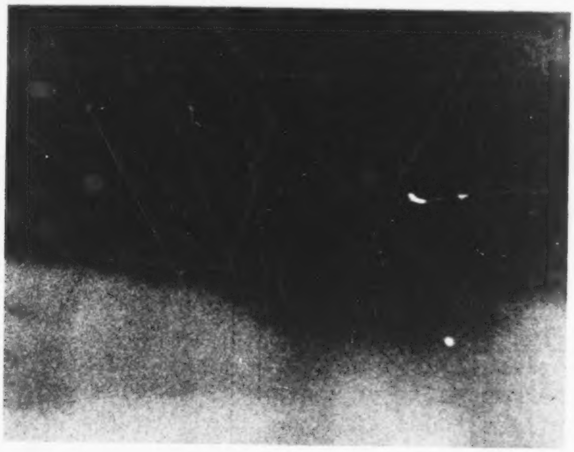
> If it be true that the impetus with which the ship moves remains indelibly impressed in the stone after it is let fall from the mast; and if it be further true that this motion brings no impediment or retardment to the motion directly downwards natural to the stone, then there ought to ensue an effect of a very wondrous nature. Suppose a ship stands still, and the time of the falling of a stone from the mast's round top to the deck is two beats of the pulse. Then afterwards have the ship under sail and let the same stone depart from the same place. According to what has been premised, it shall take up the time of two pulses in its fall, in which time the ship will have gone, say, twenty yards. The true motion of the stone then will be a transverse line [i.e., a curved line in the vertical plane], considerably longer than the first straight and perpendicular line, the height of the mast, and yet nevertheless the stone will have passed it in the same time. Increase the ship's velocity as much as you will, the falling stone shall describe its transverse lines still longer and longer and yet shall pass them all in those selfsame two pulses.

Because the stone "remembers" its initial horizontal velocity which is the ship's motion, it will strike the deck of the ship at the base of the mast.

In the film, a ball is dropped three times. Event 1: As in Galileo's discussion, the ball continues to move forward with the boat's velocity, and it falls vertically relative to the mast. Event 2: The ball is tipped off a stationary support as the boat goes by; now it has no forward velocity and it falls vertically relative to the ground. Event 3: In the final trial, a student moving with the boat picks up the ball and holds it a few seconds before releasing it. Galilean relativity is illustrated by these three events. The ship and earth are the two frames of reference which are in relative motion. The same laws for the description of projectile motion are valid in either system. Thus each of the three events can be described as viewed in either of two frames of reference; but only one set of laws for projectile motion is needed for all six descriptions. For example, Event 1 in the boat frame is described thus: "A ball, initially at rest, is released. It accelerates downward at 9.8 m/sec^2 and strikes a point directly beneath the starting point." Event 1 in the earth frame is described differently: "A ball is projected horizontally toward the left; its path is a parabola and it strikes a point below and to the left of the starting point."

To test your understanding of Galilean relativity, you should also describe, in words, the following: Event 2 in boat frame; Event 2 in earth frame; Event 3 in boat frame; Event 3 in earth frame.

Event 2, boat frame: "A ball is moving horizontally toward the right, and at the moment it is opposite an observer it is allowed to move freely as a projectile. The path of the ball is a parabola, and the ball moves downward and to the right."
Event 2, earth frame: "A ball is allowed to fall vertically from rest, and it strikes a point directly below the point of release."
Event 3, boat frame: "A ball, initially moving toward the right, is stopped by the muscular action of a student who is stationary on the mast. The student lets go the ball, which then falls vertically downward."
Event 3, earth frame: "A stationary ball is given a forward velocity to the left by the muscular action of a student. The ball is then released, and its motion, that of a projectile, takes the ball to a point downward and to the left of the starting point."



Film Loop 6: Galilean Relativity II – Object Dropped From Aircraft

A Cessna 150 aircraft 23 ft long is moving almost horizontally at about 100 ft/sec at an altitude of about 200 ft. A lighted flare is dropped from the cockpit of the aircraft; the action is filmed from the ground in slow motion. Quantitative measurements can be made at any of the "freeze frames" when the motion on the screen is stopped for a few seconds. Scene 1 shows part of the flare's motion; Scene 2, shot from a greater distance, shows the flare dropping into a lake.

Consider the motion relative to two frames of reference. In the earth frame, the motion is that of a projectile whose original velocity is the plane's velocity. The motion is a parabola in this frame of reference. Relative to the plane, the motion is that of a freely falling body which starts from rest. In this frame of reference (the plane frame) the motion is vertically downward. Scene 3 shows this vertical motion viewed head-on.

The plane is flying at uniform speed in a straight line, but its path is not necessarily a horizontal line. The flare retains the plane's original velocity both in magnitude and direction, and in addition it falls freely under the action of gravity. We might expect the displacement below the plane to be given by $d = \frac{1}{2} at^2$, but there is a slight problem. We cannot be sure that the first freeze frame occurs at the very instant the flare is dropped overboard. However, there is a way of getting around this difficulty. Suppose a time B has elapsed between the release of the flare and the first freeze frame. This time must be added to each of the freeze frame times, and so we would have

$$d = \frac{1}{2} a(t + B)^2. \qquad (1)$$

To see if the flare follows an equation such as this, take the square root of each side:

$$\sqrt{d} = (\text{constant})(t + B). \qquad (2)$$

Now if we plot $\sqrt{d}$ against t, we expect a straight line. Moreover, if $B = 0$, this straight line will also pass through the origin.

Suggested Measurements

a) Path relative to ground. Project Scene 1 on a piece of paper. At each freeze frame, mark the position of the flare and that of the aircraft cockpit. Measure the displacement d (in arbitrary units) of the flare below the plane. The times can be considered to be integers, $t = 0, 1, 2, \ldots$ designating the various freeze frames. Plot a graph of $\sqrt{d}$ versus t. Discuss your result: does the graph deviate from a straight line? What would be the effect of air resistance on the motion, and how would this show up in your graph? Does the graph pass through the origin?

b) Analyze Scene 2 in the same way. Does this graph pass through the origin? Are the effects of air resistance noticeable in the horizontal motion? Does air resistance seem to affect the vertical motion appreciably?

The slow motion factors are 2.4 for scene 1 and 5.1 for scene 2 (rounded off to 5 in the film).

This illustrates a common procedure in science, in which data are manipulated before graphs are plotted. In this case, a graph of s versus t^2 would not give nearly as much insight as does a graph of $\sqrt{s}$ versus t.

If air resistance were negligible, the horizontal displacement graph would be a straight line passing through the origin, $x = v_0t$, assuming that the correction B is 0. The vertical displacement graph would be a parabola, $s = \frac{1}{2}a\,t^2$. The horizontal motion shows a decided "droop" due to air resistance, but the vertical motion is surprisingly good, as shown by the fact that the graph of s versus t remains almost straight for the whole motion even in scene 2 which is the longer of the two. To explain this, note that air resistance depends on speed. The flare is moving at large horizontal speed from the instant it is released, but it has large vertical speed only for the latter part of the trajectory.

c) Superposition of motions. Use another piece of graph paper with time (in intervals) plotted horizontally and displacements (in squares) plotted vertically. Using the same set of axes, make two graphs for the two independent simultaneous motions in Scene 2. Use one color of pencil for the horizontal displacement as a function of time, and another color for vertical displacement as a function of time.

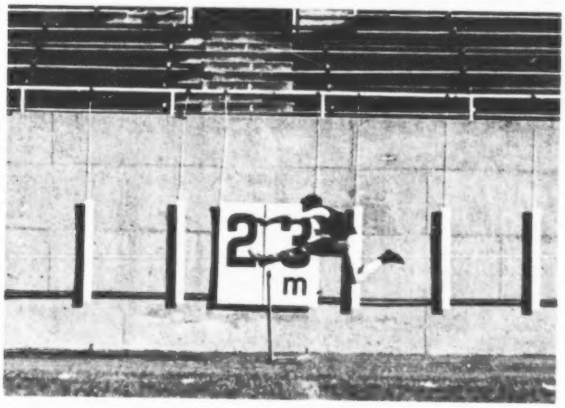
d) Acceleration due to gravity (optional). The "constant" in Eq. (2) is $\sqrt{\frac{1}{2}a}$; this is the slope of the straight line graph obtained in part a). The square of the slope gives $\frac{1}{2}a$, so the acceleration is twice the square of the slope. In this way you can obtain the acceleration in squares/(interval)^2. To convert your acceleration into ft/sec^2 or m/sec^2, you can estimate the size of a "square" from the fact that the length of the plane is 23 ft (7 m). The time interval in seconds between freeze frames can be found from the slow-motion factor.

STRONG WIND TODAY.

According to Galilean relativity, the flare retains the velocity (if any) of the ski-doo. Relative to the earth, each event is the usual parabola of projectile motion. Relative to the ski-doo, Event 2 is a vertical motion; the flare falls down again into the ski-doo (compare Figs. 3.24 and 3.25). In Event 3, the ski-doo comes to a halt after the flare is fired, so the flare lands ahead of the ski-doo. In Event 4, the ski-doo accelerates in the forward direction after the flare is fired, so the flare lands behind the ski-doo.

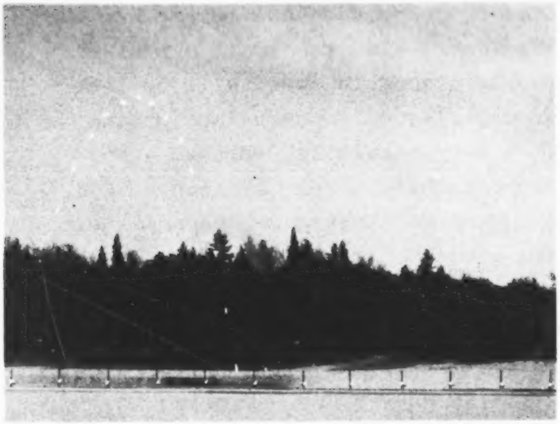
Film Loop 7 Galilean Relativity III – Projectile Fired Vertically

A rocket gun is mounted in gymbal bearings which are free to turn in any direction. When the gun is hauled along the snow-covered surface of a frozen lake by a tractor-like vehicle called a "ski-doo," the gymbals allow the gun to remain pointing vertically upward in spite of some roughness of path. Equally-spaced lamps along the path allow one to judge whether the ski-doo has constant velocity or is accelerating in either a positive or a negative sense. A preliminary run shows the entire scene; the setting, at dusk, is in the Laurentian mountains in the Province of Quebec.



Four events are photographed. In each case the flare is fired vertically upward relative to the ski-doo. Event 1: the ski-doo is stationary relative to the earth. Event 2: the ski-doo moves at uniform velocity relative to the earth. Describe the motion of the flare relative to the earth; describe the motion of the flare relative to the ski-doo. Events 3 and 4: the ski-doo's speed changes after the shot is fired; describe the flare's motion in each case relative to the earth, and also relative to

How do the events shown in this film illustrate the principle of Galilean relativity?



Film Loop 8 Analysis Of A Hurdle Race Part 1

Film Loop 9 Analysis Of A Hurdle Race Part 2

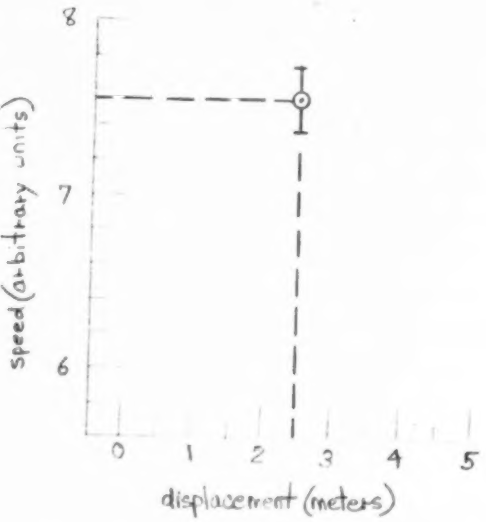
Some preliminary scenes show a hurdle race of regulation length. The hurdles, each 1 meter high, were spaced 9 meters apart. Judging from the number of hurdles knocked over, the competitors were of something less than Olympic caliber! Next, a single runner is shown in medium slow motion during a 50-meter run. Finally, three parts of the run are shown in extreme slow motion for purposes of analysis. The solo runner was Frank White, a 75-kg medical student at McGill University. His time for 50 meters was 8.1 sec.

To study the runner's motion, measure the average speed for each of the 1-meter intervals in the three slow-motion scenes.

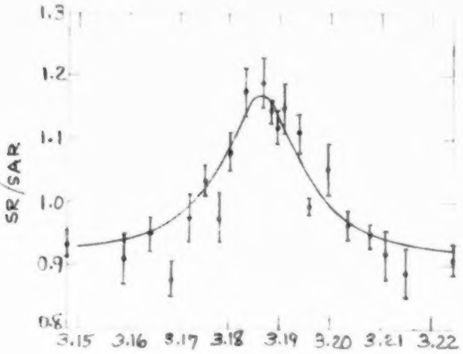
We are interested in how the speed varies during the run; therefore, you need only calculate relative values of speed, in whatever units you find convenient. The slow-motion factor of 80 is given for orientation and need not be used for this part of the analysis. We assume you are using a "dragstrip" to measure time intervals. Then the time is measured in "cm" (distance between marks on the moving piece of paper).

These two loops are intended to give the student a feeling for the power of careful measurement to reveal "structure" in a motion which seems to a casual observer to be nearly uniform; and to encourage him to speculate on the causes of the changes in motion which he observes.

Whatever method you choose for measuring the time intervals, the small but significant variations in speed will be lost in the experimental uncertainty unless you work very carefully. Repeat each measurement three times, reading to the nearest half-millimeter on the dragstrip. Use the average times to compute the speeds. For example, if the time for the interval 2 m to 3 m is measured as 13.7 cm, 12.9 cm and 13.25 cm, the average time is 13.28 units and the speed is (100 cm)/(13.28 cm) = 7.55 units. This is plotted at the mid-time of the interval. The error bar is based on the spread of the observed times. The worst values are about 0.4 unit away from the average; this error of 0.4 unit out of 13.28 units is about 3 out of 100, or 3%. The speed is subject to the same percent error as is the time, and 3% of 7.55 is 0.2. We plot the point as 7.55 ± 0.2 units (Fig. 1).



You are in the best tradition of experimental science when you pay close attention to limits of error. Only then can you decide whether some of the seemingly small changes in the runner's speed are really significant. Your graph is likely to be at least as good as those research workers draw conclusions from. Thus in Fig. 2 a research team has plotted a



Stroboscopic Coincidence Positive Muons in Bromoform
$f_{osc} = 48.63$ Mc/Sec

quantity designated as "SR/SAR" which depends on a ratio f_{osc}/f_p (to appreciate the point we are making, it is not necessary that you know anything at all about the experiment as such). The peak at 3.19 in Fig. 2 is significant, even though some of the plotted points have error bars representing limits of error as large as 5%.

Scene 1 shows the runner's motion from 0 m to 6 m. Mark the dragstrip paper when the seat of the runner's shorts just clears the far (left-hand) edge of the vertical red meter-marks. (What are some other possible reference points on the runner that could be used? Are all reference points equally useful? Would the near edge of the meter-marker allow as precise a measurement as does the far edge?) Use a ruler or meter stick to measure each of the six dragstrip intervals corresponding to 0-1, 1-2, 2-3, 3-4, 4-5 and 5-6 m. Mark the dragstrip in this way three times and average your

The graph from Physical Review Letters is shown to reassure the student who may be unhappy with a graph whose plotted points show considerable scatter. It is not necessary to go into the details of the experiment summarized in Fig. 2, except to point out that this graph is a real-life example of published work by a team of five highly capable physicists.

See the remarks earlier in the Student Activity Book on using the dragstrip for measuring time intervals. A student may suggest that a systematic error has occurred because of perspective. This was taken into account when the meter-marks were located on the wall behind the runner. The camera was positioned opposite the middle of the 6-meter interval, and the markers were "spread" somewhat so that the runner's positions are correctly indicated when his image coincides with those of the markers.

The front of the runner's shorts would be unreliable because he straightens up after the start. Using the forward edge of the vertical meter-marker is helpful because it gives the observer time to anticipate the moment of tangency.

Careful measurements give a speed graph for the first 6 m similar to Fig. 3. The "droop" at 5 m can be related to Newton's second law, as suggested in the student notes. A good student should be encouraged to study the film closely, perhaps plotting regions of "push" as in the graph shown here for your information. It is evident that the runner is practically coasting as his hip moves from "5" to "6". The initial acceleration can be found from $s = \frac{1}{2}at^2$; for $s = 1.4$ m and $t = 38/80$ sec, this gives $a = 12.5$ m/sec^2. The average acceleration during the first 1.4 m is about 1.3 times the acceleration due to gravity. Thus the ground pushes on the runner, and the runner on the ground, with a force about 1.3 times his own weight (about 100 kg, or 220 lb). If this seems unreasonable, note that the world record for weightlifting, using arm muscles only, is about 180 kg (395 lb). The acceleration is even greater at the very start, during the first 0.1 m of motion. A frame-by-frame analysis of the film gave an acceleration of about 40 m/sec^2 during this interval, corresponding to a momentary force of more than 600 lb.

results for each interval. It might improve your accuracy if you form a grand average by combining your averages with those of your lab partner (assuming that he used the same dragstrip). Calculate the average speed for each interval, and plot a graph of speed versus displacement. Estimate the limit of error for a typical point and add similar error bars to each plotted point. Draw a smooth graph through the points. Discuss any interesting features of the graph.

One might assume that any push of the runner's legs comes only between the time when a foot is directly beneath the runner's hip and the time when that foot lifts off the ground. Study the film carefully; is there any relationship between your graph of speed and the way the runner's feet push on the track?

The initial acceleration of the runner can be estimated if you find the time for him to travel from the starting point to the 1-meter mark. For this you must use a clock or a watch with sweep second hand, and you must use the slow motion factor to convert apparent time to real time. Calculate the average acceleration, in m/sec^2, during this initial interval of about 1.4 m. How does this forward acceleration compare with the magnitude of the acceleration of a freely falling body? How much force was required to give the runner this acceleration? What was the origin of this force?

Scene 2 and Scene 3 are on a second loop which is a continuation of the first loop. In Scene 2, the hurdler moves from 20 m to 26 m, clearing a hurdle at 23 m. In Scene 3, the runner moves from 40 m to 50 m, clearing a hurdle at 41 m and sprinting toward the finish line at 50 m. Plot graphs of these motions, and discuss any interesting features.

In Scene 2, the speed increases just after the runner clears the hurdle, while he is still in the air. This paradoxical result is explained by the fact that the runner straightens up after clearing the hurdle; if his center of mass maintains constant speed then his hip must come forward as his knee and torso come back relative to the center of mass. This unexpected result is clearly shown in a typical student measurement and should provoke a valuable discussion. A similar effect explains the continued rise of speed in the 2 m to 3 m interval of Scene 1; the runner is still straightening his torso following the start of the run. In Scene 3, the measurements are less precise than in Scenes 1 and 2 because the magnification is less. There is a modest rise in speed as the runner approaches the finish lines at 50 m.

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The reference materials cited below are far from complete. We hope that these listings will be helpful to you and those of your students who wish to go beyond the course materials.

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- a) Newton, pp. 30-46. A brief biographical sketch followed by quotations from the 1803 (Motte's) translation of the Principia.
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- b) Volume II, "The System of the World." projectiles and satellites, p. 551; mass relationship in gravitation, p. 414; inverse-square law of gravitation, p. 406.

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- a) Development of Gravity Pendulums in the Nineteenth Century, U. S. National Museum Bulletin 240, Paper 44 ($.75).
- b) The International System of Units, N. A. S. A. Publication ($.20).

E. Resource Letters

Teachers should know of and obtain copies of resource letters reprinted from the American Journal of Physics. Resource letters may be obtained free by sending a self-addressed, stamped envelope to the American Institute of Physics, 335 East 45th St., New York, N. Y. 10017. Name the resource letter requested.

Resource Letter SL-1, Science and Literature, prepared by Marjorie Nicholson, has many references that are appropriate to Unit 1. A useful list of article-length readings is in the resource letter: Collateral Readings in Physics Courses, prepared by Alfred Bork and Arnold Arons.

Suggested Answers to Unit 1 Tests

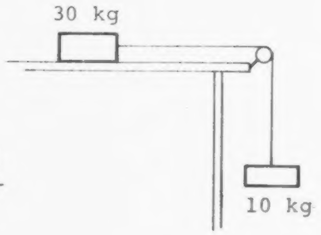
Test A

ITEM	ANSWER	SECTION OF UNIT	PROPORTION OF TEST SAMPLE ANSWERING ITEM CORRECTLY
1	A	3.4	0.472
2	C	3.2	0.674
3	B	4.10	0.416
4	E	4.11	0.483
5	D	1.6	0.944
6	D	1.6	0.876
7	E	1.0	0.404
8	A	1.5	0.640
9	E	1.5	0.933
10	B	3.5	0.607
11	C	1.0	not available
12	A	1.0	0.640
13	C	4.11	0.438
14	D	2.8	0.551
15	B	4.11	0.730

PROBLEM-AND-ESSAY QUESTIONS

Group One

1. Section of Unit: 3.10



$$F = ma$$

$$a = \frac{F}{m}$$

$$= \frac{(10\ \text{kg})(10\ \frac{\text{m}}{\text{sec}^2})}{10\ \text{kg} + 30\ \text{kg}}$$

$$= \frac{100}{40}\ \frac{\text{m}}{\text{sec}^2}$$

$$= 2.5\ \frac{\text{m}}{\text{sec}^2}$$

The 10 kg mass accelerates downwards at 2.5 m/sec^2.

The 30 kg mass accelerates to the right at 2.5 m/sec^2.

2. Section of Unit: 1.7

a) The average speed of the car is the total distance traveled from the stoplight divided by the total time traveled. The instantaneous speed is an "average" speed calculated over an infinitesimally small time interval.

b) $\bar{v} = \frac{\Delta d}{\Delta t}$

$v_i = \lim\limits_{\Delta t \to 0} \frac{\Delta d}{\Delta t}$

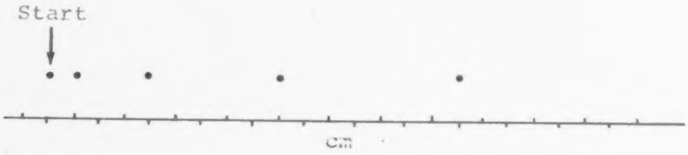
$\bar{v}$ = average speed

v_i = instantaneous speed

Δd = distance traveled

during Δt = the time interval

3. Section of Unit: 2.3



$$d = \frac{at^2}{2}$$

$$a = \frac{2d}{t^2}$$

$$= \frac{(2)(8\ \text{cm})}{(.4\ \text{sec})^2}$$

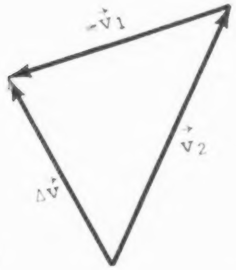
$$= \frac{16 \text{ cm}}{.16 \text{ sec}^2}$$

$$= 100 \frac{\text{cm}}{\text{sec}^2}$$

4. Sections of Unit 3.2, 4.6, programmed material

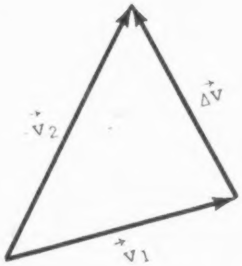
Find $\Delta\vec{v}$, where $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$.

I)


$-\vec{v}_1$
$\vec{v}_2$
$\Delta\vec{v}$

(by adding $-\vec{v}_1$ to $\vec{v}_2$)

II)


$\vec{v}_2$
$\Delta\vec{v}$
$\vec{v}_1$

(from the head of $\vec{v}_1$ to the head of $\vec{v}_2$)

PROBLEM-AND-ESSAY QUESTIONS

Group Two

5. Section of Unit 4.3

A satisfactory answer may involve:

a) a discussion in depth of one or two relevant points, or
b) a comprehensive overview of many relevant points.

A suitable answer may involve any of the following points. This list does not include all possibilities.

a) Precise description - allowing comparison, measurement, graphical representation, transmission of knowledge.
b) Functional relationships - allowing prediction, specific tests, deduction, inference, combination of independent equations, mathematical operations

6. Section of Unit 1.6

During the first 10 minutes the man walks 0.5 miles at a constant speed of $v = d/t = 0.5$ miles/10 min = 0.05 miles/min.

During the next 10 minutes he does not move.

In the following 5 minutes he walks 0.25 miles at a constant speed of 0.25 miles/5 min = 0.05 miles/min.

Beyond the 25th minute the man is at rest.

Suggested Answers to Unit 1 Tests

Test B

ITEM	ANSWER	SECTION OF UNIT	PROPORTION OF TEST SAMPLE ANSWERING ITEM CORRECTLY
1	B	1.5	0.889
2	B	1.5	0.750
3	D	3.5	0.792
4	A	2.0	not available
5	E	4.13	0.431
6	C	4.11	not available
7	D	1.7	0.500
8	B	4.6	0.458
9	C	2.7	0.736
10	D	3.5	0.944
11	E	3.12	0.375
12	A	general	0.944
13	B	3.4	0.250
14	B	3.11	0.819
15	E	3.4	0.653

PROBLEM-AND-ESSAY QUESTIONS

Group One

1. Section of Unit 2.9

The answer must depend on your assessment of the accuracy of Galileo's instruments.

The extreme values in this column of data are .00192 and .00182.

They differ by less than 5%. Assuming the crudeness of Galileo's equipment accounts for this variation, we can conclude that d/t^2 is constant.

2. Section of Unit 4.4

a) As seen by the pilot, the flare drops straight down.
b) As seen by an observer on the ground, the flare is a projectile whose trajectory is a parabola.

3. Section of Unit: 3.8

Weight is a measure of the gravitational force on an object. It is defined by Newton's 2nd Law: $w = ma_g$. Its unit is the newton

$$(= 1\ \text{kg} - \text{m/sec}^2).$$

Mass is a measure of the resistance of an object to changes in motion, a measure of inertia. It is also defined by Newton's 2nd Law: $m = w/a_g$. Its unit is the kilogram.

4. Section of Unit: 3.8

$$F = ma$$

$$a = \frac{F}{m}$$

$$= \frac{a_g}{m}$$

$$a = a_g \quad \text{or about} \quad \frac{10\ \text{m}}{\text{sec}^2}$$

PROBLEM-AND-ESSAY QUESTIONS

Group Two

5. Sections of Units 2.7 - 2.10

a) Joe's point of view is stated mathematically as $v = at$. i.e. If an object is accelerating uniformly, the longer it goes, the faster it goes.

Louis' point of view is stated mathematically as $v^2 = 2ad$. i.e. If an object is accelerating uniformly, the farther it goes, the faster it goes.

b) Both points of view are correct. Louis' equation may be derived from Joe's.

$$v = at \qquad\qquad d = \tfrac{1}{2} at^2$$

square

$$v^2 = a^2 t^2 \qquad\qquad t^2 = \frac{2d}{a}$$

substitute $\frac{2d}{a}$ for t^2

$$v^2 = a^2 \frac{2d}{a}$$

$$v^2 = 2ad$$

6. Sections of Units 2.2 - 2.4

A satisfactory answer may involve the following points. These suggestions do not exhaust the list of possibilities.

a) Precise description - allowing comparison, measurement, graphical representation, transmission of knowledge, etc.

Functional relationships - allowing prediction, tests, deduction, inference, mathematical operations, etc.

b) Thought experiments

'Ideal' motion - frictionless surface. Real situations are often too complicated.

Newton's laws, Galileo, etc.

Suggested Answers to Unit 1 Tests

Test C

ITEM	ANSWER	SECTION OF UNIT	PROPORTION OF TEST SAMPLE ANSWERING ITEM CORRECTLY	ITEM	ANSWER	SECTION OF UNIT	PROPORTION OF TEST SAMPLE ANSWERING ITEM CORRECTLY
1	C	1.4	0.857	21	D	1.6	0.847
2	B	general	0.896	22	D	1.5	0.785
3	A	3.2,4.4,4.5	0.700	23	A	3.7	0.436
4	E	1.6	0.700	24	E	4.3	0.528
5	B	1.7	0.632	25	E	4.3	0.362
6	D	3.2	0.814	26	E	3.2	0.622
7	C	4.5	0.779	27	C	2.10	0.371
8	B	3.7	0.788	28	D	1.4	0.788
9	A	3.6	0.590	29	C	1.7	0.436
10	C	general	0.697	30	C	4.5	0.928
11	A	3.10	0.554	31	A	4.4	0.678
12	B	3.10	0.482	32	A	3.2	0.436
13	B	2.3	0.423	33	D	3.8	not available
14	B	general	0.469	34	C	4.3	0.629
15	E	1.7	0.821	35	A	2.1	0.326
16	A	1.4	0.749	36	C	3.2	0.394
17	D	1.7	0.736	37	B	2.8	0.557
18	A	4.4	0.893	38	A	3.2	0.391
19	C	2.8	0.557	39	D	4.6	0.414
20	B	2.8	0.661	40	C	4.6	0.404

Suggested Answers to Unit 1 Tests
Test D

Group One

1. Sections of Unit: 4.4, 4.5

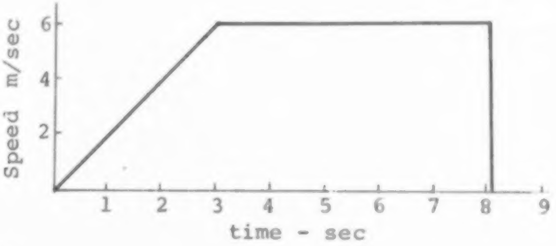
Force - central, due to gravitational interaction between earth and satellite, constant magnitude

Acceleration - in direction of force, constant magnitude

Velocity - perpendicular to acceleration, constant magnitude

Speed - constant

2. Sections of Unit: 1.5, 2.7



3. Section of Unit: 2.3

   1. Photography
      a) slow motion
      b) strobe light
      c) 'fast' motion

   2. Doppler effect

   3. Accurate measurement of time
      a) 'atomic' clocks
      b) photography

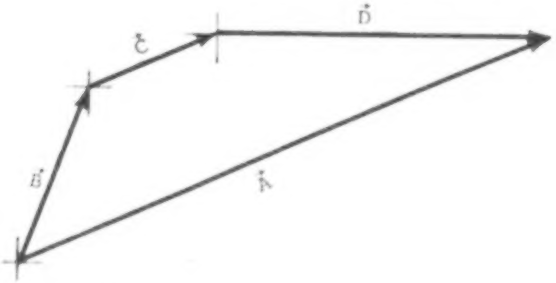
   4. Elimination of friction - approaches ideal motion
      a) dry ice puck
      b) air track

   5. Vector mathematics

   6. The calculus

4. Section of Unit: 3.2 and programmed material

a)



b) Parallelogram method

5. Section of Unit: 3.4

Newton's first law states that a body will remain at rest or in uniform motion unless acted on by an external unbalanced force.

A Newtonian would not accept an a priori statement as fact. He would expect experimental verification.

Group Two

6. Section of Unit: 4.2

a) $d = \frac{1}{2} a_g t^2$

$t^2 = \frac{2d}{a_g}$

$= \frac{(2)(2000\ m)}{10\ \frac{m}{sec^2}}$

$= 400\ sec^2$

$= 20\ sec.$

b) $d = vt$

$= 200\ \frac{m}{sec}\ (20\ sec)$

$= 4000\ m$

7. Section of Unit: 1.8

a) $surge_{av} = \frac{\Delta a}{\Delta t}$

b) $\frac{m}{sec^3}$ or distance/time^3

c) Instantaneous surge is the slope of an acceleration-time graph at any point.

8. Section of Unit: 2.3

a) Three assumptions are

1. if a heavy and light body are tied together, the natural speed of the new body will be the "average" of the natural speeds of the two original bodies
2. bodies have natural speeds that differ from body to body
3. the natural speed of a heavy body is greater than that of a light body

b) More appropriate assumptions are

1. if a heavy and light body are tied together, the natural speed of the new body will be the "sum" of the natural speeds of the two original bodies
2. bodies do not have natural speeds
3. all bodies accelerate towards the earth at the same rate
4. all bodies have the same natural speeds

c) The following are conclusions based on the assumptions suggested in part D.

1. A heavy object has a natural speed which is greater than that of a light object.
2. On the basis of this premise we can make no conclusion concerning the speeds with which heavy and light objects fall.
3. Heavy and light bodies should fall at the same speed.

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